Matrix 1

In[1]:=
```
mA = {{.8, 0}, {0, .64}}
```

Out[1]=
```
{{0.8, 0}, {0, 0.64}}
```

In[2]:=
```
MatrixPower[mA, 3].{3, 3}
```

Out[2]=
```
{1.536, 0.786432}
```

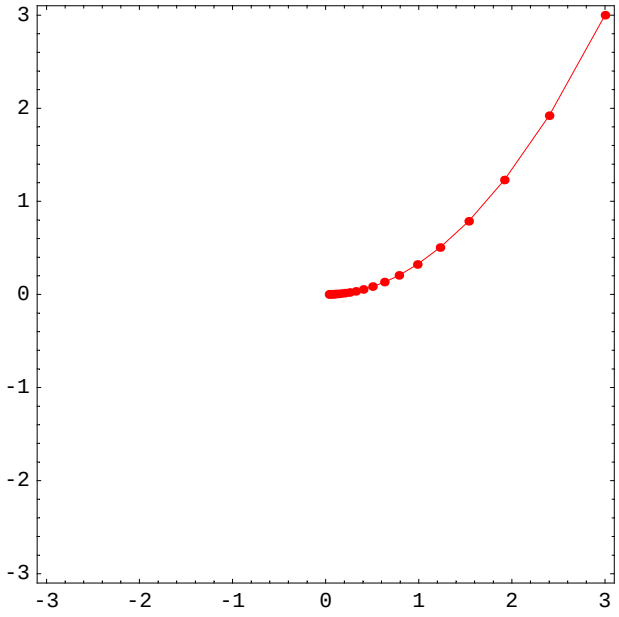
In[3]:=
```
ips = {{3, 0}, {3, 3}, {1.5, 3}, {0, 3}, {-1.5, 3},
   {-3, 3}, {-3, 0}, {-3, -3}, {-1.5, -3}, {0, -3}, {1.5, -3}, {3, -3}};
```

In[4]:=
```
ips = {{3, 3}}
```

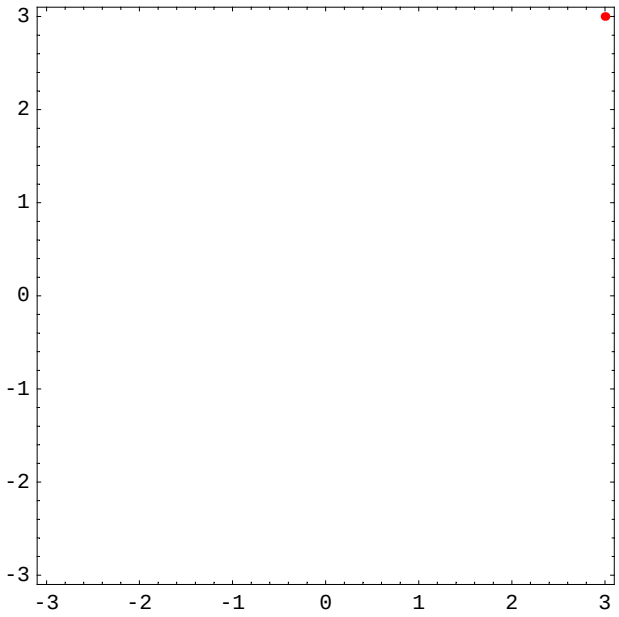
Out[4]=
```
{{3, 3}}
```

In[5]:=
```
steps = 20;
```

```
Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → Automatic, Frame → True
 ];
```



In[7]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
         {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
         {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```


3
2
1
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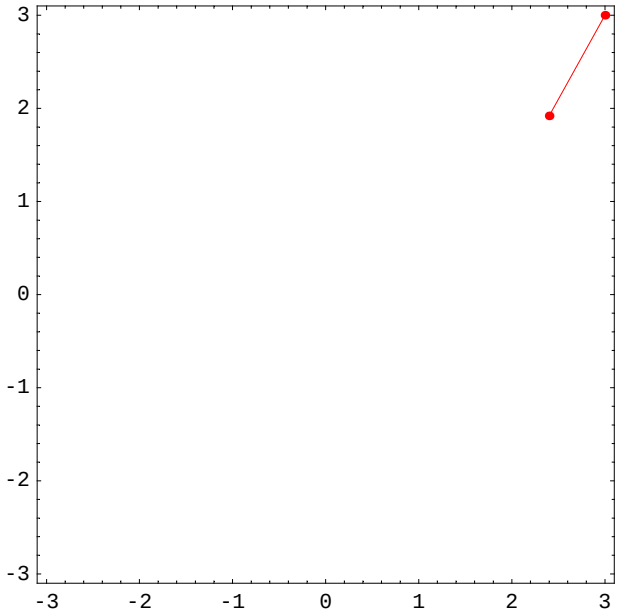
-1
-2
-3
-3 -2 -1 0 1 2 3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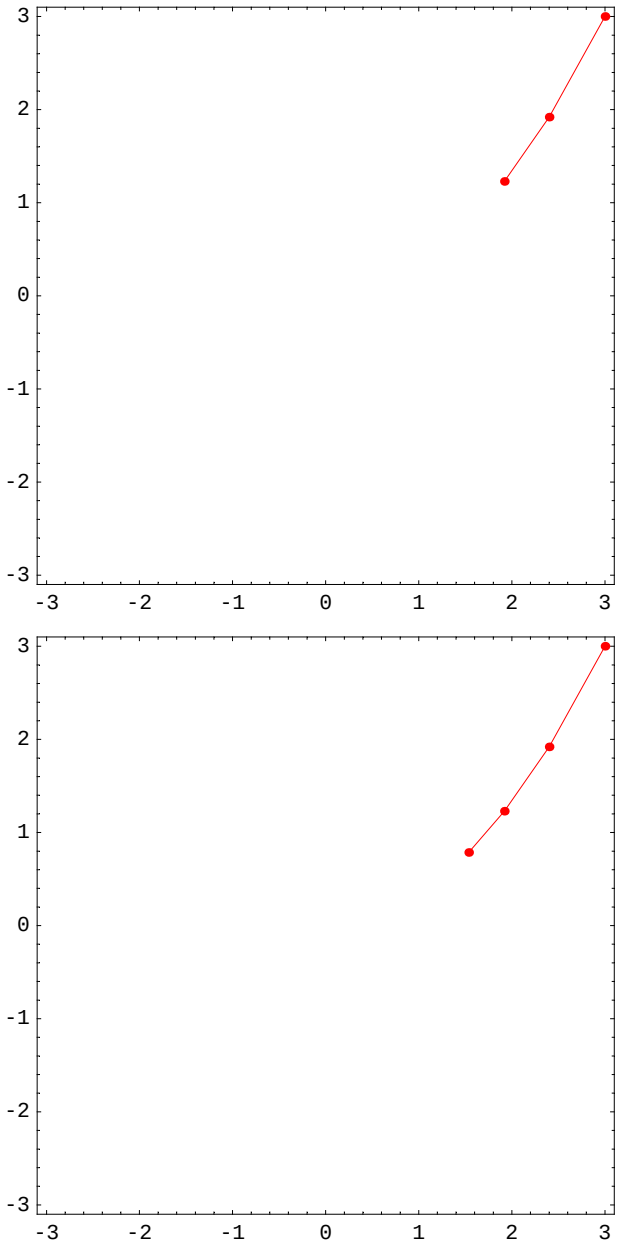
-1
-2
-3
-3 -2 -1 0 1 2 3


3
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-1
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-3 -2 -1 0 1 2 3
3
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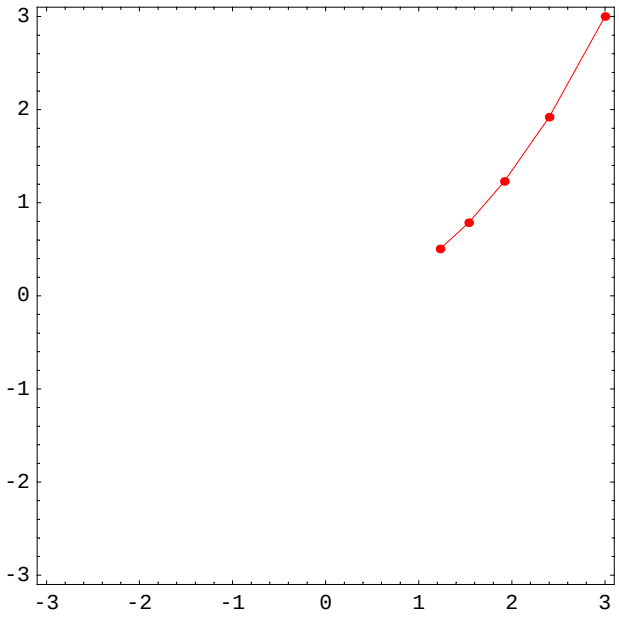
-1
-2
-3
-3 -2 -1 0 1 2 3


