MATH 226 Assignment 2b May 21, 2009 Name ______________________

Give all details of your reasoning.

Problem 1. For each natural number n the number $n!$ is defined by $n! := 1 \cdot 2 \cdot \ldots \cdot (n-1) \cdot n$. We also define $0! := 1$. Consider the infinite series

$$\sum_{n=0}^{+\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!} + \cdots \ .$$

(a) Prove that the sequence of partial sums of this series is bounded.

(b) Prove that the given series converges.

Problem 2. Use convergence tests to determine whether the series is conditionally convergent, absolutely convergent or divergent. Explain your reasoning: State clearly which test is being used and make sure that all requirements of that test are fulfilled.

(A) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n} \cos\left(\frac{1}{n}\right)$; (B) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n} \ln\left(1+\frac{1}{n}\right)$; (C) $\displaystyle\sum_{n=1}^{+\infty} \frac{\sqrt{n}}{2n^2-\sqrt{n}}$.

Problem 3. Prove the following statement:
If $a_n \geq 0$ and $\displaystyle\sum_{n=1}^{+\infty} a_n$ converges, then the series $\displaystyle\sum_{n=1}^{+\infty} a_n^2$ converges.

Problem 4. Consider the series $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n^p}$. Determine all the values of p for which the series converges absolutely, converges conditionally and diverges.