Prove $\forall x \in \mathbb{R}$ $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + 1/2 \rfloor$.

Let $x \in \mathbb{R}$ be arbitrary. $\underbrace{\lfloor 2x \rfloor}_{m} = \lfloor x \rfloor + \lfloor x + \tfrac{1}{2} \rfloor$

Known is: $y \in \mathbb{R}$

$\boxed{\ell = \lfloor y \rfloor \iff \ell \in \mathbb{Z} \wedge \ell \leq y < \ell + 1}$ (G1)

Proof of $=$. Set $m = \lfloor 2x \rfloor$. By G1, I know $m \in \mathbb{Z}$ and (G2) $\boxed{m \leq 2x < m+1}$.

$\Downarrow$ $\boxed{2 > 0}$ $\tfrac{1}{2} > 0$

(G3) $\boxed{\frac{m}{2} \leq x < \frac{m}{2} + \frac{1}{2}}$

Case 1 m is even

Case 2 m is odd

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Let $x \in \mathbb{R}$ be arbitrary.

$$\underbrace{\lfloor 2x \rfloor}_{m} = \lfloor x \rfloor + \lfloor x + \tfrac{1}{2} \rfloor$$

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G2 $\boxed{m \leq 2x < m+1}$.

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Case 1 m is even

Case 2 m is odd

Case 1 $\quad$ m is even

$\lfloor 2x \rfloor = m = 2k \quad k \in \mathbb{Z}$

G4

$k \leq x < k + \frac{1}{2} < k+1$

$k \in \mathbb{Z} \wedge k \leq x < k+1 \Rightarrow$ G1 $\lfloor x \rfloor = k$

G1
$\ell = \lfloor y \rfloor \Leftrightarrow \ell \in \mathbb{Z}$
$\ell \leq y < \ell + 1$

G4

$k \leq x < k + 1/2$

add 1/2

$\Downarrow$ ineq. resp. add

G5

$k \leq k + \frac{1}{2} \leq x + \frac{1}{2} < k + 1$ $\Rightarrow$ $k \leq x + \frac{1}{2} < k+1$ $\Rightarrow$ $\lfloor x + \frac{1}{2} \rfloor = k$

G5

This Proves case 1

$$\lfloor 2x \rfloor \overset{G}{=} m \overset{G}{=} 2k \overset{G}{=} k+k \overset{G}{=} \lfloor x \rfloor + \lfloor x + 1/2 \rfloor$$

MS

Case 2 $m = 2k+1$ odd

I pod this

What is a proof.

Identify Red $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + 1/2 \rfloor$

Identify green stuff Prop $\lfloor x \rfloor$ yeR

Have a brilliant idea

Case 1 $m = \lfloor 2x \rfloor$, m even

Case 2 $m = \lfloor 2x \rfloor$, m is odd.

Prove Triangle Inequality

$\forall a, b \in \mathbb{R}$ $\quad |a+b| \leq |a| + |b|$

$a+b \in \mathbb{R}$

$|a+b|$

$|a|$, $|b|$, $|a|+|b|$

known?

$|x| = \max\{-x, x\} \Rightarrow$ $x \leq |x|$, $-x \leq |x|$