3
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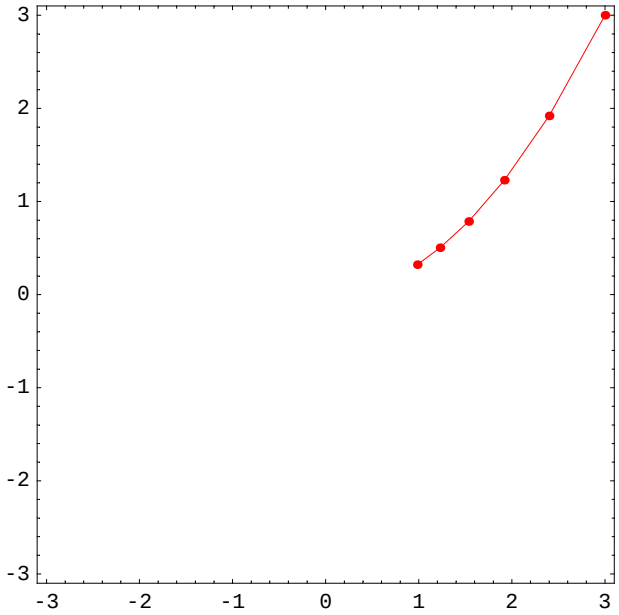
-1
-2
-3
-3 -2 -1 0 1 2 3


3
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-1
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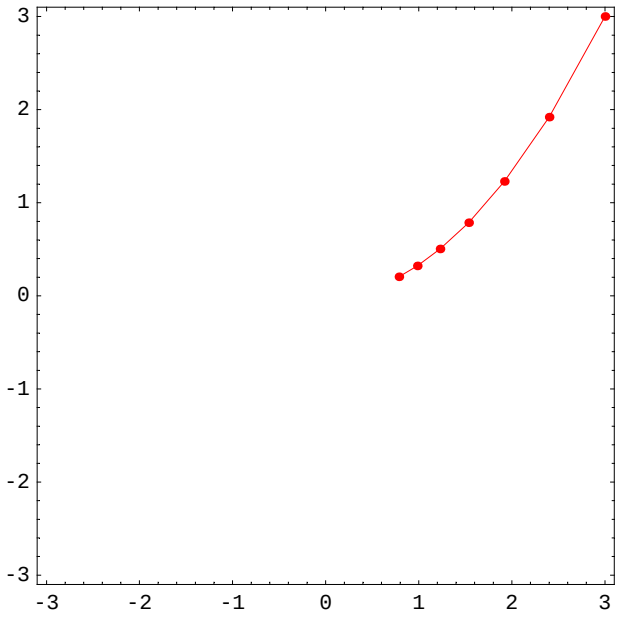
-3 -2 -1 0 1 2 3


3
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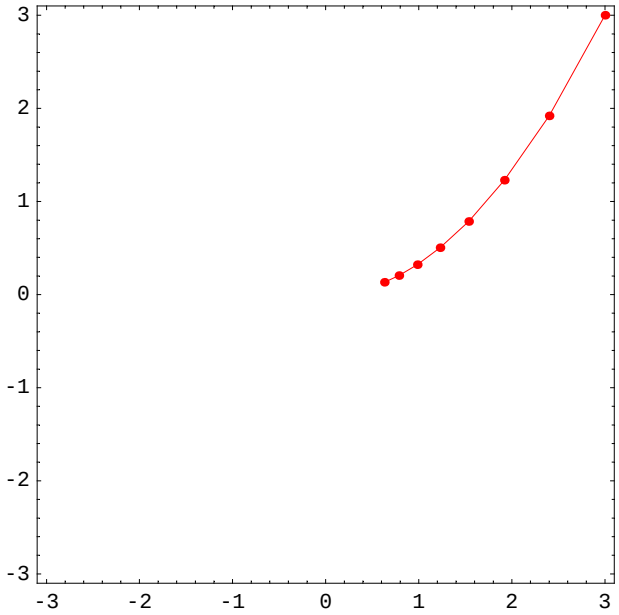
-3
-3
-2
-1
0
1
2
3


3
2
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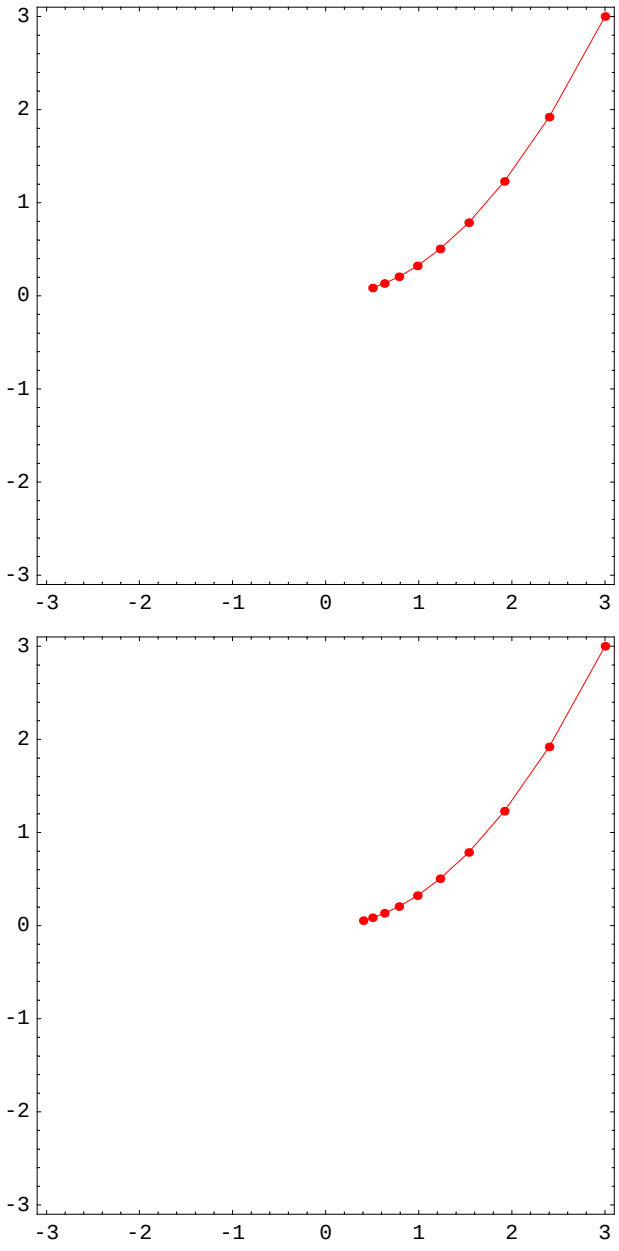
-1
-2
-3
-3
-2
-1
0
1
2
3


3
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-3 -2 -1 0 1 2 3
3
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-1
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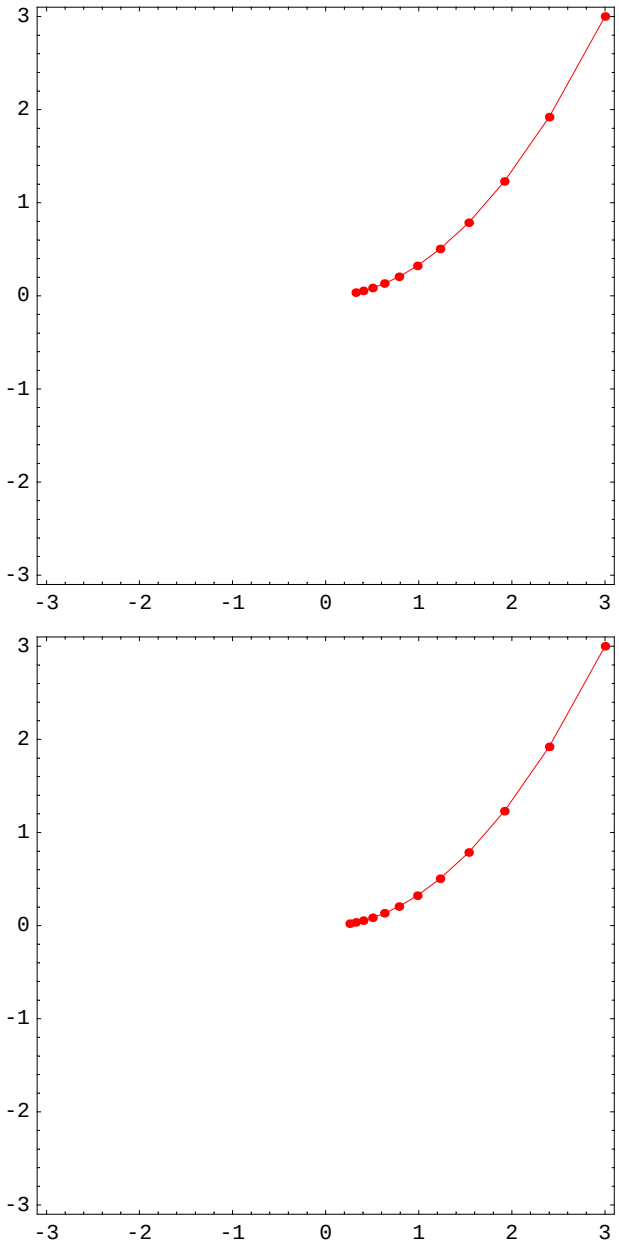
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
3
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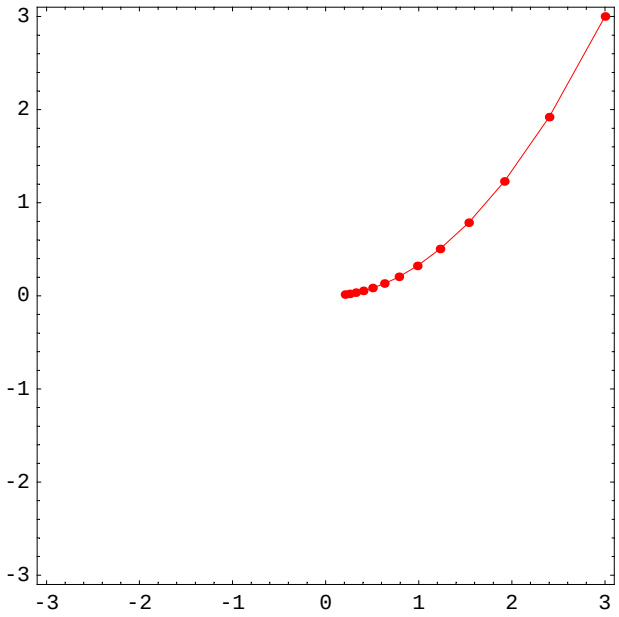
-3 -2 -1 0 1 2 3


