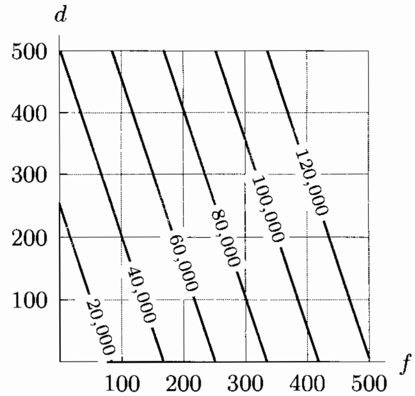


Figure 12.62: Revenue as a function of full and discount fares, $R = 239f + 79d$

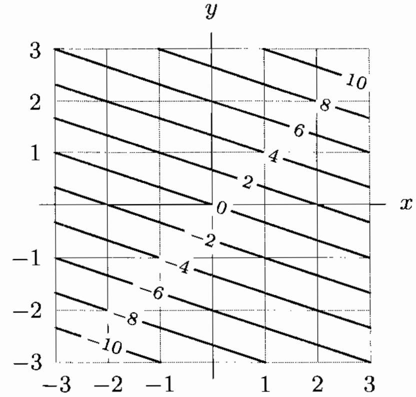


Figure 12.63: Contour map of linear function $f(x, y)$

Example 4 Find the equation of the linear function whose contour diagram is in Figure 12.63.

Solution Suppose we start at the origin on the $z = 0$ contour. Moving 2 units in the y direction takes us to the $z = 6$ contour; so the slope in the y direction is $\Delta z/\Delta y = 6/2 = 3$. Similarly, a move of 2 in the x-direction from the origin takes us to the $z = 2$ contour, so the slope in the x direction is $\Delta z/\Delta x = 2/2 = 1$. Since $f(0,0) = 0$, we have $f(x, y) = x + 3y$.

Exercises and Problems for Section 12.4

Exercises

Problems 1–2 each contain a partial table of values for a linear function. Fill in the blanks.

1.

$x\backslash y$	0.0	1.0
0.0		1.0
2.0	3.0	5.0

2.

$x\backslash y$	−1.0	0.0	1.0
2.0	4.0		
3.0		3.0	5.0

Which of the tables of values in Exercises 3–6 could represent linear functions?

3.

$x \backslash y$	0	1	2
0	0	1	4
1	1	0	1
2	4	1	0

4.

$x \backslash y$	0	1	2
0	10	13	16
1	6	9	12
2	2	5	8

5.

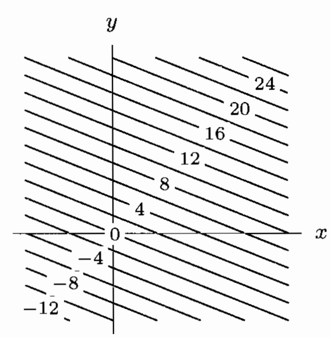
$x \backslash y$	0	1	2
0	0	5	10
1	2	7	12
2	4	9	14

6.

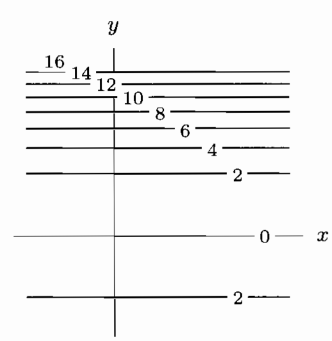
$x \backslash y$	0	1	2
0	5	7	9
1	6	9	12
2	7	11	15

Which of the contour diagrams in Exercises 7–8 could represent linear functions?

7.



8.



9. Find the equation of the linear function $z = c + mx + ny$ whose graph contains the points $(0,0,0)$, $(0,2,-1)$, and $(-3,0,-4)$.

10. Find the linear function whose graph is the plane through the points $(4,0,0)$, $(0,3,0)$ and $(0,0,2)$.

11. Find an equation for the plane containing the line in the xy-plane where $y = 1$, and the line in the xz-plane where $z = 2$.

