More Limits

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = L$$

$$\lim_{x \to -\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

$$\lim_{x \to a} f(x) = L$$

Pr 7 Case 1 $L \geq 1/2$?

Case 2. $L < 1/2$

$L \in \mathbb{R}$ $\lim_{x \to +\infty} f(x) = L$ (finite limit)

$f: D \to \mathbb{R}$

(I) $\exists X_0 \in D$ s.t. $[X_0, +\infty) \subseteq D$

(II) $\forall \varepsilon > 0 \ \exists X(\varepsilon) \geq X_0$ s.t. $x > X(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$

$$\boxed{\lim_{x \to +\infty} f(x) = +\infty}$$

Def. of Let $D \subseteq \mathbb{R}$. A function $f: D \to \mathbb{R}$ has the limit $+\infty$ if the following two cond. are satisfied

(I) $\exists X_0 \in D$ s.t. $[X_0, +\infty) \subseteq D$

(II) $\forall M > 0 \quad \exists X(M) \geq X_0$ s.t. $x > X(M) \Rightarrow f(x) > M$

silent $\forall x$

$$\boxed{\lim_{x \to +\infty} f(x) = -\infty}$$

this means

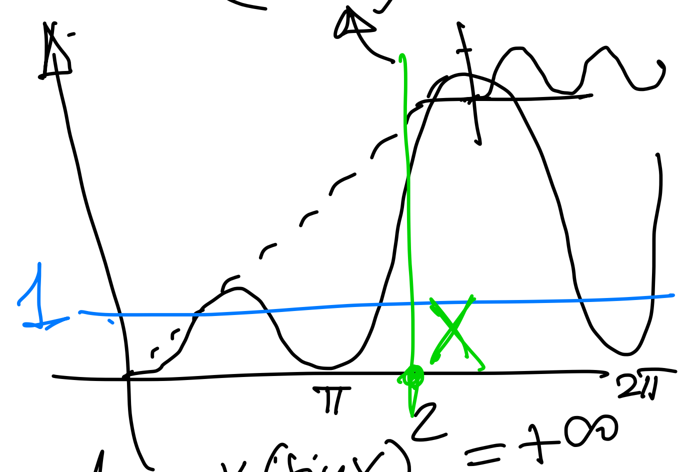
Let $D \subseteq \mathbb{R}$, $f: D \to \mathbb{R}$

(I) $\exists X_0 \in D$ s.t. $[X_0, +\infty) \subseteq D$

(II) $\forall M < 0 \quad \exists X(M) \geq X_0$ s.t.

$x > X(M) \Rightarrow f(x) < M$

$f(x) = x(\sin x)^2$



$$\lim_{x \to +\infty} x(\sin x)^2 = +\infty$$

$L \in \mathbb{R}$ $\boxed{\lim_{x \to -\infty} f(x) = L}$

What does this mean?

$D \subseteq \mathbb{R}$

$f: D \to \mathbb{R}$

(I) $\exists X_0 \in D$ s.t. $(-\infty, X_0] \subseteq D$

(II) $\forall \varepsilon > 0 \quad \exists X(\varepsilon) \leq X_0$ s.t. $\quad x < X(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$

$$\boxed{\lim_{x\to-\infty} f(x) = +\infty}$$

(I) $\exists x_0 \in D$ s.t. $(-\infty, x_0] \subseteq D$

(II) $\forall M>0 \quad \exists X(M) \le x_0$ s.t. $x < X(M) \Rightarrow f(x) > M$

$$\boxed{\lim_{x\to-\infty} f(x) = -\infty}$$ Do as exercise

<u>Exercise</u> Prove $\lim_{x\to+\infty} x(\sin x)^2 = +\infty$ is NOT TRUE

the negation of (II) in the def. of $\lim_{x\to+\infty} f(x)=+\infty$ is

$\exists M>0$ s.t. $\forall X>0 \quad \exists x > X$ s.t. $x(\sin x)^2 < M$

I want x to be $x = k\pi$ $k\in\mathbb{Z}$

Set $M=1$. Let $X>0$ be arbitrary

$k\pi > X$ solve for $k\in\mathbb{Z}$

$k > X/\pi$

$k = \lceil X/\pi \rceil + 1$

Set $M=1$. Let $X>0$ be arbitrary. Choose

$$x = \left(\left\lceil \frac{X}{\pi} \right\rceil + 1\right)\pi .$$ Then $\sin x = 0$ so

$$x(\sin x)^2 = 0 < 1$$ Yes it is!

<u>Definition</u> Let $D \subseteq \mathbb{R}$, let $a, L \in \mathbb{R}$. A function $f: D \to \mathbb{R}$ approaches L as x approaches a if the following two conditions are satisfied.

(I) $\exists\ \delta_0 > 0$ such that $(a-\delta_0, a) \cup (a, a+\delta_0) \subseteq D$

(II) $\forall \varepsilon > 0 \quad \exists\, \delta(\varepsilon)$ s.t. $0 < \delta(\varepsilon) \leq \delta_0$ and

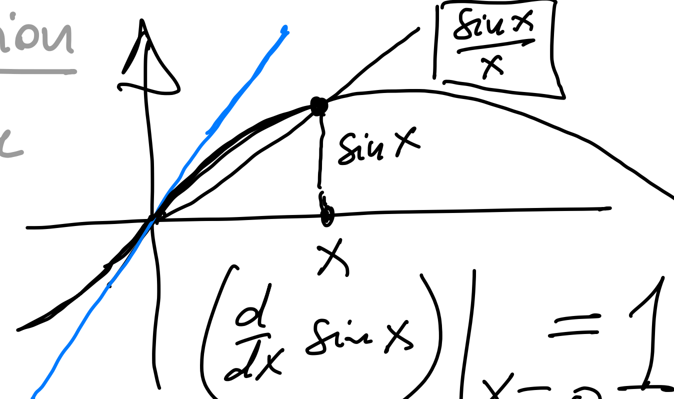
$$0 < |x-a| < \delta(\varepsilon) \Longrightarrow |f(x) - L| < \varepsilon$$

close to a

motivation

$$\lim_{x \to \underset{a}{0}} \underbrace{\frac{\sin x}{x}}_{f(x)} = \underset{L}{1}$$

we will prove this later



$$\left(\frac{d}{dx}\sin x\right)\Big|_{x=0} = 1$$

$$D = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$$

setminus