3
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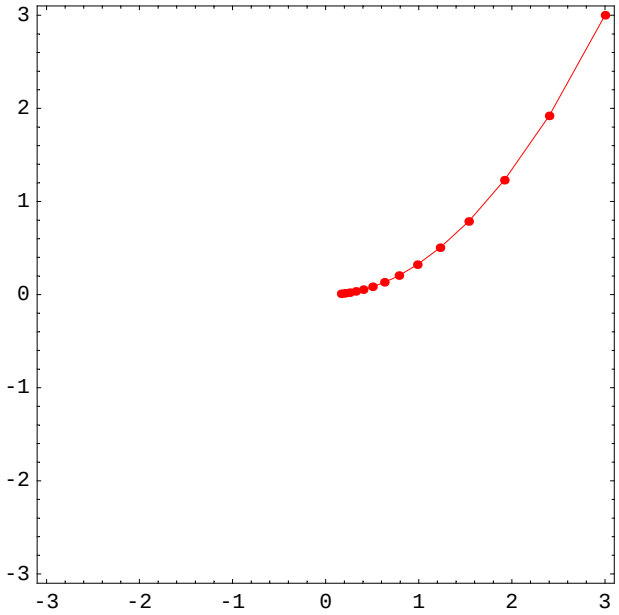
-2
-3
-3 -2 -1 0 1 2 3


3
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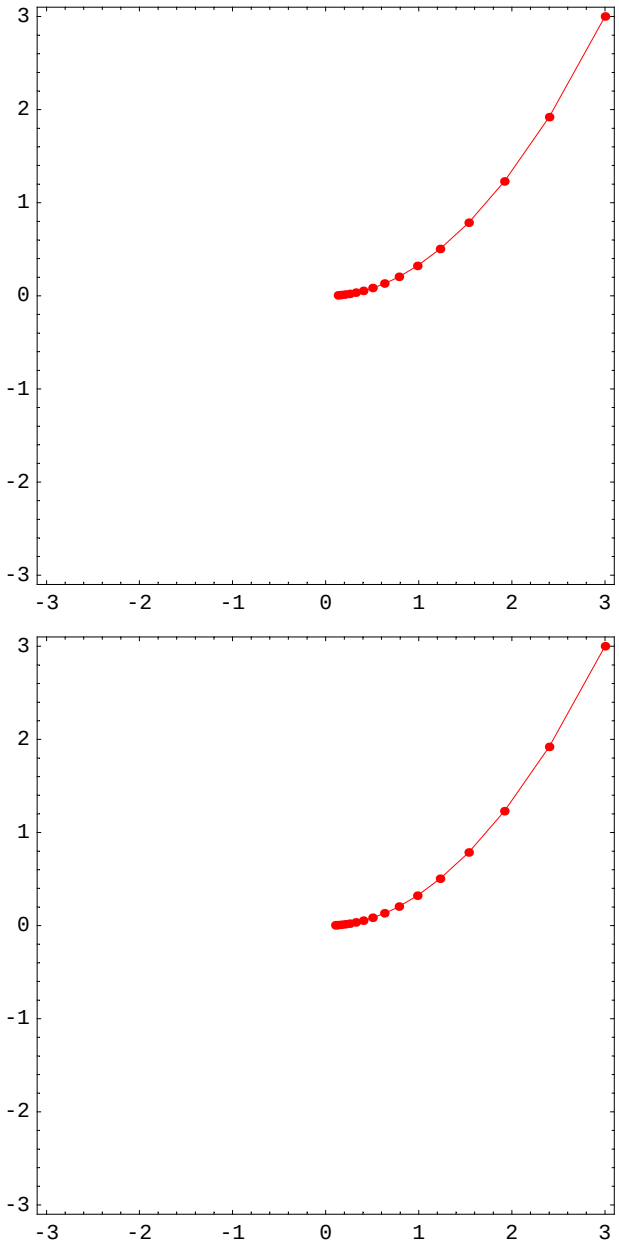
-1
-2
-3
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
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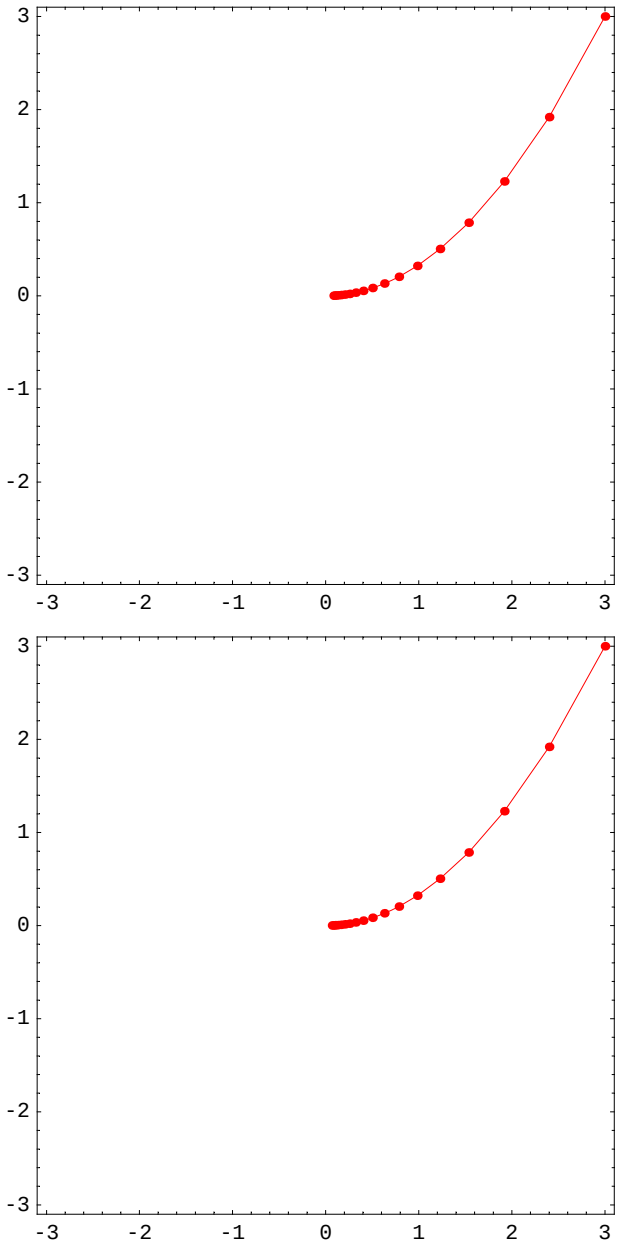
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3

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-1
-2
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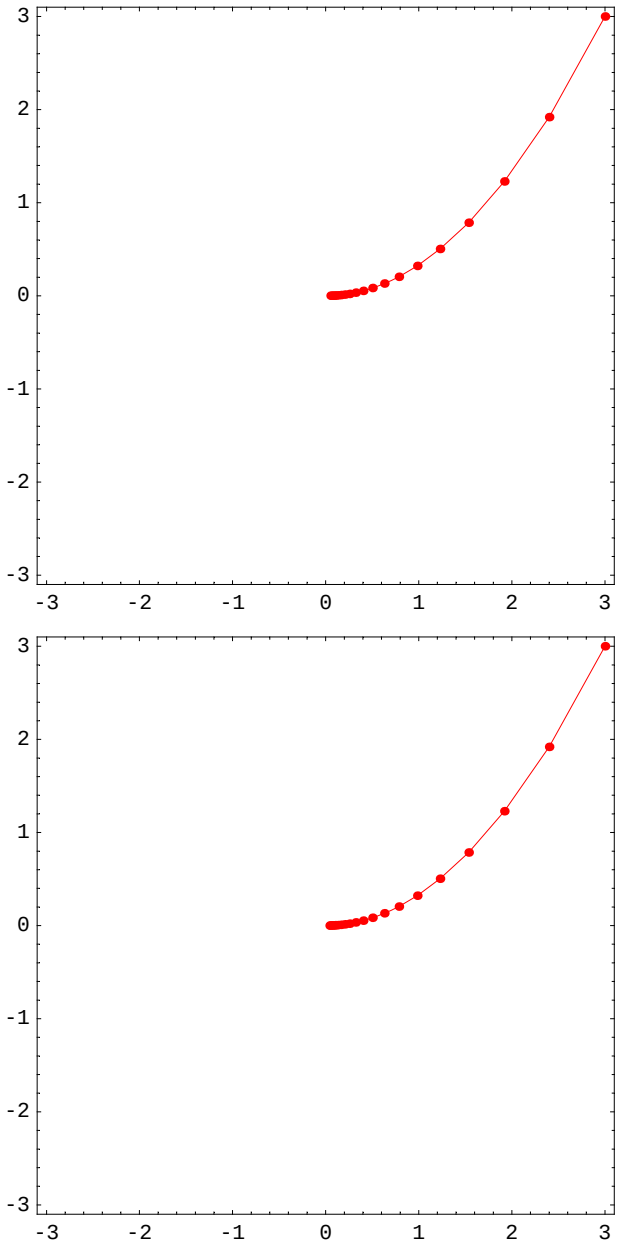
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3

