MATH 224 Examination 1
April 19, 2010

Name Key

GIVE DETAILED EXPLANATIONS FOR YOUR ANSWERS.
There are five problems. Each is worth 20 points.

1. (a) Use lines $y = mx$ to show that the function

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

does not have a limit as $(x, y) \to (0, 0)$.

(b) The graph of the function

$$g(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$$

is given in Figure 1. What does this graph suggest about the existence of a limit of $g(x, y)$ as $(x, y) \to (0, 0)$? Explain your answer.

(c) Show that the reasoning that you used in (1a) when applied to the function $g(x, y)$ supports your answer in (1b).

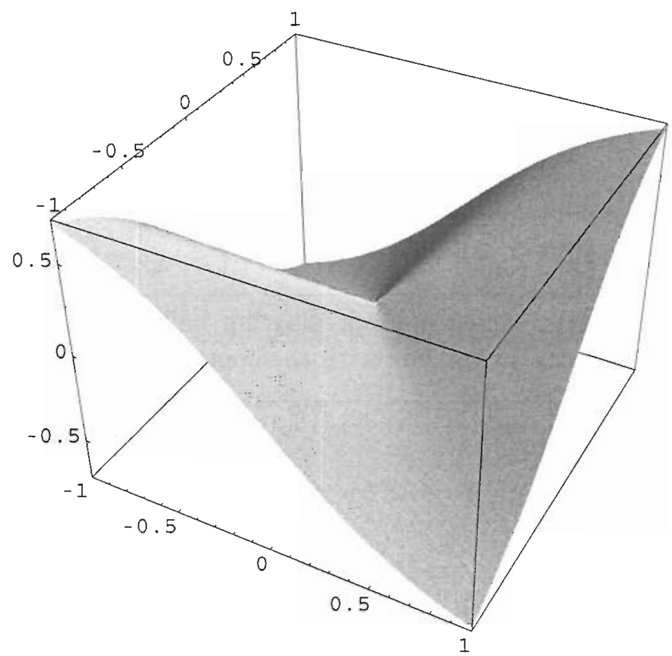


Figure 1: The graph of $g(x, y)$

2. Given points $A = (3, 2, 1)$ and $B = (3, 0, 1)$ answer the following questions:

(a) Write the vector $\overrightarrow{OB}$ as the sum of two vectors, one parallel to the vector $\overrightarrow{OA}$ and one perpendicular to $\overrightarrow{OA}$.

(b) Based on (2a) calculate the distance from the point B to the line OA.

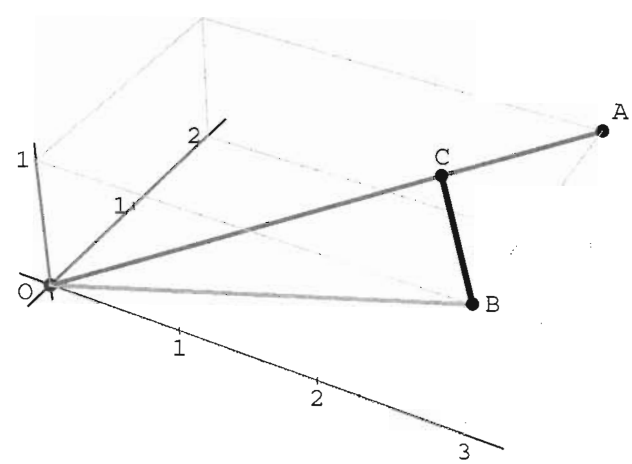


Figure 2: A picture for Problem 2

3. Given three points $A = (3, 4, 2), B = (1, 6, 3), C = (1, 9, 6)$ answer the following questions:

(a) Find an equation of the plane passing through these three points.

(b) Find an equation of the plane which is parallel to the plane determined in (3a) and passes through the origin.

(c) Find the area of the triangle ABC.

OVER

4. The temperature in a heated plane is given by a linear function of x and y. A bug walks on this plane at a constant speed. When the bug moves East the temperature increases at the rate of 5 degrees per foot. When the bug moves North there is no change in temperature. What is the rate of change of temperature when the bug moves Northeast?

5. The contour diagrams below show contour lines at indicated levels. I used six out of the following eight functions

(A) $\cancel{z = \dfrac{y}{1+x^2}}$	(B) $\cancel{z = \sqrt{2-(y-x)}}$	(C) $\cancel{z = \dfrac{e^y}{e^x}}$	(D) $z = x + \sqrt{y}$
(E) $\cancel{z = \dfrac{e^x}{e^y}}$	(F) $\cancel{z = \sqrt{4-(x-y)^2}}$	(G) $\cancel{z = \dfrac{y^2}{1+x^2}}$	(H) $\cancel{z = \sqrt{2-(x-y)}}$

Identify which contour diagram belongs to which function. Place the letter corresponding to the function in the box with the contour diagram of that function.

