Winter 2014 Math 226 Topics for the final

Limits. Know:

- definition and properties of absolute value function with rigorous proofs, avoiding cases whenever possible
- fifteen different definitions of limits
- how to prove limits based on definitions
- how to prove that limit does not exist in simple cases
- two versions of the squeeze theorem with proofs
- how to use geometric arguments and squeeze theorems to formally prove three trigonometric limits and one logarithmic limit
- the rules of the algebra of limits and the proof of the sum rule

Continuity. Know

- ϵ-δ definition of continuity
- how to use ϵ-δ definition of continuity to prove that simple algebraic functions are continuous
- how to use a geometric argument to prove that the logarithm, exponential, sine and cosine are continuous functions
- a rigorous proof that a composition of continuous functions is continuous

Sequences. Know

- the definitions of convergence, boundedness, monotonicity of a sequence
- how to use the definition of convergence to prove that some simple sequences converge or diverge
- proofs related to sequences defined by a simple formula (Theorems 2.1.7 and 2.1.8);
- relationships between convergence and boundedness with the corresponding proof
- the Completeness axiom
- the monotone convergence theorem and its proof
- how to use the monotone convergence theorem to prove convergence of three sequences: Example 2.1.18, Example 2.1.3(b), and $S_n = \sum_{k=0}^{n} \frac{1}{k!}$, $n \in \mathbb{N}$

Infinite series. Know

- the definition of convergence for series
- all about geometric series with proofs, also examples from exercises
- the Harmonic series diverges
- the application of geometric series to decimal expansions with the proof of convergence of decimal expansions
- a rigorous proof that the harmonic series diverges
- how to recognize telescopic series and prove its convergence
- a very important necessary condition for the convergence of infinite series with the proof (Theorem 2.2.8)
- how to apply the algebra of convergent series to determine convergence or divergence of series
- how to apply the direct comparison test, the limit comparison test and the integral test to determine convergence of various series (p-series is an important application)
- how to apply the alternating series test to determine convergence of alternating series
- the concept of absolute and conditional convergence
- how to prove that every absolutely convergent series converges
- how to apply the ratio and the root test to determine absolute convergence of series

Power series. Know

- how to determine the interval of convergence of a given power series, in particular how to decide whether the endpoints of the interval belong to the interval of convergence
- how to apply Theorem 4.8 to find sums of some simple power series (Examples 4.10, 4.11, 4.12)