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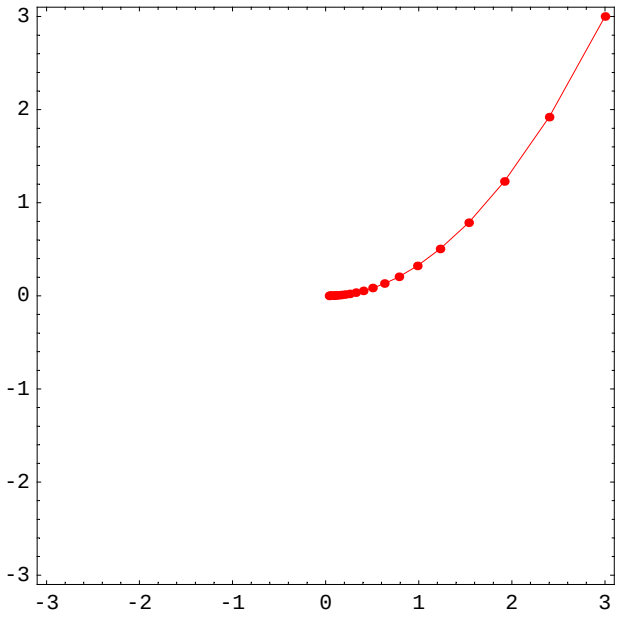
-3 -2 -1 0 1 2 3



Matrix 1a

```
In[8]:= mA = {{1, 0}, {0, .8}}
Out[8]= {{1, 0}, {0, 0.8}}

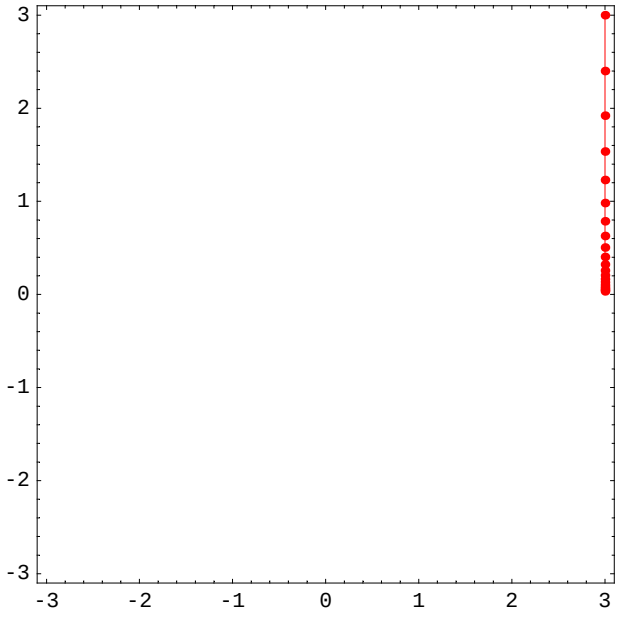
In[9]:= MatrixPower[mA, 3].{3, 3}
Out[9]= {3., 1.536}

In[10]:= ips = {{3, 0}, {3, 3}, {1.5, 3}, {0, 3}, {-1.5, 3},
            {-3, 3}, {-3, 0}, {-3, -3}, {-1.5, -3}, {0, -3}, {1.5, -3}, {3, -3}};

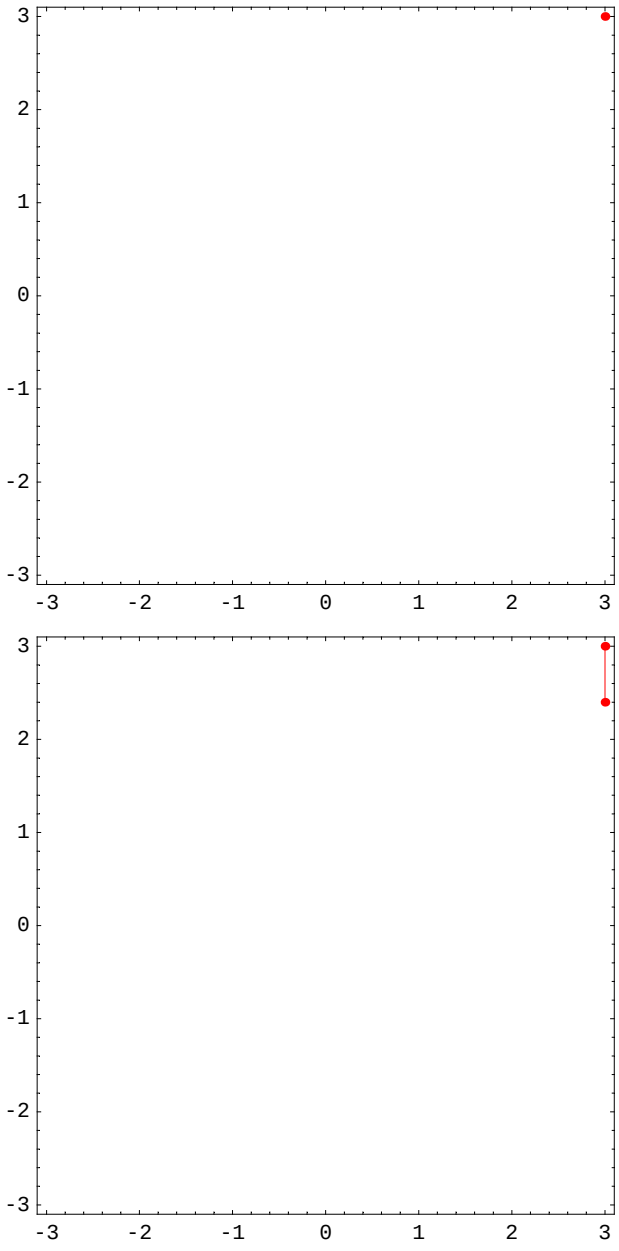

In[11]:= ips = {{3, 3}}
Out[11]= {{3, 3}}
```

In[12]:=
```
steps = 20;
```

```
Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → Automatic, Frame → True
 ];
```



In[14]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
         {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
         {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```


3
2
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-1
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-3 -2 -1 0 1 2 3
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2
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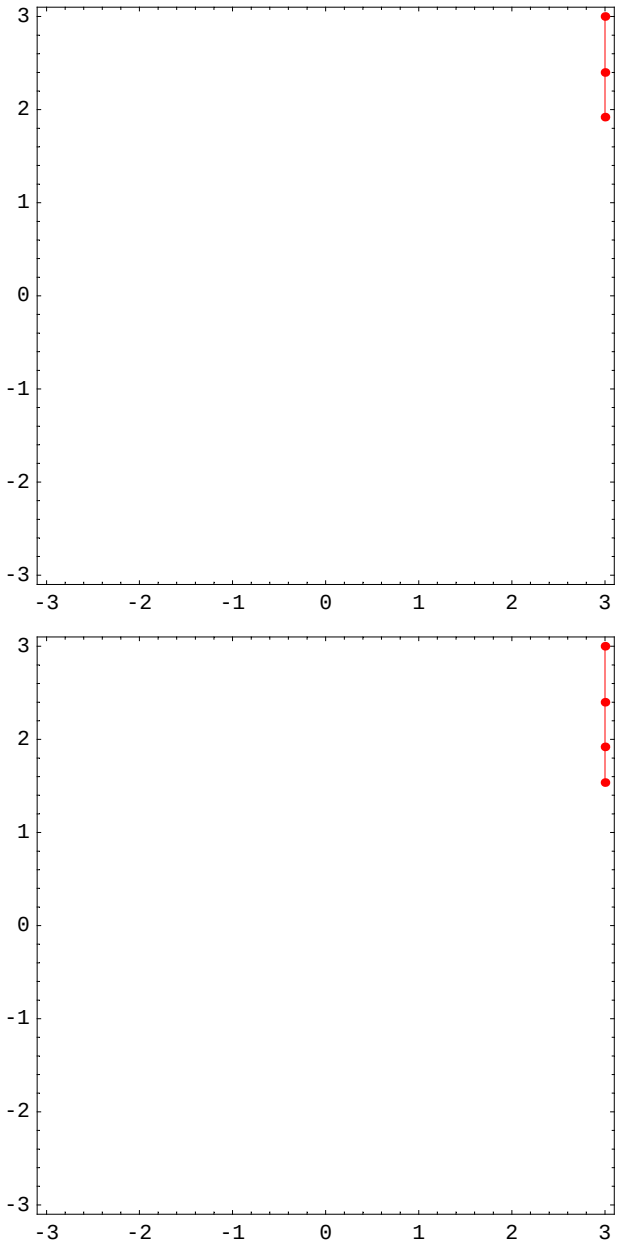
-3
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
3
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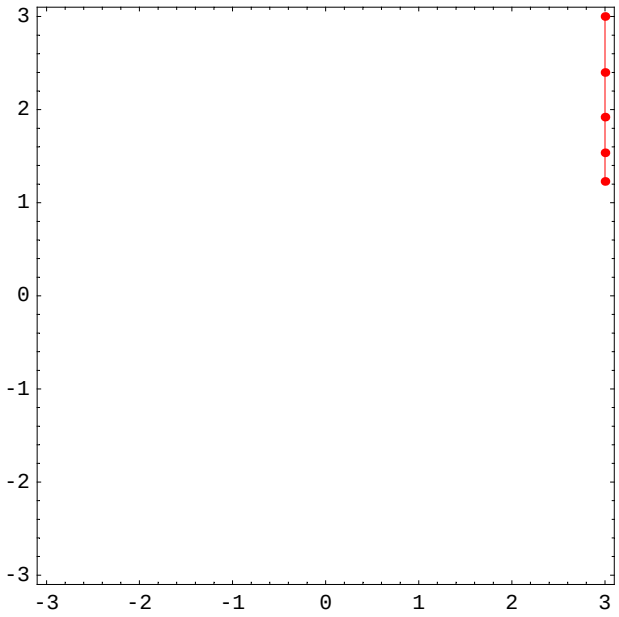
-3 -2 -1 0 1 2 3


