Fall 2015 Math 138 Topics for the exam

Preliminaries about functions. Know:

- Formal definition of a function.
- Formal definitions of injection, surjection and bijection; important examples of bijections and definitions of the important inverse functions (website posts on September 25 and October 7)
- The definition of the composition of functions and have understanding of website post on September 29 related to this definition

Limits, continuity Know:

- The ϵ-δ definition of a limit and both definitions of continuity of a function at a point.
- Proofs posted on the website on September 30.
- The Intermediate Value Theorem and the Extreme Values Theorem and how to apply them in simple situations.
- How to decide whether a piecewise function is continuous or whether it can be extended to be a continuous function, Section 2.6 in Notes on Calculus and related exercises.
- Little Oh notation, Section 1.9 in Notes on Calculus and related exercises.

Derivatives. Know:

- The formal definition of differentiability of a function and its connection to local linearity and the concept of the tangent line to a graph, Section 2.4 in Notes on Calculus.
- Use properties of limits to prove that a function which is differentiable at a point is continuous at that point.
- The concept of derivative function and the derivatives of a quadratic polynomial, the exponential function e^x and the natural logarithm function.
- The geometric relationship between the derivative of a bijection and its inverse.
- How to interpret the derivative in applied problems, Section 2.2 in Notes on Calculus and related exercises.
- How to decide whether a piecewise defined function is differentiable or whether it can be extended to be a differentiable function, Sections 2.5 and 2.6 in Notes on Calculus and related exercises.

Differentiation. Know:

- The statement and the geometric interpretation of the mean value theorem and its consequences and how to apply them to do related problems
- How to use the definition of derivative to calculate derivatives of power functions, reciprocal function, trigonometric functions, and inverses of these functions and how to do related problems
- How to do implicit differentiation and how to use it to analyze simple implicit equations
- How to solve optimization problems using properties of differentiable functions
- How to find higher order approximations and the osculating circle for a function at a point
- How to construct parametric equations of simple planar curves and their tangent lines

Integration. Know:

- The definition of a Riemann sum of a function, definitions of special Riemann sums: Left, Right, Middle, Lower, Upper
- The formal definition of a Riemann integrable function and the definite integral of a function on an interval $[a, b]$.

- How to use the formal definition to prove that $f(x) = x$ is integrable on $[a, b]$.
- How to use the Left, Right, Middle, Lower, Upper Riemann sums and the Trapezoidal rule and the Simpson's rule to find approximations for definite integrals
- How to use known areas to find definite integrals
- How to interpret sums as areas and use this to find approximations and limits for sums
- The concept of the average value of a function
- The formal statement of the Fundamental Theorem of Calculus and how to use it solve related problems