12. Find the equation of the linear function $z = c + mx + ny$ whose graph intersects the xz-plane in the line $z = 3x + 4$ and intersects the yz-plane in the line $z = y + 4$.

13. Suppose that z is a linear function of x and y with slope 2 in the x direction and slope 3 in the y direction.

(a) A change of 0.5 in x and -0.2 in y produces what change in z?

(b) If $z = 2$ when $x = 5$ and $y = 7$, what is the value of z when $x = 4.9$ and $y = 7.2$?

14. **(a)** Find a formula for the linear function whose graph is a plane passing through point $(4,3,-2)$ with slope 5 in the x-direction and slope -3 in the y-direction.

(b) Sketch the contour diagram for this function.

Problems

15. A store sells CDs at one price and DVDs at another price. Figure 12.64 shows the revenue (in dollars) of the music store as a function of the number, *c*, of CDs and the number, *d*, of DVDs that it sells. What is the price of a CD? What is the price of a DVD?

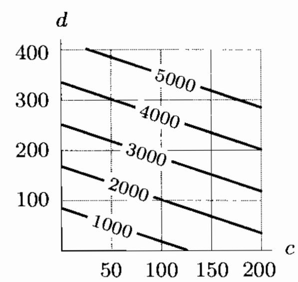


Figure 12.64

16. A college admissions office uses the following linear equation to predict the grade point average of an incoming student:

$$z = 0.003x + 0.8y - 4,$$

where z is the predicted college GPA on a scale of 0 to 4.3, and x is the sum of the student's SAT Math and SAT Verbal on a scale of 400 to 1600, and y is the student's high school GPA on a scale of 0 to 4.3. The college admits students whose predicted GPA is at least 2.3.

(a) Will a student with SATs of 1050 and high school GPA of 3.0 be admitted?

(b) Will every student with SATs of 1600 be admitted?

(c) Will every student with a high school GPA of 4.3 be admitted?

(d) Draw a contour diagram for the predicted GPA z with $400 \le x \le 1600$ and $0 \le y \le 4.3$. Shade the points corresponding to students who will be admitted.

(e) Which is more important, an extra 100 points on the SAT or an extra 0.5 of high school GPA?

17. A manufacturer makes two products out of two raw materials. Let q_1, q_2 be the quantities sold of the two products, p_1, p_2 their prices, and m_1, m_2 the quantities purchased of the two raw materials. Which of the following functions do you expect to be linear, and why? In each case, assume that all variables except the ones mentioned are held fixed.

(a) Expenditure on raw materials as a function of m_1 and m_2.

(b) Revenue as a function of q_1 and q_2.

(c) Revenue as a function of p_1 and q_1.

Problems 18–20 concern Table 12.11, which gives the number of calories burned per minute for someone roller-blading, as a function of the person's weight and speed.[5]

Table 12.11

Calories burned per minute				
Weight	8 mph	9 mph	10 mph	11 mph
120 lbs	4.2	5.8	7.4	8.9
140 lbs	5.1	6.7	8.3	9.9
160 lbs	6.1	7.7	9.2	10.8
180 lbs	7.0	8.6	10.2	11.7
200 lbs	7.9	9.5	11.1	12.6

18. Does the data in Table 12.11 look approximately linear? Give a formula for B, the number of calories burned per minute in terms of the weight, w, and the speed, s. Does the formula make sense for all weights or speeds?

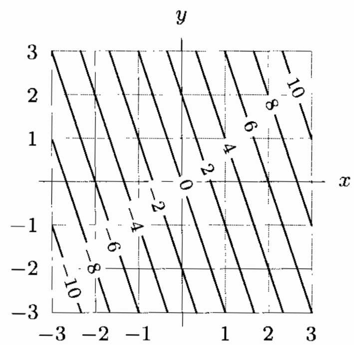
19. Who burns more total calories to go 10 miles: A 120 lb person going 10 mph or a 180 lb person going 8 mph? Which of these two people burns more calories per pound for the 10-mile trip?

20. Use Problem 18 to give a formula for P, the number of calories burned per pound, in terms of w and s, for a person weighing w lbs roller-blading 10 miles at s mph.