3
2
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-1
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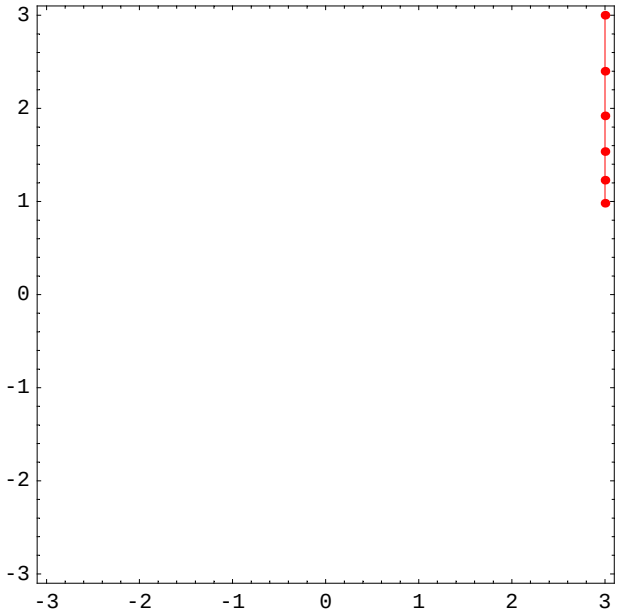
-3 -2 -1 0 1 2 3


3
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-1
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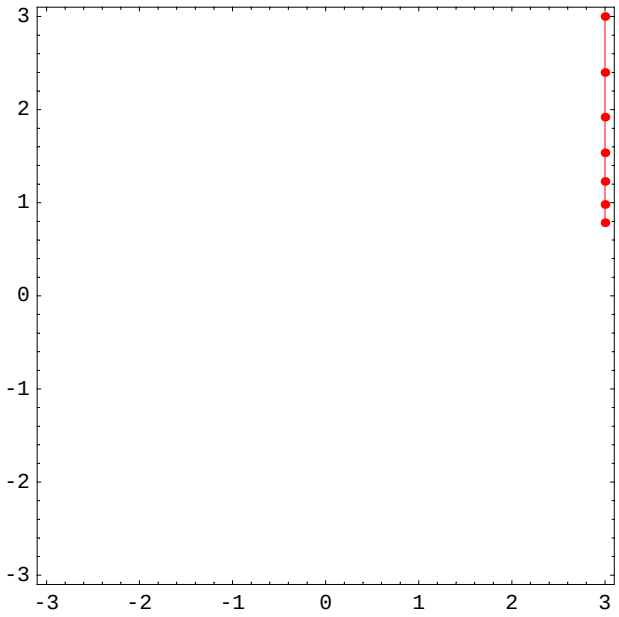
-3
-3 -2 -1 0 1 2 3


3
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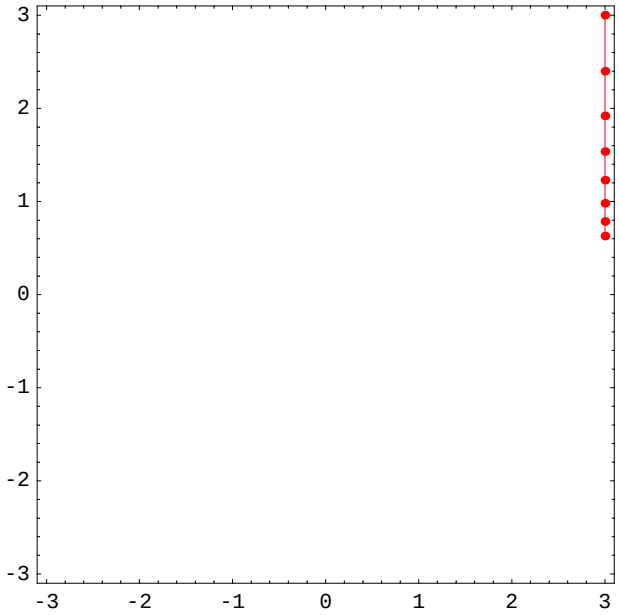
-1
-2
-3
-3 -2 -1 0 1 2 3


3
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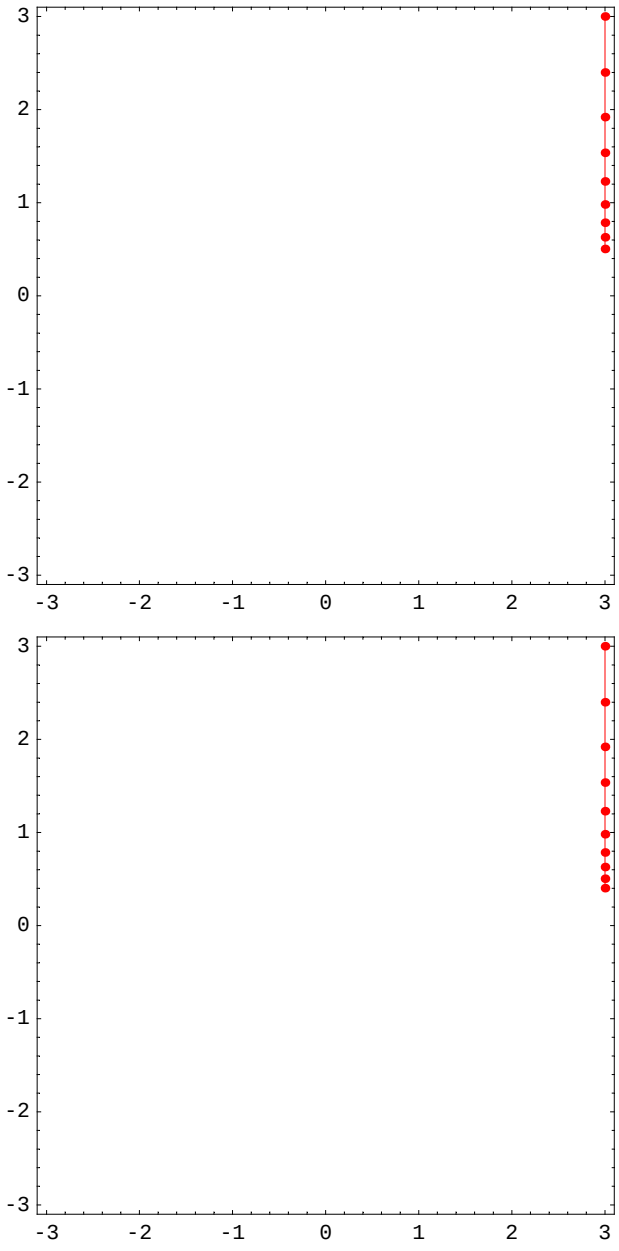
-3 -2 -1 0 1 2 3


