MATH 209

Examination 2
May 14, 2012

Name _____

1. (a) Let A and B be nonempty sets. Let $f : A \to B$ be a function. State the definitions of: an injection (one-to-one function), a surjection (onto function) and a bijection. Formal definitions must include quantifiers either in English sentences or logical statements.

   (b) Consider the function defined by $g(n) = \left\lfloor \frac{4\,n}{3} \right\rfloor + 1$ for all $n \in \mathbb{Z}$.

   Describe in words the range of g. Denote this range by S.

   (c) Prove that $g : \mathbb{Z} \to S$ is a bijection.

2. Recall that the Fibonacci sequence is defined as

$$f_0 = 0, \quad f_1 = 1, \quad f_{n+1} = f_n + f_{n-1}, \quad n \in \mathbb{Z}_+.$$

   Recall that the Golden ratio ϕ is a positive solution of the equation $\phi^2 = \phi + 1$.

   Prove that for all $n \in \mathbb{Z}_+$ we have $\phi^n = f_{n-1} + \phi f_n$.

3. In this problem we consider bit strings of length 6.

   (a) What is cardinality of the set of all bit strings of length six?

   (b) How many bit strings of length six have exactly three consecutive 0s?

   (c) How many bit strings of length six have at least three consecutive 0s?

   (d) How many bit strings of length six have at least three consecutive 0s and at least three consecutive 1s?

   (e) How many bit strings of length six have either at least three consecutive 0s or at least three consecutive 1s?

4. The universe of discourse in this problem is the set of integers. In this problem we study devisability by 5. Recall that when divided by 5 an integer leaves a remainder $0, 1, 2, 3$ or 4.

   (a) Given any distinct six integers a, b, c, d, e, f prove that there exists two of them whose difference is divisible by 5.

   (b) Given any distinct six integers a, b, c, d, e, f prove or disprove that there exists two of them whose sum is divisible by 5.

   (c) Given any distinct four integers a, b, c, d prove that there exists two of them whose sum **or** difference is divisible by 5.

   This problem involves the Pigeonhole principle. Please be clear what are pigeons and what are pigeonholes in your solutions.