Fall 2019 Math 204 Topics for Exam 1

1.1 Linear systems. Know:

- ➢ The definition of a linear equation and the definition of a linear system;
- ➢ What is the coefficient matrix and the augmented matrix of a linear system;
- ➢ What are equivalent linear systems and what are three basic operations which transform a linear system into an equivalent simpler system;
- ➢ The existence and uniqueness questions for linear system.

1.2 Row reduction, row echelon form (REF) and reduced row echelon form (RREF). Know:

- ➢ The definitions of REF and RREF of a matrix and how to use the Row Reduction Algorithm to transform a matrix to RREF.
- ➢ The concepts of a pivot position and a pivot column in a matrix and the connection with the basic and free variables of a system.
- ➢ How to use row reduction to find the general solution of a linear system and how to write this solution in parametric vector form.
- ➢ The Existence and Uniqueness Theorem (Theorem 2 page 21).

1.3 Vector equations. Know:

- ➢ How to write a linear system as one vector equation.
- ➢ It is essential to know how to verify whether row reduction has been performed correctly: **The linear relationships among vectors in the given matrix and its RREF are the same.**
- ➢ Algebraic operations in the vector space $\mathbb{R}^n$, their geometric illustrations in $\mathbb{R}^2$ and $\mathbb{R}^3$.
- ➢ The concepts of a linear combination of vectors and a span of vectors and the geometric interpretation of a span in $\mathbb{R}^2$ and $\mathbb{R}^3$.

1.4 The matrix equation $A\mathbf{x} = \mathbf{b}$. Know:

- ➢ The definition of matrix-vector product and its basic properties: $A(\mathbf{u}+\mathbf{v}) = A\mathbf{u} + A\mathbf{v}$, $A(c\mathbf{u}) = cA\mathbf{u}$ (Theorem 5).
- ➢ The matrix equation, the vector equation and the linear system which have the same solution set (Theorem 3).
- ➢ Four equivalent ways of saying: For every $\mathbf{b} \in \mathbb{R}^m$ the equation $A\mathbf{x} = \mathbf{b}$ has a solution (Theorem 4).

1.5 Solutions sets of linear systems. Know:

- ➢ The geometric illustration of the expression $\mathbf{p} + t\mathbf{v}$ where t is an arbitrary scalar and $\mathbf{p}$ and $\mathbf{v}$ are fixed vectors in $\mathbb{R}^n$.
- ➢ The geometric illustration of the expression $\mathbf{p} + s\mathbf{u} + t\mathbf{v}$ where t and s are arbitrary scalars and $\mathbf{p}$, $\mathbf{u}$ and $\mathbf{v}$ are fixed vectors in $\mathbb{R}^n$.
- ➢ How to write a solution of a linear system in parametric vector form.
- ➢ The relationship between the solution sets of a nonhomogeneous equation $A\mathbf{x} = \mathbf{b}$ and the corresponding homogeneous equation $A\mathbf{x} = \mathbf{0}$ (Theorem 6).

1.7 Linear independence. Know:

- ➢ The definitions of linear independence and linear dependence and how to implement them to decide whether given vectors are linearly dependent or independent.
- ➢ The meaning of linear independence/dependence in the case of one vector and two vectors.
- ➢ Two simple sufficient conditions for the linear dependence (Theorems 8 and 9).

- ➢ Characterization of linearly dependent sets (Theorems 7).

1.8 Linear transformations. Know:

- ➢ That in this context the words transformation, mapping and function are synonyms.
- ➢ The definition of a **linear transformation**. Let n and m be positive integers. A transformation T defined on $\mathbb{R}^n$ and with the values in $\mathbb{R}^m$ is **linear** if the following two conditions are satisfied: $T(\mathbf{x}+\mathbf{y}) = T\mathbf{x} + T\mathbf{y}$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and $T(c\mathbf{x}) = cT\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n$ and all $c \in \mathbb{R}$.
- ➢ How to associate pictures to formulas and formulas to pictures.

1.9 Matrix of a linear transformations. Know:

- ➢ The most important theorem on the **standard matrix for a linear transformation**: If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then there exists a unique $m \times n$ matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n$. In fact, the jth column of A is $T(\mathbf{e}_j)$, where $\mathbf{e}_j$ is the jth column of the identity matrix.
- ➢ How to use the above theorem to get the standard matrix for a rotation about the origin in $\mathbb{R}^2$ and a reflection through a line passing through the origin in $\mathbb{R}^2$ (which are linear transformations of $\mathbb{R}^2$).
- ➢ How to use the above theorem to get the standard matrices of transformations in Tables 1, 2, 3, 4 in Section 1.9. Also Exercises 1-22 all use this idea.