3
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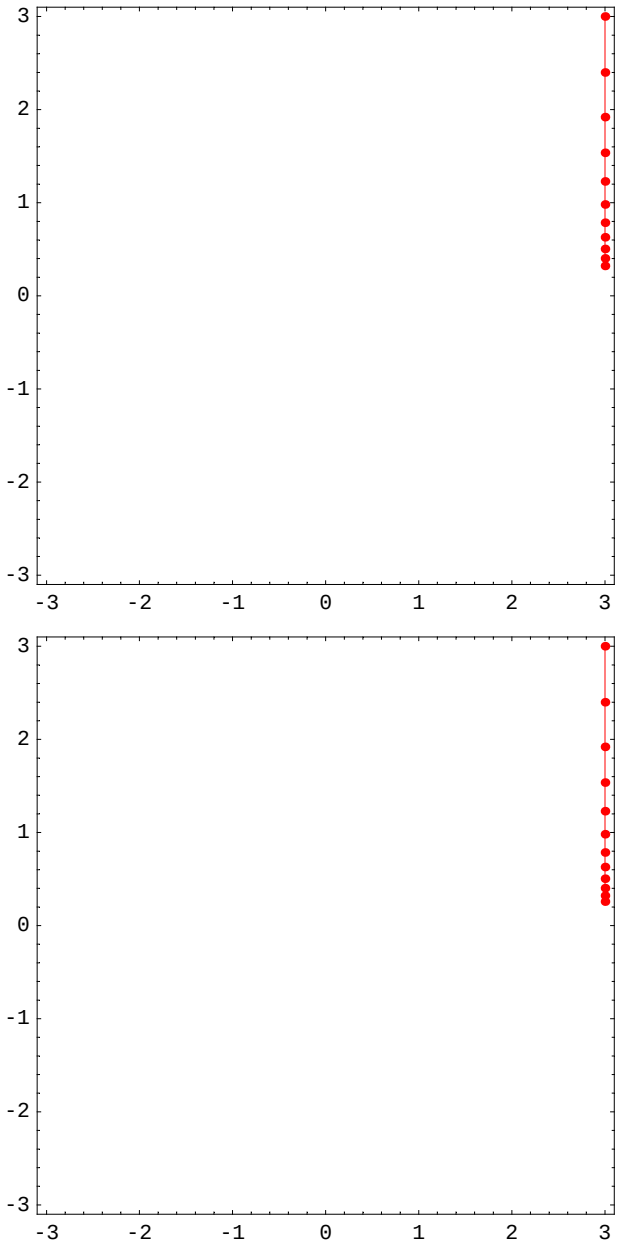
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
3
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-1
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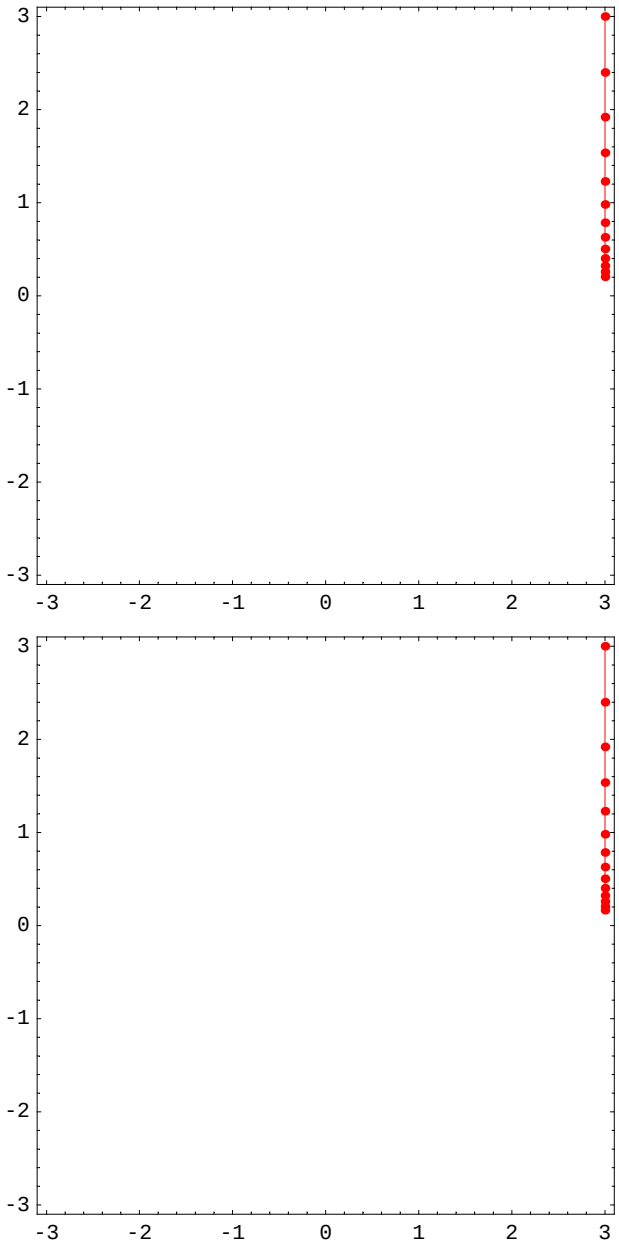
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
3
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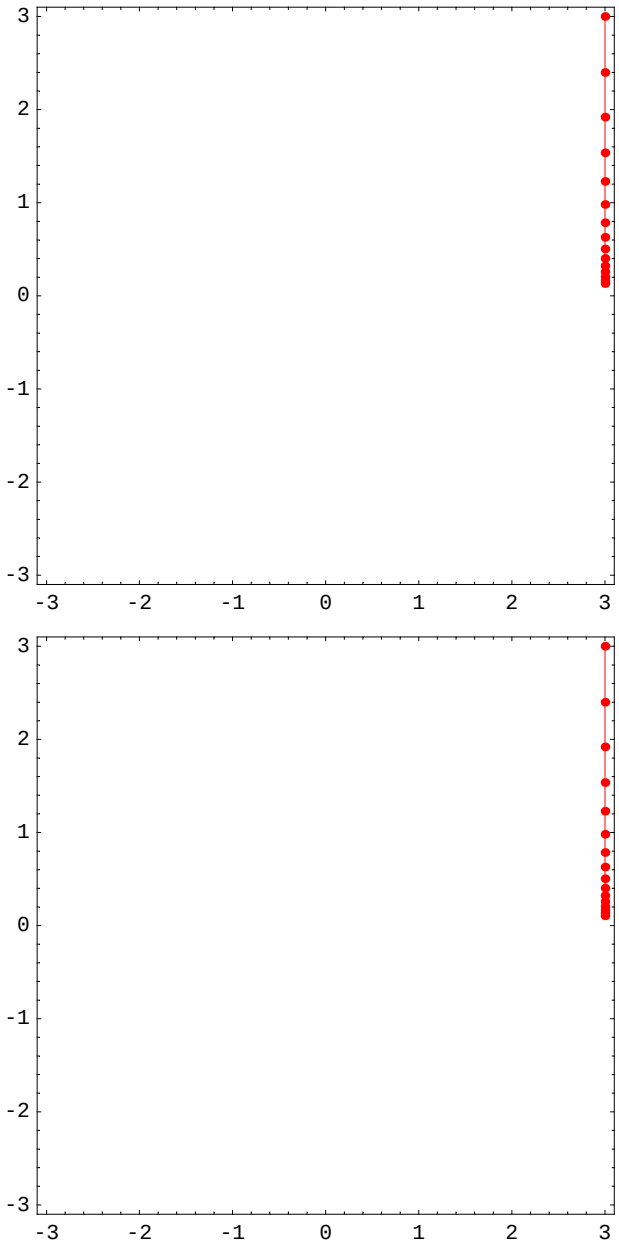
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
3
2
1
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-2
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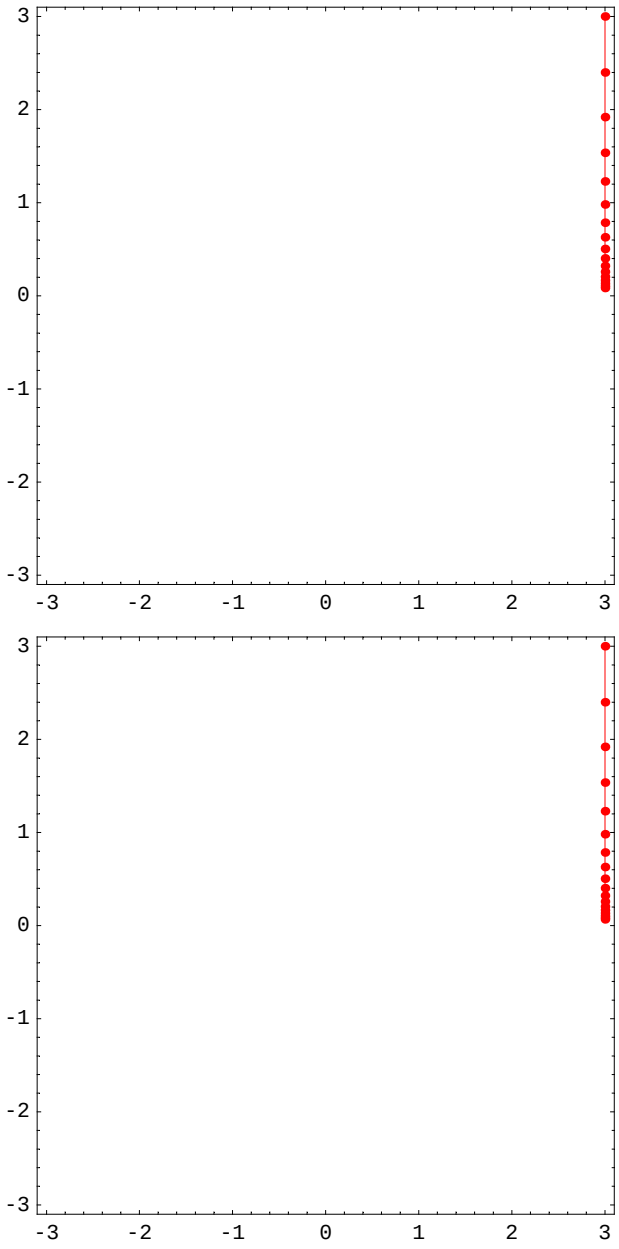
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
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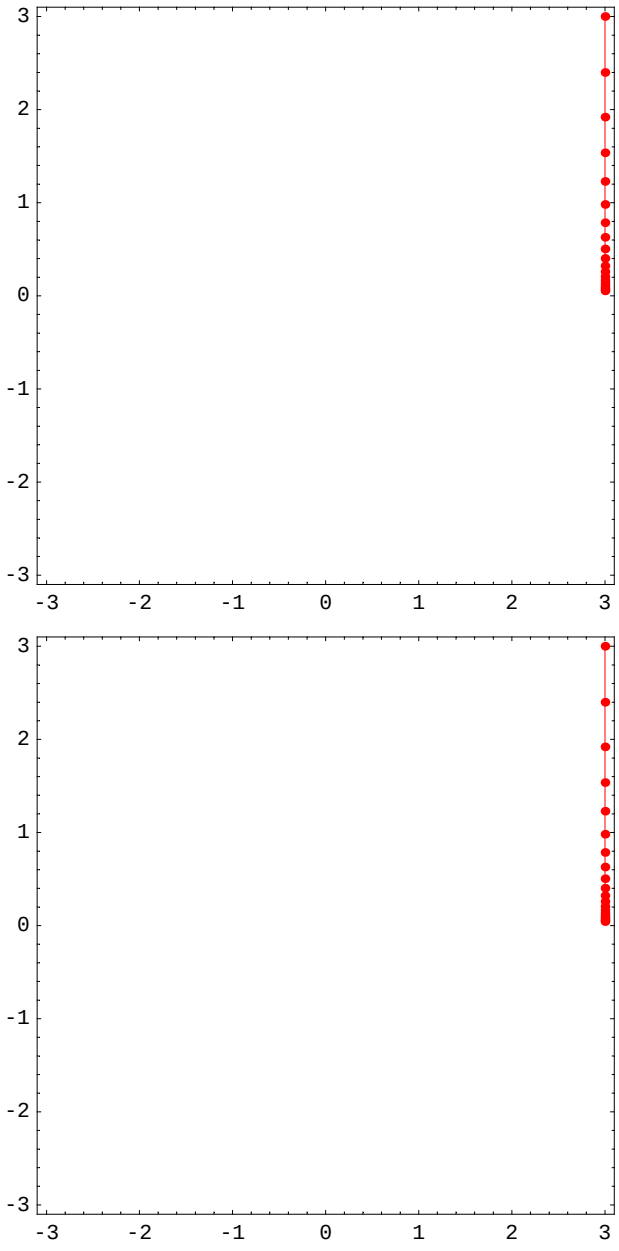
-2
-3
-3 -2 -1 0 1 2 3


