You have saved this notebook in a directory. The following command will tell you in which directory you saved this notebook.

To execute the command below, place the cursor in the cell below and press Shift+Enter

In[1]:=
```
NotebookDirectory[]
```

Out[1]=
```
C:\Dropbox\Work\COURSES\225\2015\
```

The following command will set the above directory as a working directory for this notebook. Since the graph that I create below will be exported to this directory.

In[2]:=
```
SetDirectory[NotebookDirectory[]]
```

Out[2]=
```
C:\Dropbox\Work\COURSES\225\2015
```

I experimented I found out that a good view of the graph that I produced is from the following point. I use this in the last command below.

In[3]:=
```
VP = {0.9309251513656115`, -3.031838195362718`, 1.1795488627838389`}
```

Out[3]=
```
{0.930925, -3.03184, 1.17955}
```

In the following cell I produce several graphs and then put them together in a “house” that I displayed at the class web site.

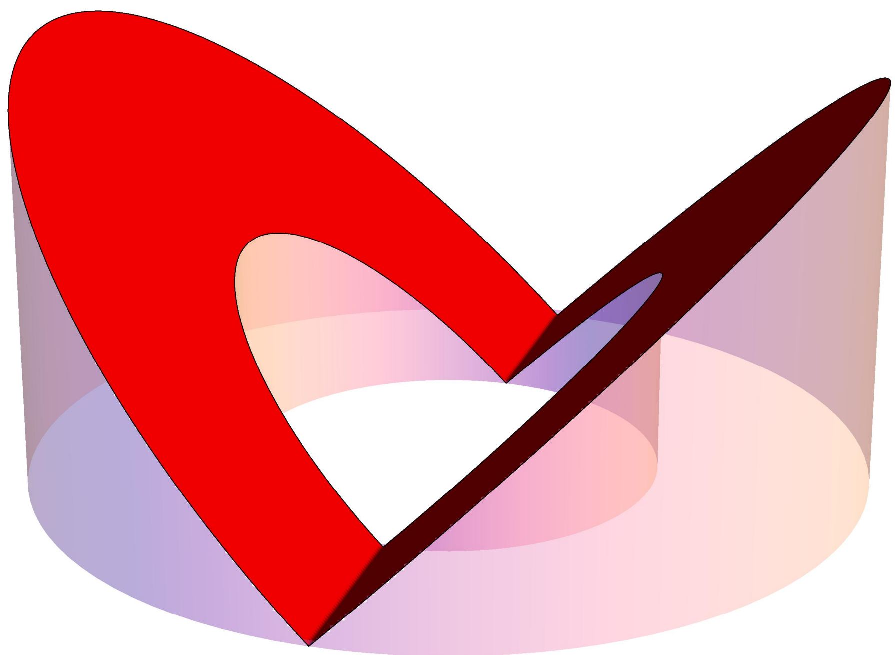
In[4]:=
```
s1 = Plot3D[Abs[x], {x, -2, 2}, {y, -2, 2},
    RegionFunction → Function[{x, y, z}, And[1 < x^2 + y^2, x^2 + y^2 < 4]],
    PlotStyle → {Red}, PlotRange → All, BoxRatios → {1, 1, 1 / 2},
    PlotPoints → {100, 100}, Mesh → None];


s2 = ParametricPlot3D[{Cos[t], Sin[t], s}, {t, 0, 2 Pi}, {s, 0, 1},
    RegionFunction → Function[{x, y, z}, And[z < Abs[x]]], PlotStyle → {Opacity[.5]},
    PlotRange → All, BoxRatios → {1, 1, 1 / 2}, PlotPoints → {100, 100}, Mesh → None];

s3 = ParametricPlot3D[{2 Cos[t], 2 Sin[t], s}, {t, 0, 2 Pi}, {s, 0, 2},
    RegionFunction → Function[{x, y, z}, And[z < Abs[x]]], PlotStyle → {Opacity[.4]},
    PlotRange → All, BoxRatios → {1, 1, 1 / 2}, PlotPoints → {100, 100}, Mesh → None];

annulus12house = Show[s1, s2, s3, Boxed → False, Axes → False, ImageSize → 600, ViewPoint → VP]
```

Out[7]=



Next, I will export the above graph as a png file that I can use on my web site.

