The negation of the definition of limit

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The statement (II) in the def of limit is

$\forall \varepsilon > 0 \; \exists X(\varepsilon) \geq X_0$ s.t. $x > X(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$ -

$\forall x$ silent

The logical negation of: minute

$\forall$ day $\exists$ min $\in$ day s.t. it rains that minute

negation

$\exists$ day $\forall$ min $\in$ day no rain that minute

negation

$\exists \varepsilon > 0 \quad \forall X \geq X_0 \quad \ldots$ negate the implication

bad epsilon

finish two pages down

a statement $p \Rightarrow q$ a statement

Red square

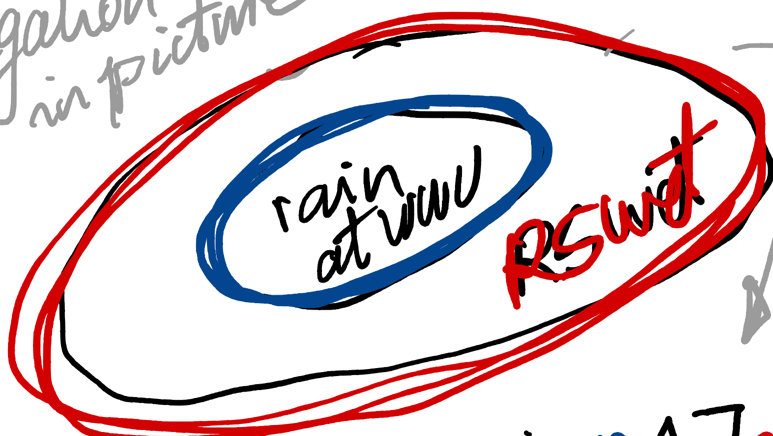
WWU story → rain at campus $\Rightarrow$ RS wet

negation:

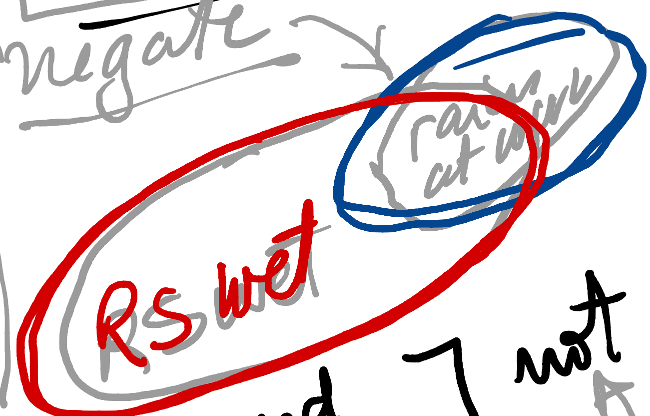
negation:

rain and RS dry

negation in picture



negate



negation of $p \Rightarrow q$ is $p \wedge \neg q$ ..... $\wedge$ and $\neg$ not

symbol symbol

$$\exists \varepsilon > 0 \ \forall X \geq X_0 \ \exists x \text{ s.t.}$$

$$x > X \wedge |f(x) - L| \geq \varepsilon$$

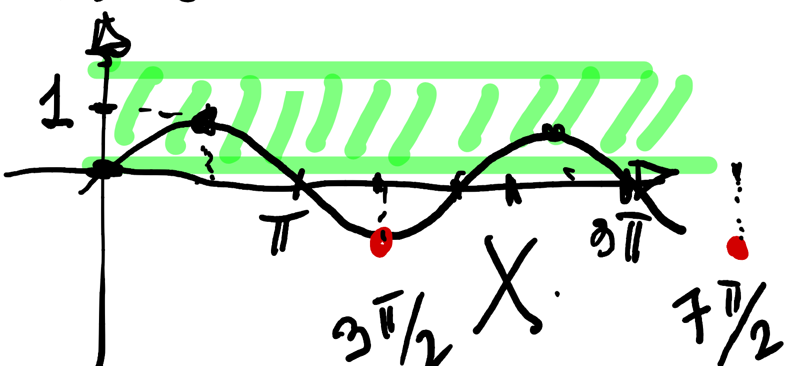
$\lim_{x \to +\infty} \sin x = 1$ <u>not true</u> $\quad X_0 = 0$

$\varepsilon = 1$

$\forall X \geq 0 \ \exists x$

$x > X \wedge |f(x) - 1| \geq 1$

$x = (2k+1)\pi + \frac{\pi}{2}$



X find larger $\underbrace{(2k+1)\pi} + \frac{\pi}{2}$

$$X < (2k+1)\pi$$

integer

$$\frac{X}{\pi} < 2k+1$$

$$\left\lceil \frac{\frac{X}{\pi} - 1}{2} \right\rceil = k$$

$$X < \left(2\left\lceil \frac{\frac{X}{\pi}-1}{2} \right\rceil + 1\right)\pi + \frac{\pi}{2}$$

true!

$$\sin\left(\quad\right) = -1$$

$\forall X \geq 0$ take $x = \left(2\left\lceil \frac{\frac{X}{\pi}-1}{2} \right\rceil + 1\right)\pi + \frac{\pi}{2}$, then

$\sin(x) = -1$ and $x > X$ and

$$|\sin(x) - 1| = 2 > 1$$

In fact, to prove $\lim_{x \to +\infty} \sin x$ does not exist

To prove this claim we need to prove that $\lim_{x \to +\infty} \sin x = L$ is wrong $\forall L \in \mathbb{R}$.

Consider three cases:

$L > 0$, $L = 0$, $L < 0$

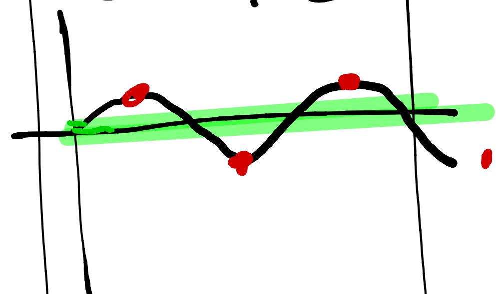
$L > 0$:

$\varepsilon = 1$

Same reasoning as $L = 1$

$L = 0$:

$\varepsilon = 1/2$



$L < 0$:

$\varepsilon = 1$

Similar reasoning as