CALCULUS, Fourth Edition, Hughes-Hallett, Gleason, McCallum, et al.

## Chapter 2

| Section | Suggested Problems |
|---|---|
| 2.1 | 1, 3, 5-8, 11, 12, 14, 15. |
| 2.2 | 1, 3, 5, 8-13, 15, 16, 21. |
| 2.3 | 1-13 odd, 17, 19, 33, 36, 40-42. |
| 2.4 | 1, 3, 6, 7, 10, 11, 14, 15, 17. |
| 2.5 | 1-5, 7, 9, 11, 14, 15, 18 - 23. |
| 2.6 | 1-5, 6, 10, 15. |
| Review | 7, 8, 19, 20, 28. |

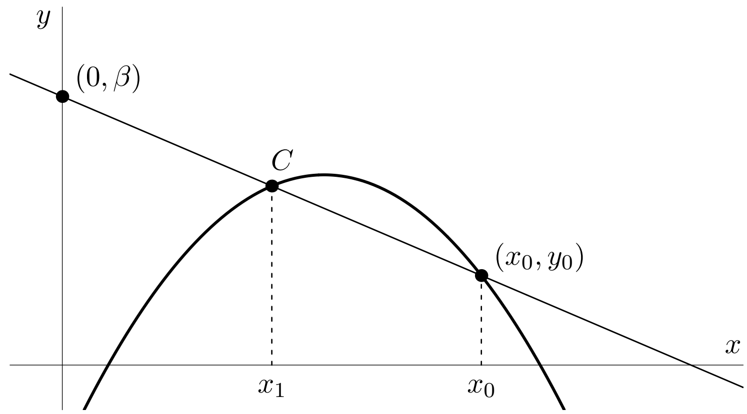


Figure 1: $y = ax^2 + bx + c$

Please recall Problem 46 on page 60 and what we did in class related to this problem. Here we replace the function $y = x^2$ with a general parabola $y = ax^2 + bx + c$ and repeat what we did in class to find the slope of the tangent line to this parabola at the point $\big(x_0, y_0\big)$ where $y_0 = ax_0^2 + bx_0 + c$. Let $(0, \beta)$ be a point on $y$-axes. The slope of the line through the points $(0, \beta)$ and $\big(x_0, y_0\big)$ is

$$\frac{y_0 - \beta}{x_0 - 0} = \frac{ax_0^2 + bx_0 + c - \beta}{x_0}.$$

The equation of the line through the points $(0, \beta)$ and $\big(x_0, y_0\big)$ is

$$y = \frac{ax_0^2 + bx_0 + c - \beta}{x_0}\, x + \beta.$$

To find where this line intersects the parabola $y = ax^2 + bx + c$ we solve for $x$ the equation:

$$ax^2 + bx + c = \frac{ax_0^2 + bx_0 + c - \beta}{x_0}\, x + \beta.$$

Multiplying through by $x_0$ we get

$$ax_0x^2 + bx_0x + cx_0 = \big(ax_0^2 + bx_0 + c - \beta\big)\, x + \beta\, x_0.$$

Grouping the terms in the last equation and factoring gives

$$\begin{aligned} 0 &= ax_0x^2 + bx_0x + cx_0 - axx_0^2 - bxx_0 - cx + \beta x - \beta\, x_0 \\ &= ax_0x(x - x_0) - c(x - x_0) + \beta(x - x_0) \\ &= \big(ax_0x - c + \beta\big)(x - x_0). \end{aligned}$$

Hence the $x$-coordinates of the points of intersection of the line and the parabola are

$$x = x_1 = \frac{c - \beta}{ax_0} \quad \text{and} \quad x = x_0.$$

These two points coincide if $c - \beta = ax_0^2$, that is if $\beta = c - ax_0^2$. In this case the line is the tangent line to the parabola at the point $(x_0, y_0)$ and the slope of the tangent line is

$$\frac{ax_0^2 + bx_0 + c - \beta}{x_0} = \frac{ax_0^2 + bx_0 + c - c + ax_0^2}{x_0} = \frac{2ax_0^2 + bx_0}{x_0} = 2ax_0 + b.$$

Thus the slope of the tangent line to the parabola $y = ax^2 + bx + c$ at the point $\big(x_0, y_0\big)$ where $y_0 = ax_0^2 + bx_0 + c$ is given by the formula $2ax_0 + b$.

In Figure 1 I used $a = -2, b = 5, c = -1, x_0 = 2$ and $\beta = 3$. What is the slope of the tangent line to this parabola at the point $(2, 1)$? (Notice that the horizontal and vertical units are different.)