In[8]:=
```
Export["annulus12house.png", annulus12house]
```

Out[8]= annulus12house.png

I could have used gif or jpg or svg, all popular image file-types for use at web sites. Interestingly, *Mathematica* does not produce a very good svg file.

In[9]:=
```
Export["annulus12house.gif", annulus12house]
```

Out[9]= annulus12house.gif

In[10]:= `Export["annulus12house.jpg", annulus12house]`

Out[10]= annulus12house.jpg

In[11]:= `Export["annulus12house.svg", annulus12house]`

Out[11]= annulus12house.svg

The volume of the house pictured above is

In[12]:= `2 Integrate[Integrate[r Cos[θ] r, {r, 1, 2}], {θ, -Pi/2, Pi/2}]`

Out[12]= $\frac{28}{3}$

The average distance is to the fixed radius is

In[13]:= $\frac{\texttt{2 Integrate[Integrate[r Cos[}\theta\texttt{] r, \{r, 1, 2\}], \{}\theta\texttt{, -}\frac{Pi}{2}\texttt{, }\frac{Pi}{2}\texttt{\}]}}{\texttt{(2\^{}2 - 1\^{}2) Pi}}$

Out[13]= $\frac{28}{9\pi}$

Interestingly this number is very close to 1

In[14]:= $N\left[\frac{28}{9\pi}\right]$

Out[14]= 0.990297

We could have done this calculation for any inner radius a and outer radius b. Then the average would be

In[15]:= $\texttt{FullSimplify}\left[\frac{\texttt{2 Integrate[Integrate[r Cos[}\theta\texttt{] r, \{r, a, b\}], \{}\theta\texttt{, -}\frac{Pi}{2}\texttt{, }\frac{Pi}{2}\texttt{\}]}}{(b^2 - a^2)\ \texttt{Pi}}\right]$

Out[15]= $\frac{4\left(a^2 + a\,b + b^2\right)}{3\,(a + b)\,\pi}$

Can you explain the following limit?

In[16]:= $\texttt{Limit}\left[\frac{4\left(a^2 + a\,b + b^2\right)}{3\,(a + b)\,\pi},\ b \to a\right]$

Out[16]= $\frac{2\,a}{\pi}$

Here is a question related to our first calculation. Taking the inner radius 1, which outer radius would produce the average value of the distance exactly equal to 1? Using *Mathematica* the answer is:

In[17]:= $\texttt{Solve}\left[\frac{4\left(1 + b + b^2\right)}{3\,(1 + b)\,\pi} == 1,\ b\right]$

Out[17]= $\left\{\left\{b \to \frac{1}{8}\left(-4 + 3\pi - \sqrt{-48 + 24\pi + 9\pi^2}\right)\right\}, \left\{b \to \frac{1}{8}\left(-4 + 3\pi + \sqrt{-48 + 24\pi + 9\pi^2}\right)\right\}\right\}$

In[18]:= `N[%]`

Out[18]= {{b → -0.669497}, {b → 2.02569}}

In[19]:= $\frac{4\left(a^2+a\,b+b^2\right)}{3\,(a+b)\,\pi}$ /. {b → 2}

Out[19]= $\frac{4\left(4+2\,a+a^2\right)}{3\,(2+a)\,\pi}$

In[20]:= Solve[$\frac{4\left(4+2\,a+a^2\right)}{3\,(2+a)\,\pi}$ == 1, a]

Out[20]= $\left\{\left\{a\to\frac{1}{8}\left(-8+3\,\pi-\sqrt{-192+48\,\pi+9\,\pi^2}\right)\right\},\left\{a\to\frac{1}{8}\left(-8+3\,\pi+\sqrt{-192+48\,\pi+9\,\pi^2}\right)\right\}\right\}$

In[21]:= N[%]

Out[21]= {{a → -0.684519}, {a → 1.04071}}