Fall 2009   Math 224   Chapter 14

Section 14.1 Partial derivative

- Pay attention to the notation: $f_x(x,y)$, $f_y(x,y)$, $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$ and several other forms
- Visualizing partial derivatives on a graph
- Estimating partial derivatives using difference quotients
- Estimating partial derivatives from a contour diagram
- Using units to interpret partial derivatives

Section 14.1, Exercises and Problems: 1, 2, 4, 5, 6, 8, 9 - 12, 16, 17, 20 - 23, 30

Section 14.2 Computing partial derivatives algebraically

- Review all derivative rules from Math 124
- What partial derivatives mean in practical terms?

Section 14.2, Exercises and Problems: 1 - 35 (do as many as you can), 36, 37, 38, 40, 42, 43, 44, 45

Section 14.3 Local linearity and the differential

- Let $z = f(x,y)$ be a function of two variables. Let (a,b) be a point in the domain of f. Then the point $\big(a,b,f(a,b)\big)$ is on the graph of f. The graph of f is the surface $z = f(x,y)$. The equation of the tangent plane to the surface $z = f(x,y)$ at the point $\big(a,b,f(a,b)\big)$ is

$$z = f(a,b) + f_x(a,b)(x - a) + f_y(a,b)(y - b)$$

- The tangent plane approximation to $f(x,y)$ near (a,b) - the local linearization
- The differential

Section 14.3, Exercises and Problems: 1 - 12, 17, 18, 20, 21, 22, 23, 24, 30

Section 14.4 Gradients and the directional derivative in the plane

Let $z = f(x,y)$ be a differentiable function of two variables. Let (a,b) be a point in the domain of f.

- The **gradient vector** of f at the point (a,b) is

$$(\overrightarrow{\text{grad}\, f})(a,b) = (\overrightarrow{\nabla} f)(a,b) = f_x(a,b)\overrightarrow{i} + f_y(a,b)\overrightarrow{j}$$

- The **directional derivative** of f at the point (a,b) in the direction of the unit vector $\overrightarrow{u} = u_1 \overrightarrow{i} + u_2 \overrightarrow{j}$ is

$$f_{\overrightarrow{u}}(a,b) = \big((\overrightarrow{\nabla} f)(a,b)\big) \cdot \overrightarrow{u} = f_x(a,b)u_1 + f_y(a,b)u_2$$

Section 14.4, Exercises and Problems: 1 - 43 (do as many as you can), 46, 48, 53 - 57, 63, 65 - 70, 73, 76, 78, 80

Section 14.5 Gradients and the directional derivative in space

Let $w = f(x, y, z)$ be a differentiable function of three variables. Let (a, b, c) be a point in the domain of f.

- The **gradient vector** of f at the point (a, b, c) is

$$(\overrightarrow{\text{grad}}\, f)(a, b, c) = (\vec{\nabla} f)(a, b, c) = f_x(a, b, c)\,\vec{i} + f_y(a, b, c)\,\vec{j} + f_z(a, b, c)\,\vec{k}$$

- The **directional derivative** of f at the point (a, b, c) in the direction of the unit vector $\vec{u} = u_1\,\vec{i} + u_2\,\vec{j} + u_3\,\vec{k}$ is

$$f_{\vec{u}}(a, b, c) = ((\vec{\nabla} f)(a, b, c)) \cdot \vec{u} = f_x(a, b, c)u_1 + f_y(a, b, c)u_2 + f_z(a, b, c)u_3$$

Section 14.5, Exercises and Problems: 1 - 40 (do as many as you can; choose various types), 41, 42, 44, 46, 48, 49, 51, 54, 55, 58.

Section 14.6 The chain rule

Section 14.6, Exercises and Problems: 1 - 14, 16, 18, 20, 27, 29, 30, 31

Section 14.7 Second order partial derivatives

Section 14.7, Exercises and Problems: 1 - 30 (do most), 32, 33, 35, 36, 40, 41, 44, 45.

Section 14.8 Differentiability

Section 14.8, Exercises and Problems: 2, 3, 4, 9, 10, 12, 17, 20

Chapter 14, Review Exercises and Problems: 45, 48, 49, 54, 55, 57, 58, 59, 102