Summer 2015 Math 225 Topics covered

Chapter 16 Know:

- How to set-up double and triple integrals based on geometric description of a region of integration.
- How to convert integrals between different coordinate systems (rectangular, polar, cylindrical, spherical).
- How to assess whether a result of your calculation is reasonable.

Section 17.1 Parameterized Curves Know:

- How vector-valued function $\vec{r}(t) = x(t)\vec{\imath} + y(t)\vec{\jmath} + z(t)\vec{k}$ describes a curve in $\mathbb{R}^3$.
- How to find parametric equations of simple geometric objects (lines, circles, helices) based on given geometric information.
- How to find whether parameterized geometric objects intersect or not.

Section 17.2 Motion, velocity and acceleration Know:

- How to use the concept of velocity and acceleration vector to solve various problems about motion along parameterized curves.
- The relationship between the velocity vector and the arc length of a curve and how to use it to calculate lengths of simple curves. (see related problems on the class web-site)

Section 17.3 Vector fields Know:

- "must know" vector fields (one of the components is 0, variations on an "exploding" vector field, variations on a rotational vector field, and $y\vec{\imath} + x\vec{\jmath}$ and its variations).
- Hot to sketch a vector field from a given formula and how to recognize which formula belongs to a vector field given graphically.
- That vector fields can be interpreted as velocity fields and force fields
- That in Math 224 we defined a gradient field of a function of two and three variables; a gradient field is a special kind of a vector field.

Section 17.4 Flow of a vector field Know:

- Definition of a flow line and how to verify is a given parametric curve is a flow line of a given vector field.
- How to find flow lines for a given simple vector field (for example for a field with one component constant).
- How to approximate a flow line numerically.

Section 17.5 Parameterized surfaces Know:

- How to parameterize simple surfaces (plane, a graph of $z = f(x, y)$, cylinder, sphere, cone, surfaces of revolution, torus)
- the meaning of parameter curves

Section 18.1 The idea of a line integral Know:

- The definition of a line integral
- How to estimate and compare line integrals for a given vector field and given curves without calculating them
- Two important applications of line integrals: work and circulation
- Properties of line integrals

Section 18.2 Computing line integrals over parameterized curves Know:

- How to parameterize familiar curves (circles, lines, helices, ...)
- How to compute line integrals over parameterized curve
- The differential notation for line integrals

Section 18.3 Gradient fields and path-independent fields Know:

- Fundamental Theorem of Calculus for line integrals
- How to calculate line integrals for gradient fields
- That a continuous vector field $\vec{F}$ defined on an open region R is path-independent if and only if there exists f such that $\vec{F} = \operatorname{grad} f$.

Section 18.4 Path independent vector fields and Green's theorem Know:

- Green's theorem
- How to use Green's theorem to calculate line integrals over simple piecewise closed curves
- The curl test for vector fields in 2-space
- The curl test for vector fields in 3-space

Section 19.1 The idea of a flux integral Know:

- How to calculate flux of a constant vector field through a flat surface (the idea of the area vector)
- The definition of a flux integral
- How to determine if the flux of a given vector field through a given oriented surface is positive, negative or zero (without calculating it)
- How to compare two flux of given vector fields through given oriented surfaces (without calculating them)
- How to use the idea of the unit normal vector to calculate flux, $d\vec{A} = \vec{n}\, dA$

Section 19.2 Know:

- How to calculate flux of a given vector field $\vec{F}(x, y, z)$ through a surface S given as the graph of $z = f(x, y)$ where $(x, y) \in D$:

$$\iint_S \vec{F} \cdot d\vec{A} = \iint_D \vec{F}\big(x, y, f(x, y)\big) \cdot \big(-f_x\vec{\imath} - f_y\vec{\jmath} + \vec{k}\big)dx\, dy$$

- How to calculate flux of a given vector field $\vec{F}(x, y, z)$ through a cylinder S of radius R centered on the z-axes between $z = a$ and $z = b$, $a < b$, and oriented away from z-axis:

$$\iint_S \vec{F} \cdot d\vec{A} = \int_a^b \int_0^{2\pi} \vec{F}\big(R\cos\theta, R\sin\theta, z\big) \cdot \big((\cos\theta)\vec{\imath} + (\sin\theta)\vec{\jmath}\big)R\, d\theta\, dz$$

