MATH 125 Examination 2
February 17, 2009

Name: Key

Answers which are not clearly supported by the presented work will receive a minimal credit.

1. Find the following integrals:

$$\text{(A)} \int \frac{x-1}{\sqrt{x}}\,dx, \qquad \text{(B)} \int \sqrt{x}\,(x+1)\,dx, \qquad \text{(C)} \int \frac{1}{w(w+1)}\,dw.$$

2. Find the following integrals:

$$\text{(A)} \int \frac{1}{x \ln x}\,dx, \qquad \text{(B)} \int e^x \cos x\,dx,$$

3. Find the following integrals:

$$\text{(A)} \int \frac{x}{x^2+2x+2}\,dx, \qquad \text{(B)} \int \frac{1}{1+e^x}\,dx.$$

4. For each of the four functions represented by the graphs below I calculated LEFT(5), RIGHT(5), MID(5) and TRAP(5). I also calculated the numerical value of the integral. For each function these five numbers are in one of the tables below. Determine which table belongs to which function. (You do not have to explain your reasoning here.)

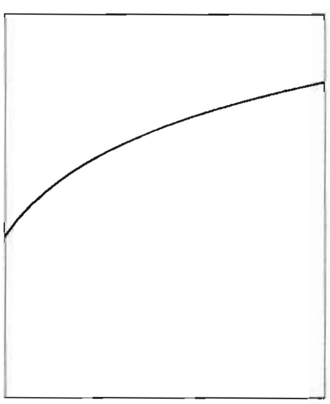

Figure 1: Function 1

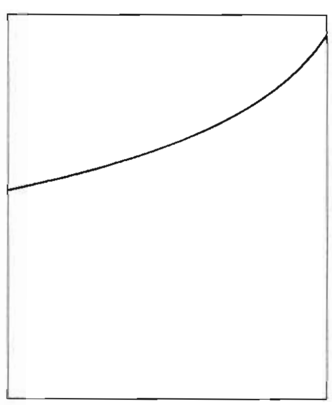

Figure 2: Function 2

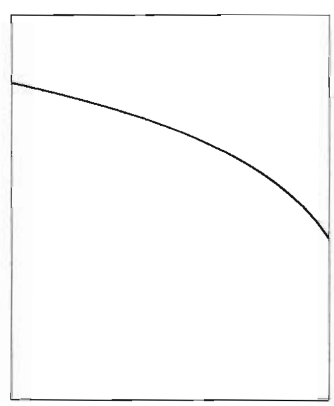

Figure 3: Function 3

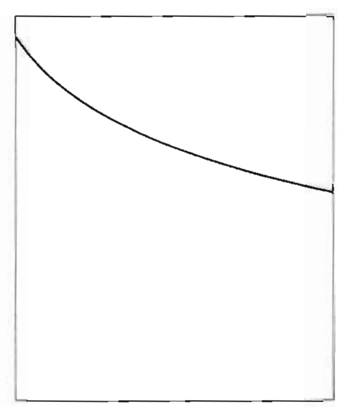

Figure 4: Function 4

LEFT	1.54602
RIGHT	1.74061
MID	1.63114
TRAP	1.64332
Integral	1.63511

Table 1:

F2

$L < R$

$M < T$

LEFT	1.74061
RIGHT	1.54602
MID	1.63114
TRAP	1.64332
Integral	1.63511

Table 2:

F4

$L > R$

$M < T$

LEFT	1.52961
RIGHT	1.72421
MID	1.63909
TRAP	1.62691
Integral	1.63511

Table 3:

F1

$L < R$

$M > T$

LEFT	1.72421
RIGHT	1.52961
MID	1.63909
TRAP	1.62691
Integral	1.63511

Table 4:

F3

$L > R$

$M > T$

① Ⓐ

$\boxed{1}$

$$\int \frac{x-1}{\sqrt{x}}\,dx = \int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx =$$

$$= \int \sqrt{x}\,dx - \int \frac{1}{\sqrt{x}}\,dx$$

$$= \int x^{1/2}dx - \int x^{-1/2}dx$$

$$= \frac{2}{3}x^{3/2} - 2x^{1/2} + C$$

Ⓑ $$\int \sqrt{x}(x+1)dx = \int\left(x^{3/2} + \sqrt{x}\right)dx$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$$