C

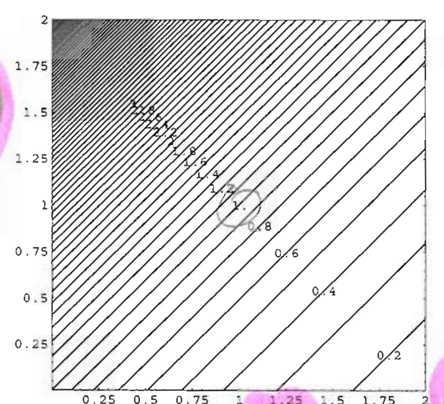


Figure 3

H

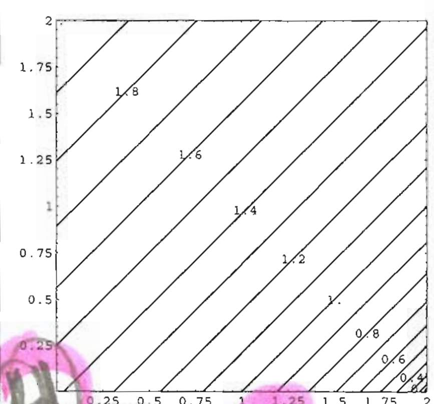


Figure 4

G

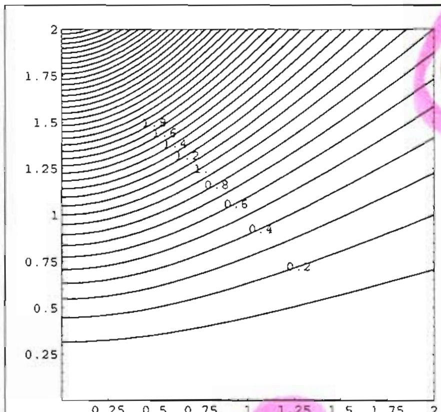


Figure 5

F

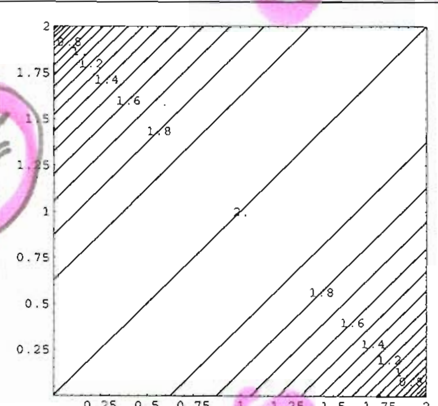


Figure 6

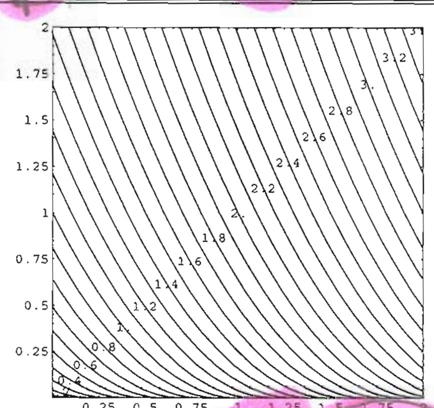


Figure 7

D

A

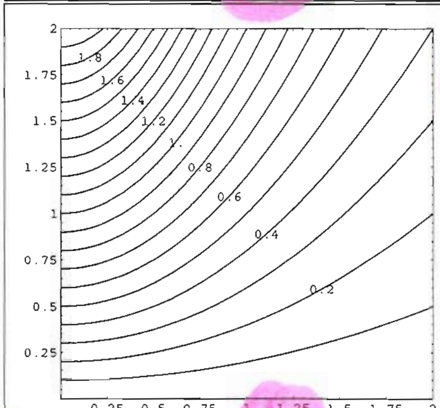


Figure 8

3 ↔ C
4 ↔ H
5 ↔ G

6 ↔ F
7 ↔ D
8 ↔ A

1 (a) Set $x = y$

$$f(x,x) = \frac{x^2}{2x^2} = \frac{1}{2} \quad (x \neq 0)$$

Set $y = -x$

$$f(x,-x) = \frac{-x^2}{2x^2} = -\frac{1}{2}$$

So f is constantish to both $1/2$ and $-1/2$, so it is not constantish to one number. Thus f does not have a limit as $(x,y) \to (0,0)$.

