The most famous limit at $+\infty$ is

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x} = e$$

e-shirt

$$\left(1+\frac{1}{n}\right)^{n}$$

see Exercise 1.3.5

$$\lim_{x\to+\infty} \ln\left(1+\frac{1}{x}\right)^{x} = 1$$

$$\lim_{x\to+\infty} f(x) = L$$

(I) $\exists X_0 \in \mathbb{R}$ s.t. $f(x)$ is defined $\forall x \geq X_0$ $+\infty$

$[X_0, +\infty) \subseteq \mathrm{dom}(f)$

(II) $\forall \varepsilon > 0\ \exists X(\varepsilon) \geq X_0$ s.t. $x > X(\varepsilon) \Rightarrow \underbrace{|f(x) - L|}_{\text{how close is } f \text{ to } L} < \varepsilon$

must be in the domain

I want you to state this def!

$$\lim_{x\to-\infty} f(x) = L$$

(I) $\exists X_0$ s.t. $f(x)$ is defined $\forall x \leq X_0$ $-\infty$

(II) $\forall \varepsilon > 0\ \exists X(\varepsilon) \leq X_0$ s.t. $x < X(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$

must be in the domain

$$\lim_{x \to +\infty} f(x) = +\infty$$ Define this $\forall$

f(x) surpases any large number

(I) $\exists X_0 \in \mathbb{R}$ s.t. $f(x)$ is defined for $\forall x \in X_0$.

(II) $\forall M > 0 \ \exists X(M) \geq X_0$ s.t. $x > X(M) \Rightarrow f(x) > M$

$$\lim_{x \to +\infty} f(x) = -\infty$$

(I) the same

(II) $\forall M < 0 \ \exists X(M) \geq X_0$ s.t. $x \geq X(M) \Rightarrow f(x) < M$

$$\lim_{x \to -\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

Done with limits at infinity!

Limit at a, where $a \in \mathbb{R}$

The most famous limit of this kind is

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$f : D \to \mathbb{R}$, $D \subseteq \mathbb{R}$, $L \in \mathbb{R}$, $a \in \mathbb{R}$

$$\lim_{x \to a} f(x) = L$$

(I) $\exists \delta_0 > 0$ s.t. $(a-\delta_0, a) \cup (a, a+\delta_0) \subseteq D$

(II) $\forall \varepsilon > 0 \; \exists \delta(\varepsilon) > 0$ s.t. $\delta(\varepsilon) \le \delta_0$ and $0 < |x-a| < \delta(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$

exclude a

close to a

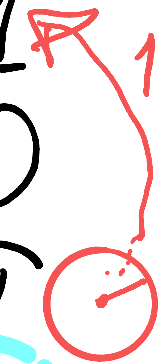
can approx. f(x) w. L

1

0

π

area



e

i

The hall of fame of numbers