MATH 209

Examination 2
February 11, 2011

Name ______________________

1. Consider the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ given by the formulas

$$f(x) = \left\lfloor \frac{x}{2} \right\rfloor + \left\lceil \frac{x}{2} \right\rceil \qquad \text{and} \qquad f(x) = \sqrt{\lfloor x \rfloor \lceil x \rceil}.$$

Sketch as accurately as you can the graphs of f and g on the separate sheets of graph paper printed on the back of this page.

2. The universe of discourse in this problem is the set of all positive integers $\mathbb{Z}_+$. Prove or disprove each of the following statements:

(A) $\forall n \in \mathbb{Z}_+ \ \lfloor \sqrt{n-1} \rfloor + 1 = \lfloor \sqrt{n} \rfloor$ (B) $\forall n \in \mathbb{Z}_+ \ \lfloor \sqrt{n-1} \rfloor + 1 = \lceil \sqrt{n} \rceil$

(C) $\forall n \in \mathbb{Z}_+ \ \lceil \sqrt{n-1} \rceil + 1 = \lfloor \sqrt{n} \rfloor$ (D) $\forall n \in \mathbb{Z}_+ \ \lceil \sqrt{n-1} \rceil + 1 = \lceil \sqrt{n} \rceil$

3. As usual if A is a set, $\mathcal{P}(A)$ denotes the power set of A and $\emptyset$ denotes the empty set.

(a) Write the following power sets: $\mathcal{P}(\emptyset)$, $\mathcal{P}\big(\mathcal{P}(\emptyset)\big)$, $\mathcal{P}\Big(\mathcal{P}\big(\mathcal{P}(\emptyset)\big)\Big)$.

(b) Decide which of the following statements are true:

(i) $\mathcal{P}(\emptyset) \in \mathcal{P}\big(\mathcal{P}(\emptyset)\big)$ (ii) $\mathcal{P}(\emptyset) \in \mathcal{P}\Big(\mathcal{P}\big(\mathcal{P}(\emptyset)\big)\Big)$ (iii) $\mathcal{P}\big(\mathcal{P}(\emptyset)\big) \in \mathcal{P}\Big(\mathcal{P}\big(\mathcal{P}(\emptyset)\big)\Big)$

(iv) $\mathcal{P}(\emptyset) \subseteq \mathcal{P}\big(\mathcal{P}(\emptyset)\big)$ (v) $\mathcal{P}(\emptyset) \subseteq \mathcal{P}\Big(\mathcal{P}\big(\mathcal{P}(\emptyset)\big)\Big)$ (vi) $\mathcal{P}\big(\mathcal{P}(\emptyset)\big) \subseteq \mathcal{P}\Big(\mathcal{P}\big(\mathcal{P}(\emptyset)\big)\Big)$

Explain your answers to (iii) and (vi).

4. I decided to include the problem about outfits even without pictures. Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ be a set of five distinct shirts and let $P = \{p_1, p_2, p_3, p_4\}$ be a set of four distinct pants. The table below contains outfits worn during a week.

(a) Do the listed outfits define a function from S to P? Why?

(b) Do the listed outfits define a function from P to S? Why?

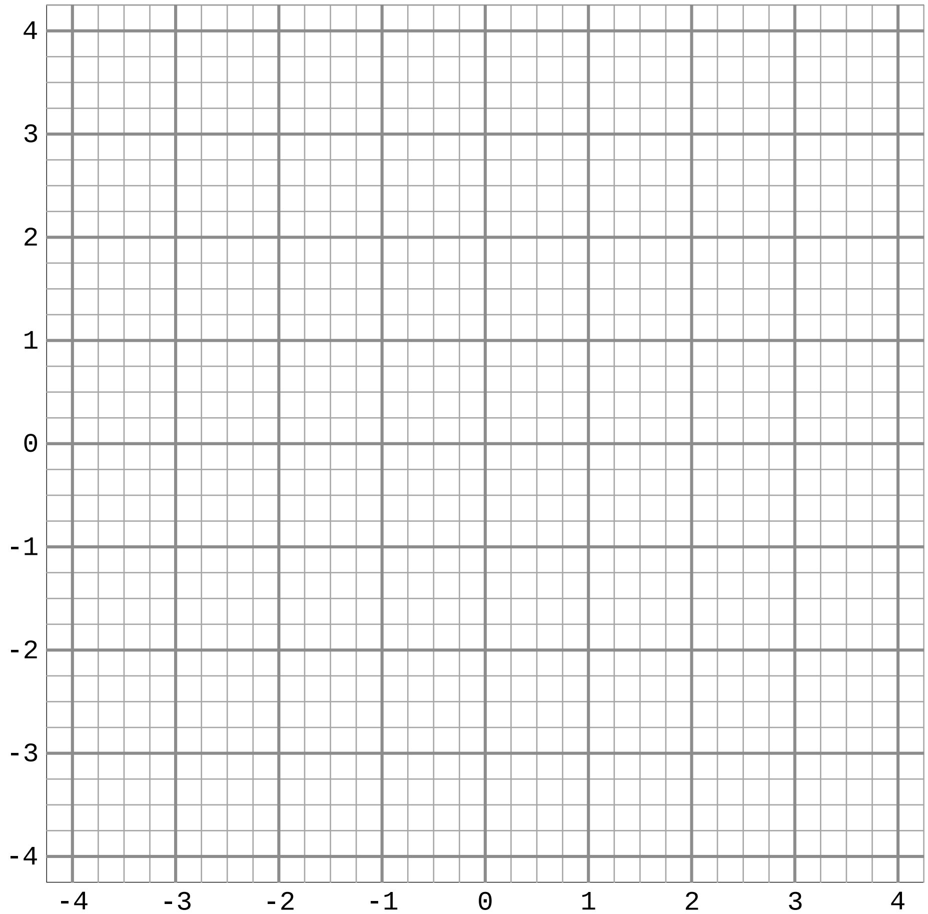
Outfits:

day	M	T	W	R	F	Sa	Su
shirt	s_2	s_3	s_1	s_4	s_3	s_5	s_4
pants	p_3	p_2	p_1	p_4	p_1	p_2	p_2

Is it possible to select days of the week so that the outfits worn during those days do define:

(c) a function from S to P? Explain?
(d) an injection from S to P? Explain?
(e) a surjection from S to P? Explain?
(f) a function from P to S? Explain?
(g) an injection from P to S? Explain?
(h) a surjection from P to S? Explain?

5. Prove that there exists **exactly** one prime p such that $p + 2$ and $p + 4$ are primes.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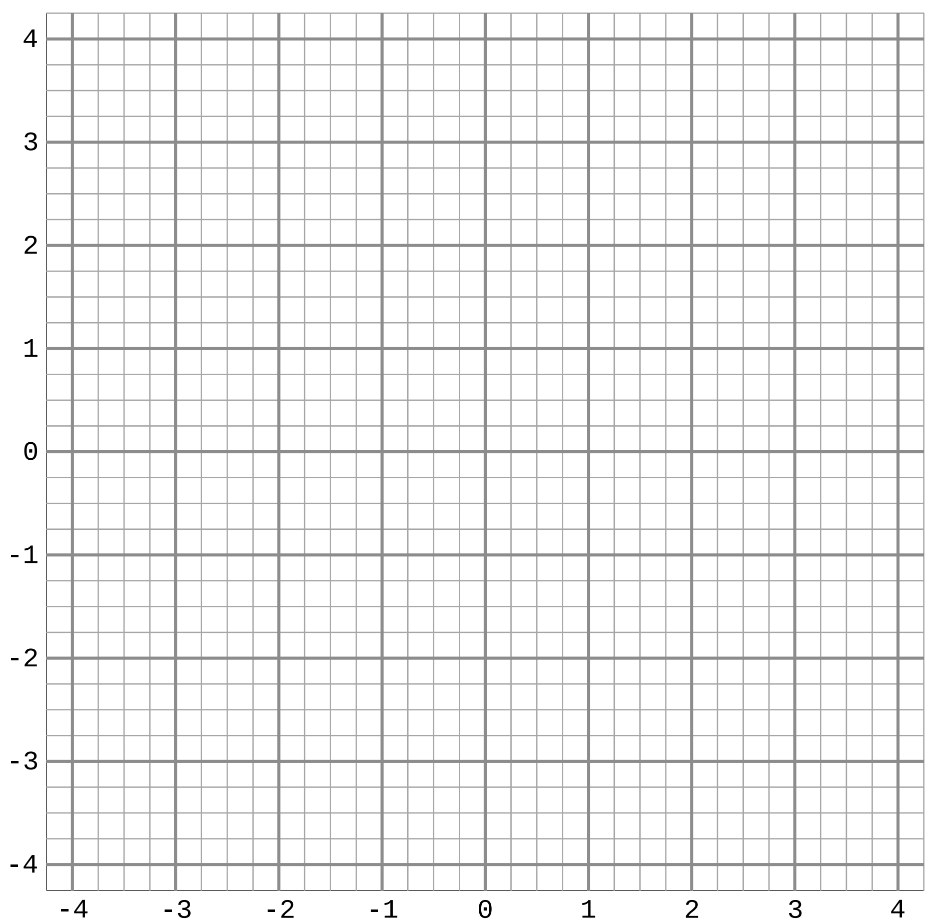
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