Ⓒ $$\frac{1}{w(w+1)} = \frac{1}{w} - \frac{1}{w+1}$$

Hence $$\int \frac{1}{w(w+1)}dw = \int \frac{1}{w}dw - \int \frac{1}{w+1}dw$$

$$= \ln|w| - \ln|w+1| + C$$

$$= \ln\frac{|w|}{|w+1|} + C$$

$$\frac{d}{dw}(\text{ans}) = \frac{(w+1)}{w} \cdot \frac{w+1-w}{(w+1)^2} = \frac{1}{w(w+1)}$$

ok ✓

(2) (A) $\displaystyle\int \frac{1}{x\ln x}\,dx = \left| \begin{array}{l} \ln x = w \\ \dfrac{dw}{dx} = \dfrac{1}{x} \\ dw = \dfrac{1}{x}\,dx \end{array} \right|$ $\boxed{2}$

$$= \int \frac{1}{w}\,dw = \ln|w| + C$$

$$= \ln|\ln x| + C$$

(B) $\displaystyle\boxed{\int e^x \cos x\,dx} = \left| \begin{array}{l} u' = e^x \\ v = \cos x \\ u = e^x \\ v' = -\sin x \end{array} \right|$

$$= e^x \cos x + \int e^x \sin x\,dx = \left| \begin{array}{l} u' = e^x \\ v = \sin x \\ v' = \cos x \\ u = e^x \end{array} \right|$$

$$= e^x \cos x + \left(e^x \sin x - \int e^x \cos x\,dx\right)$$

$$= e^x \cos x + e^x \sin x - \boxed{e^x \cos x}$$

$$2\int e^x \cos x\,dx = e^x(\cos x + \sin x)$$

$$\boxed{\int e^x \cos x\,dx = \frac{1}{2}e^x(\cos x + \sin x) + C}$$

$$\frac{d}{dx}(\text{ans}) = \frac{1}{2}e^x(c\,x + \underline{s\,x}) + \frac{1}{2}e^x(\underline{-s\,x} + c\,x) = e^x\,c(x)$$

ok✓

③ Ⓐ $\int \frac{x}{x^2+2x+2}\,dx =$ $\boxed{3}$

$= \int \frac{x}{(x+1)^2+1}\,dx = \left| \begin{array}{c} x+1=w \\ dx = dw \\ x = w-1 \end{array} \right| =$

$= \int \frac{w-1}{w^2+1}\,dw = \int \frac{w}{w^2+1}\,dw - \int \frac{1}{w^2+1}\,dw$

$= \left| \begin{array}{l} w^2+1 = z \\ 2w\,dw = dz \\ \frac{1}{2}dz = w\,dw \end{array} \right| = \frac{1}{2}\int \frac{1}{z}\,dz - \int \frac{1}{w^2+1}\,dw$

$= \frac{1}{2}\ln|z| - \arctan w + C$

$= \frac{1}{2}\ln(w^2+1) - \arctan w + C$

$= \frac{1}{2}\ln(x^2+2x+2) - \arctan(x+1) + C$

$= \ln\sqrt{x^2+2x+2} - \arctan(x+1) + C$

Verify: $\frac{d}{dx}(\text{ans}) = \frac{1}{2}\,\frac{1}{x^2+2x+2}\,(2x+2) - \frac{1}{x^2+2x+2}$

$= \frac{x+1}{x^2+2x+2} - \frac{1}{x^2+2x+2}$ ✓ ok

③ Ⓑ

$$\int \frac{1}{1+e^x}\,dx = \left|\begin{array}{l} w = e^x+1 \\ e^x = w-1 \\ \frac{dw}{dx} = e^x \\ dw = e^x\,dx = (w-1)\,dx \\ dx = \frac{1}{w-1}\,dw \end{array}\right|$$

$\boxed{4}$

$$= \int \frac{1}{w}\frac{1}{w-1}\,dw$$

$$= \int \left(-\frac{1}{w} + \frac{1}{w-1}\right) dw$$

$$= \int \frac{1}{w-1}\,dw - \int \frac{1}{w}\,dw$$

$$= \ln|w-1| - \ln|w| + C$$

$$= \ln\left|\frac{w-1}{w}\right| + C = \cancel{\ln\left|1-\frac{1}{w}\right|} + C$$

$$= \ln \frac{e^x+1-1}{e^x+1} + C$$

$$= \ln\left(\frac{e^x}{e^x+1}\right) + C$$

Verify

$\frac{d}{dx}$ (answer)

$\|$

$$\frac{e^x+1}{e^x} \cdot \frac{e^x(e^x+1)-e^{2x}}{(e^x+1)^2}$$

$$= \frac{e^x+1-e^x}{e^x+1} = \frac{1}{e^x+1} \quad \text{ok} \checkmark$$