MATH 124

Assignment 2
November 17, 2008

Name _____

GIVE DETAILED EXPLANATIONS FOR YOUR ANSWERS.

1. Consider the function $f(x) = \dfrac{x}{2} + \cos(x),\ 0 \le x \le 4$.

   (a) Determine the exact x-values of all critical points of this function. Find the exact and approximate coordinates of all critical points.

   (b) Determine the exact and approximate values of the global minimum and the global maximum of f. Justify your claims without resorting to a calculator.

   (c) Find the exact coordinates of all inflection points of f. Write the equation of the tangent line to the graph of f at the inflection point(s).

2. For $a > 0$ consider the function $g_a(x) = x^a\, a^{-x},\ x \ge 0$.

   (a) For which a does the function g_a have a critical point? Is the critical point local maximum or minimum?

   (b) Find a such that the function g_a has the global maximum equals to 1.

3. A bird forages among identical patches of berries, obtaining energy in each patch at a rate which declines the longer the bird remains in the patch. But to go off and find a new patch requires both time and energy (in travel). The problem is to decide how long to stay in each patch to maximize its net energy gain per unit time.

   Suppose that if the bird forages in a patch for t minutes, it will gain $E(t)$ joules of energy, where

   $$E(t) = \frac{3000\,t}{t+4}.$$

   (a) Draw a rough graph of E against t.

   (b) Suppose that to travel from one patch to another requires 2 minutes and consumes 300 joules of energy. Find an expression for $F(t)$, the net energy gain per minute, and find a positive value of t at which $F'(t) = 0$.

   (c) Find a geometric interpretation of $F(t)$ on the graph of E with respect to t, and use this to argue that the point in (3b) must be a maximum of $F(t)$.

4. Let $a > 1$. Consider the exponential function $f_a(x) = a^x$.

   (a) Find the equation of the tangent line to the graph of f_a at y-intercept.

   (b) Find the equation of the tangent line to the graph of f_a which contains the origin.

   (c) Find a formula for the tangent of the angle between the tangent lines in (4a) and (4b).

   (d) Which base a will yield the maximum angle between the tangent lines in (4a) and (4b)?

5. The standard can for packaging frozen orange juice has a cylindrical shape with the top and bottom made from metal and the cylindrical side made from cardboard. The cost of material to make such a can is the sum of the cost of cardboard and the cost of metal. Suppose that the dimensions of the can are chosen so that for a given volume the total cost of the material is minimal. Calculate the ratio of the cost of cardboard to the total cost of material and the ratio of the cost of metal to the total cost of material.