(b) The picture tells me that there is no sudden change in g near $(0,0)$. In fact the values of g near $(0,0)$ are constantish to 0. Thus ~~it as~~ as the picture suggests

$$\lim_{(x,y)\to(0,0)} g(x,y) = 0.$$

ⓒ Put $y = x$ $\boxed{2}$

$$g(x,x) = \frac{x^2}{|x|} = |x| \quad (x \neq 0)$$

For ~~small~~ x near 0 $g(x,x)$ is near 0.

Put $y = -x$

$$g(x,-x) = \frac{-x^2}{|x|} = -|x|$$

But for small x near 0 $-|x|$ is also close to 0.

$\lim_{(x,y)\to(0,0)} g(x,y) = 0$ is also suggested by the above calculations.

② ⓐ $\vec{OA} = 3\vec{i} + 2\vec{j} + \vec{k} = \vec{a}$

$\vec{OB} = 3\vec{i} + \vec{k} = \vec{v}$

$\vec{v} = \lambda \vec{a} + (\vec{v} - \lambda \vec{a})$, $(\vec{v} - \lambda \vec{a}) \cdot \vec{a} = 0$

$$\lambda = \frac{\vec{v} \cdot \vec{a}}{\vec{a} \cdot \vec{a}}$$

$\lambda = \frac{10}{9+4+1} = \frac{5}{7}$. $\vec{v} = \frac{5}{7}\vec{a} + (\vec{v} - \frac{5}{7}\vec{a})$

(2) (a)

$$\vec{OB} = \underbrace{\tfrac{5}{7}\vec{OA}}_{\vec{OC}} + \underbrace{\tfrac{6}{7}\vec{i} - \tfrac{10}{7}\vec{j} + \tfrac{2}{7}\vec{k}}_{\vec{CB}}, \quad \boxed{3}$$

$\vec{OC} = \frac{5}{7}\vec{OA}$

$\vec{CB} = \frac{2}{7}(3\vec{i} - 5\vec{j} + \vec{k})$

$$3\vec{i} + \vec{k} - \frac{5}{7}(3\vec{i} + 2\vec{j} + \vec{k})$$

$$\boxed{3 \cdot \frac{2}{7}\vec{i} - \frac{10}{7}\vec{j} + \frac{2}{7}\vec{k}}$$

$\perp \vec{OA}$

$$\frac{6}{7}\vec{i} - \frac{10}{7}\vec{j} + \frac{2}{7}\vec{k} = \boxed{\frac{2}{7}(3\vec{i} - 5\vec{j} + \vec{k})}$$

(b) The distance is

$$\|\vec{CB}\| = \frac{1}{7}\sqrt{36 + 100 + 4} = \frac{\sqrt{140}}{7}$$

$$= \frac{\sqrt{20}}{\sqrt{7}} = 2\sqrt{\frac{5}{7}} \approx 1.69031$$

(3) (a)

$$\vec{BA} = -2\vec{i} + 2\vec{j} + \vec{k}$$
$$\vec{CA} = -2\vec{i} + 5\vec{j} + 4\vec{k}$$

$$\vec{BA} \times \vec{CA} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & 1 \\ -2 & 5 & 4 \end{vmatrix} = 3\vec{i} - (-8+2)\vec{j} + (-10+4)\vec{k}$$

$$= \boxed{3\vec{i} + 6\vec{j} - 6\vec{k}}$$

~~The~~ An equation for the plane is $\boxed{4}$

$$3x+6y-6z = 3\cdot 3+6\cdot 4-6\cdot 2$$
$$= 9+12 = 21$$

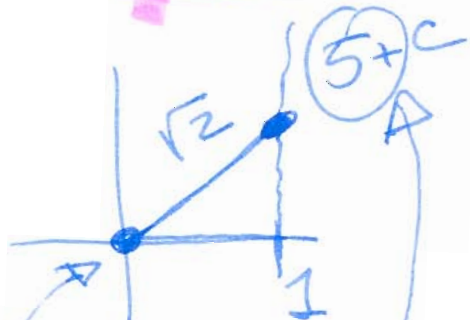
$$\boxed{x+2y-2z=7}$$

(b) $\boxed{x+2y-2z=0}$

(c) $\frac{1}{2}\sqrt{9+36+36} = \frac{1}{2}\sqrt{81}$
$$= \boxed{9/2}$$

The area is $9/2$

(4) $z = c+5x$



rise $5+c-c$

run $\sqrt{2}$

(c) temp

The temperature increases at the rate $\boxed{5/\sqrt{2}}$ ≈ 3.53553