- How to calculate flux of a given vector field $\vec{F}(x, y, z)$ through a sphere of radius R centered at the origin and oriented away from the origin:

$$\iint_S \vec{F} \cdot d\vec{A} =$$
$$\int_0^{\pi} \int_0^{2\pi} \vec{F}\big(R\cos\theta\sin\phi, R\cos\theta\sin\phi, R\cos\phi\big) \cdot \big((\cos\theta\sin\phi)\,\vec{\imath} + (\cos\theta\sin\phi)\,\vec{\jmath} + (\cos\phi)\,\vec{k}\big)R^2\, \sin\phi\, d\theta\, d\phi \tag{1}$$

- ➢ How to calculate the surface area of a surface S given as the graph of $z = f(x, y)$ where $(x, y) \in D$:

$$\iint_D \left\| -f_x(x,y)\vec{\imath} - f_y(x,y)\vec{\jmath} + \vec{k} \right\| dx\, dy$$

Section 19.3 Know:

- ➢ How to calculate flux of a given vector field $\vec{F}(x, y, z)$ through a surface S parameterized by

$$\vec{r}(s,t) = u(s,t)\vec{\imath} + v(s,t)\vec{\jmath} + w(s,t)\vec{k}$$

where $(s, t) \in D$ oriented by the cross product of tangent vectors to parameter curves:

$$\iint_S \vec{F} \cdot d\vec{A} = \iint_D \vec{F}\big(u(s,t), v(s,t), w(x,y)\big) \cdot \big(\vec{r}_s \times \vec{r}_t\big) ds\, dt$$

- ➢ How to calculate the surface area of a surface S parameterized by $\vec{r}(s,t)$ where $(s, t) \in D$:

$$\iint_D \left\| \vec{r}_s(s,t) \times \vec{r}_t(s,t) \right\| ds\, dt$$

- ➢ How to parameterize
  - graphs, for example $z = f(x, y)$:

$$\vec{r}(x,y) = x\,\vec{\imath} + y\,\vec{\jmath} + f(x,y)\,\vec{k}$$

  - cylinders, for example a cylinder of radius R centered on the y-axis:

$$\vec{r}(\theta,y) = (R\cos\theta)\,\vec{\imath} + y\,\vec{\jmath} + (R\sin\theta)\,\vec{k}$$

  - spheres
  - rotational surfaces: for example the surface formed by rotating the graph of $y = f(x)$ around the x-axis.

Section 20.1 The divergence of a vector field Know:

- ➢ Geometric and coordinate definitions of divergence and why they represent the same quantity.
- ➢ How to calculate divergence for a vector field given by a formula and how to estimate divergence for a vector field given by a picture.
- ➢ That the divergence is defined only for vector fields and that the divergence of a vector field is a scalar quantity; for example the expression $\operatorname{div}\big(\operatorname{div}\vec{F}\big)$ does not make sense.

Section 20.2 The divergence theorem Know:

- ➢ The statement of divergence theorem; what is involved, (a potato, its skin, a vector field and its divergence defined in an open region containing the potato)
- ➢ How to use the divergence theorem to calculate flux of a vector field through a closed surface
- ➢ The importance of divergence free vector fields

Section 20.3 The curl of a vector field Know:

- ➢ The concept of the circulation density of a vector field around the direction of a unit vector $\vec{n}$ and the relationship of this quantity to the curl of a vector field
- ➢ The geometric and the coordinate definition of the curl of a vector field
- ➢ That the curl is defined only for vector fields and that the curl of a vector field is a vector quantity; for example the expression $\operatorname{curl}\big(\operatorname{curl}\vec{F}\big)$ does make sense, but $\operatorname{curl}\big(\operatorname{div}\vec{F}\big)$ does not make sense

- The importance of the nabla operator notation $\vec{\nabla} = \vec{\imath}\frac{\partial}{\partial x} + \vec{\jmath}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$ and how it fits into the definitions of the gradient, the divergence and the curl

Section 20.4 Stokes' Theorem Know:

- The statement to Stokes' Theorem and why it is true;
- The importance of curl free vector field
- The concept of a curl field and its vector potential

Section 20.5 The three fundamental theorems Know:

- How the Fundamental Theorem of Calculus (from Math 125), the Fundamental Theorem of Calculus for line integrals, Stokes' Theorem and the Divergence theorem fit together
- The Curl Test for gradient vector fields
- The Divergence Test for curl fields