MATH 226 Assignment 3 February 14, 2012

Name ______________________________

Problem 1. (a) Prove the following theorem.

Theorem. *Let f be a function which is defined on the interval $[1, +\infty)$. Define the sequence $a\colon \mathbb{N} \to \mathbb{R}$ by*

$$a_n = f(n) \quad \textit{for every} \quad n \in \mathbb{N}.$$

If $\lim_{x\to+\infty} f(x) = L$, then $\lim_{n\to+\infty} a_n = L$.

(b) Is the converse of the above theorem true? That is, is the following theorem true:

Theorem. *Let f be a function which is defined on the interval $[1, +\infty)$. Define the sequence $a\colon \mathbb{N} \to \mathbb{R}$ by*

$$a_n = f(n) \quad \textit{for every} \quad n \in \mathbb{N}.$$

If $\lim_{n\to+\infty} a_n = L$, then $\lim_{x\to+\infty} f(x) = L$.

Problem 2. (a) Prove the following theorem.

Theorem. *Let f be a function which is defined on the interval $(0, 1]$. Define the sequence $a\colon \mathbb{N} \to \mathbb{R}$ by*

$$a_n = f(1/n) \quad \textit{for every} \quad n \in \mathbb{N}.$$

If $\lim_{x\downarrow 0} f(x) = L$, then $\lim_{n\to+\infty} a_n = L$.

(b) Is the converse of the above theorem true? That is, is the following theorem true:

Theorem. *Let f be a function which is defined on the interval $(0, 1]$. Define the sequence $a\colon \mathbb{N} \to \mathbb{R}$ by*

$$a_n = f(1/n) \quad \textit{for every} \quad n \in \mathbb{N}.$$

If $\lim_{n\to+\infty} a_n = L$, then $\lim_{x\downarrow 0} f(x) = L$.

Problem 3. Let $a\colon \mathbb{N} \to \mathbb{R}$ and $b\colon \mathbb{N} \to \mathbb{R}$ be given sequences. Define the sequence $c\colon \mathbb{N} \to \mathbb{R}$ by

$$c_n = a_n + b_n \quad \text{for every} \quad n \in \mathbb{N}.$$

Prove: If $\lim_{n\to+\infty} a_n = L$ and $\lim_{n\to+\infty} b_n = K$, then $\lim_{n\to+\infty} c_n = L + K$.