MATH 124

Examination 2
November 4, 2008

Name ____________________

For a full credit give your answers as exact numbers, not decimal approximations.

1. We all know that $\pi \approx 3.14159$. A very popular rational approximation of π is $\frac{22}{7}$. This is the best approximation for π by a fraction with a two-digit denominator. The best rational approximation of π using a fraction with a three-digit denominator is $\frac{355}{113}$. Notice that $\pi < \frac{355}{113} < \frac{22}{7}$. Use an appropriate linear approximation of the function $f(x) = \sin x$ to find a rational approximation of the number $\sin\left(\frac{22}{7}\right)$. Your answer should look like $\sin\left(\frac{22}{7}\right) \approx \frac{a}{b}$, where a is an integer and b is a three-digit positive integer.

2. An object is launched vertically into the air and its distance from the ground (in feet) at any time $t \leq \ln 10$ (t is in seconds) is given by $h(t) = 100(1 - e^{-t})$. The object is equipped with a remote operated cruise control device (i.e. we can fix the velocity of the object at any moment). Assume that the velocity of the object has been fixed at time $t = \ln 10$.

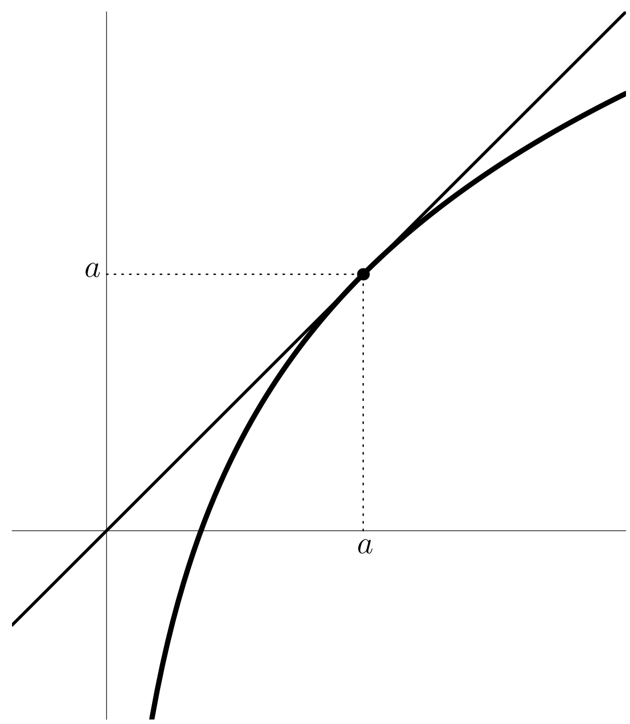
   (a) What is the velocity of the object at time $t = \ln 10$?

   (b) What is the height of the object at time $t = \ln 10$?

   (c) Assuming that the velocity has been fixed at the time $\ln 10$, give a formula for the height of the object for $t > \ln 10$.

   (d) When will the object reach the height of 100 ft?

3. The picture on the right shows the function $y = f(x) = k \ln x$ and its tangent line. The function and the tangent line touch at the point (a, a). The point $(0, 0)$ belongs to the tangent line. Determine the numbers k and a.



4. Consider the function $f(x) = x\, 2^x$.

   (a) Calculate the first and the second derivative of f.

   Use the derivatives found in (4a) to identify the maximum intervals where:

   (b) (i) f is increasing; (ii) f is decreasing;

   (c) (i) f is concave up; (ii) f is concave down.

5. Differentiate each of the following functions:

   (A) $\sin(\sqrt{x})$ (B) $\sqrt{\sin(\sqrt{x})}$ (C) $\arctan\left(\frac{1}{x}\right)$ (D) $\sqrt{1 + \sqrt{1 + x^2}}$

   For the full credit show all your work.