3
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-3 -2 -1 0 1 2 3
3
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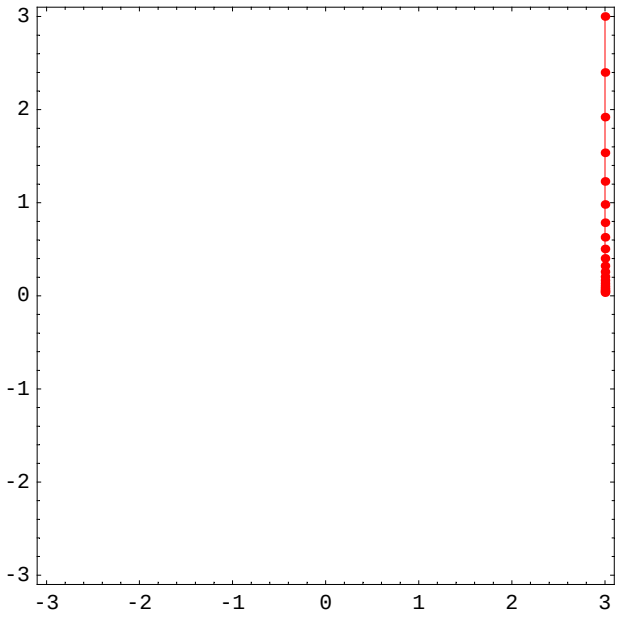
-3 -2 -1 0 1 2 3



Matrix 1b

```
In[15]:= Det[{{1, 2}, {3, 4}}]
Out[15]= -2

In[16]:= mA = {{1, 2}, {3, 4}}.{{1, 0}, {0, .8}}.Inverse[{{1, 2}, {3, 4}}]
Out[16]= {{0.4, 0.2}, {-1.2, 1.4}}

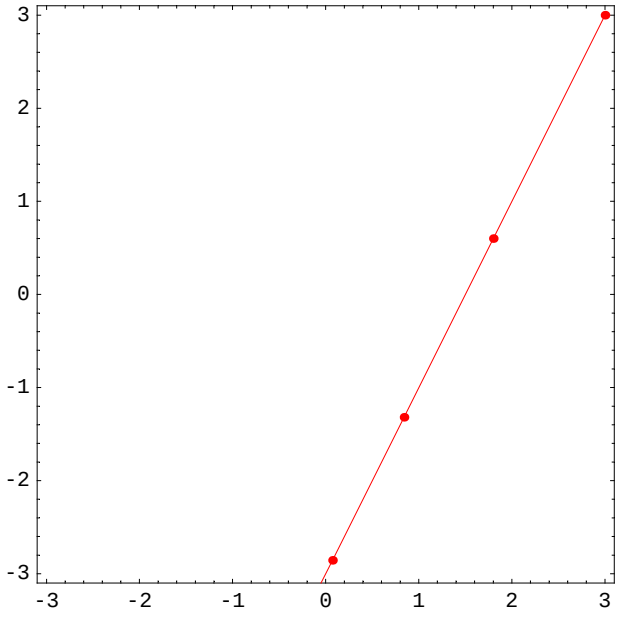
In[17]:= MatrixPower[mA, 3].{3, 3}
Out[17]= {0.072, -2.856}

In[18]:= ips = {{3, 0}, {3, 3}, {1.5, 3}, {0, 3}, {-1.5, 3},
            {-3, 3}, {-3, 0}, {-3, -3}, {-1.5, -3}, {0, -3}, {1.5, -3}, {3, -3}};

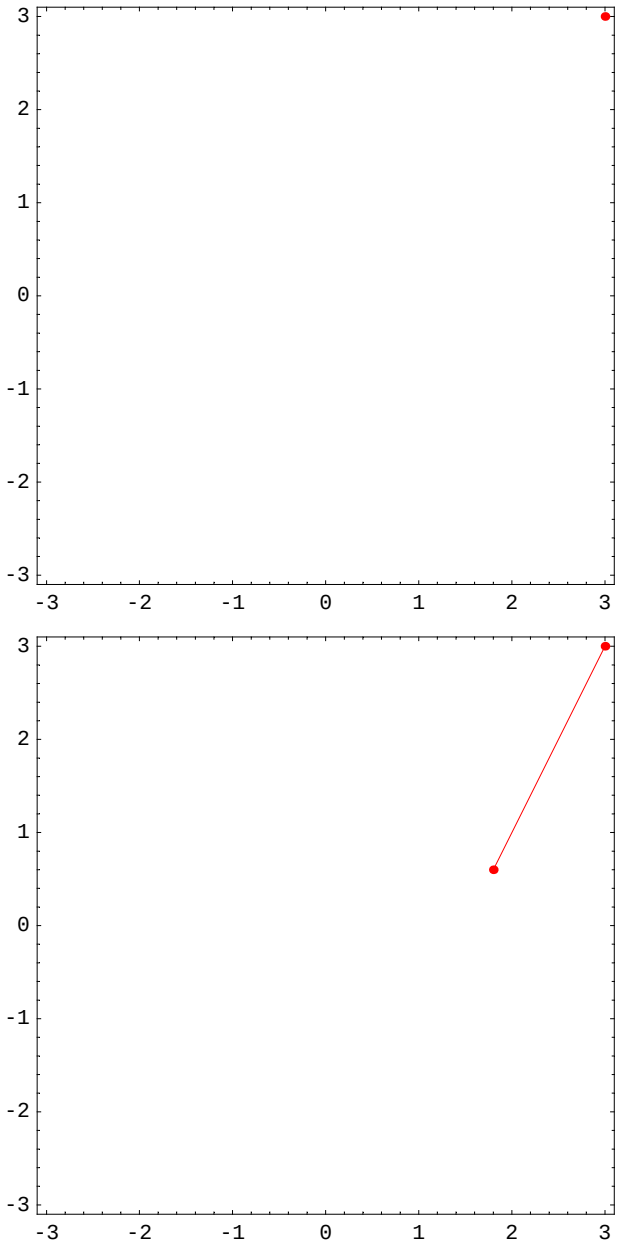

In[19]:= ips = {{3, 3}}
Out[19]= {{3, 3}}
```

In[20]:=
```
steps = 20;
```

```
Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → Automatic, Frame → True
 ];
```



In[22]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
         {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
         {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```