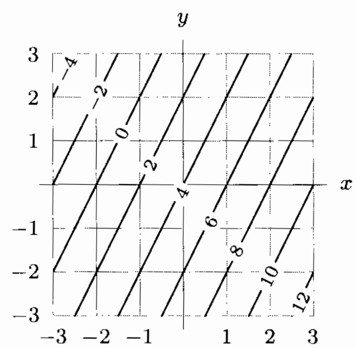
[5]From the August 28,1994, issue of *Parade Magazine*.

For Problems 21–22, find possible equations for linear functions with the given contour diagrams.

21.



22.



For Problems 23–24, find equations for linear functions with the given values.

23.

$x\backslash y$	−1	0	1	2
0	1.5	1	0.5	0
1	3.5	3	2.5	2
2	5.5	5	4.5	4
3	7.5	7	6.5	6

24.

$x\backslash y$	10	20	30	40
100	3	6	9	12
200	2	5	8	11
300	1	4	7	10
400	0	3	6	9

It is difficult to graph a linear function by hand. One method that works if the x, y, and z-intercepts are positive is to plot the intercepts and join them by a triangle as shown in Figure 12.65; this shows the part of the plane in the octant where $x \geq 0$, $y \geq 0$, $z \geq 0$. If the intercepts are not all positive, the same method works if the x, y, and z-axes are drawn from a different perspective. Use this method to graph the linear functions in Problems 25–28.

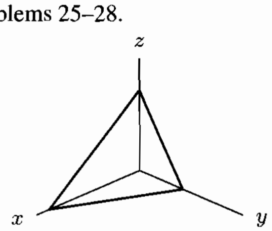


Figure 12.65

25. $z = 2 - 2x + y$

26. $z = 2 - x - 2y$

27. $z = 4 + x - 2y$

28. $z = 6 - 2x - 3y$

29. Let f be the linear function $f(x, y) = c + mx + ny$, where c, m, n are constants and $n \neq 0$.

(a) Show that all the contours of f are lines of slope $-m/n$.
(b) For all x and y, show $f(x + n, y - m) = f(x, y)$.
(c) Explain the relation between parts (a) and (b).

Problems 30–31 refer to the linear function $z = f(x, y)$ whose values are in Table 12.12.

Table 12.12

$x \backslash y$	4	6	8	10	12
5	3	6	9	12	15
10	7	10	13	16	19
15	11	14	17	20	23
20	15	18	21	24	27
25	19	22	25	28	31

30. Each column of Table 12.12 is linear with the same slope, $m = \Delta z/\Delta x = 4/5$. Each row is linear with the same slope, $n = \Delta z/\Delta y = 3/2$. We now investigate the slope obtained by moving through the table along lines that are neither rows nor columns.

(a) Move down the diagonal of the table from the upper left corner ($z = 3$) to the lower right corner ($z = 31$). What do you notice about the changes in z? Now move diagonally from $z = 6$ to $z = 27$. What do you notice about the changes in z now?
(b) Move in the table along a line right one step, up two steps from $z = 19$ to $z = 9$. Then move in the same direction from $z = 22$ to $z = 12$. What do you notice about the changes in z?
(c) Show that $\Delta z = m\Delta x + n\Delta y$. Use this to explain what you observed in parts (a) and (b).

31. If we hold y fixed, that is we keep $\Delta y = 0$, and step in the positive x-direction, we get the x-slope, m. If instead we keep $\Delta x = 0$ and step in the positive y-direction, we get the y-slope, n. Fix a step in which neither $\Delta x = 0$ nor $\Delta y = 0$. The slope in the Δx, Δy direction is

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta z}{\text{Length of step}} = \frac{\Delta z}{\sqrt{(\Delta x)^2 + (\Delta y)^2}}.$$

(a) Compute the slopes for the linear function in Table 12.12 in the direction of $\Delta x = 5, \Delta y = 2$.
(b) Compute the slopes for the linear function in Table 12.12 in the direction of $\Delta x = -10, \Delta y = 2$.