3
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-3 -2 -1 0 1 2 3
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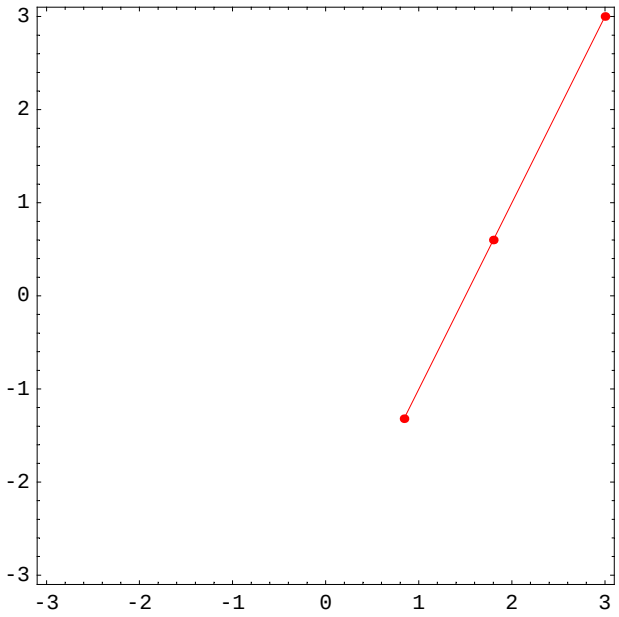
-1
-2
-3
-3 -2 -1 0 1 2 3


3
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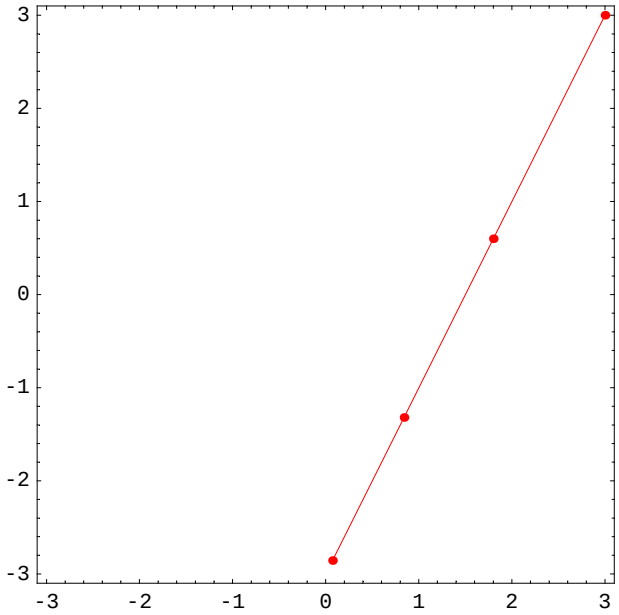
-3 -2 -1 0 1 2 3


3
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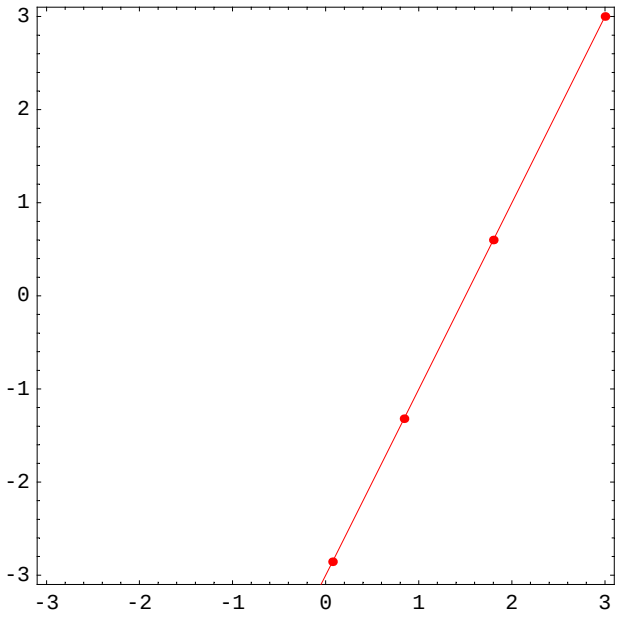
-3 -2 -1 0 1 2 3


3
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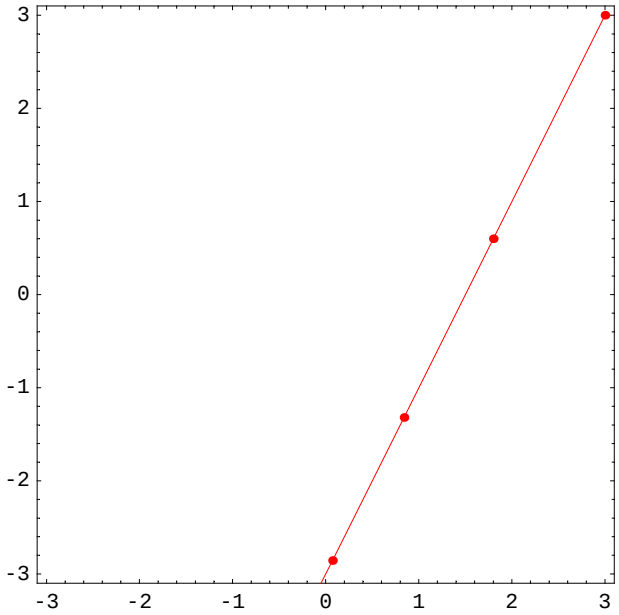
-3
-2
-1
0
1
2
3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-2
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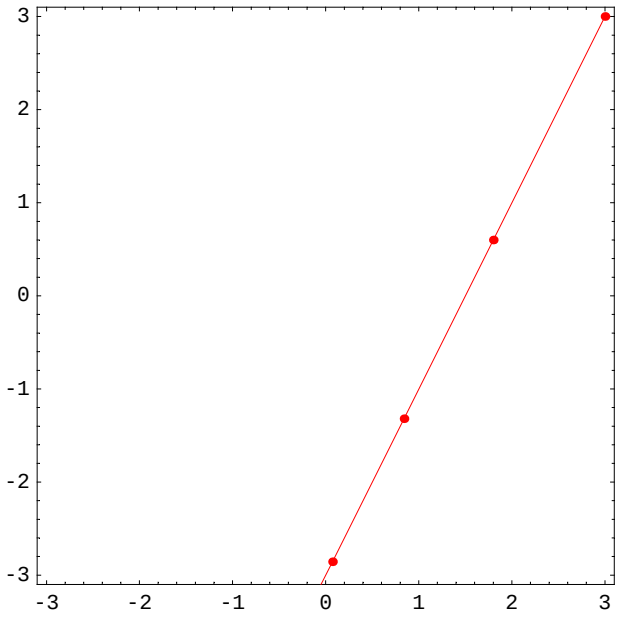
-3
-2
-1
0
1
2
3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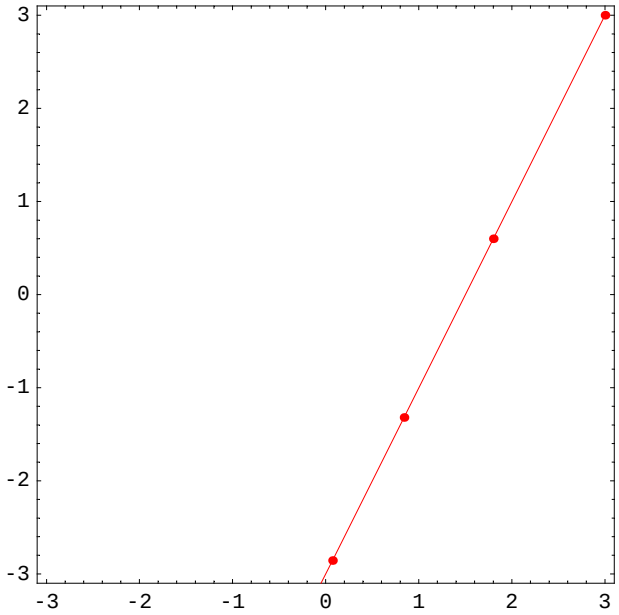
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-2
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0
1
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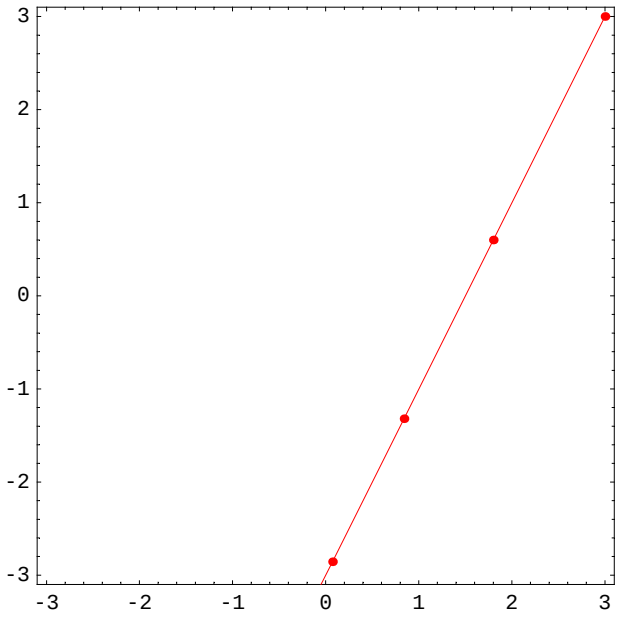
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0
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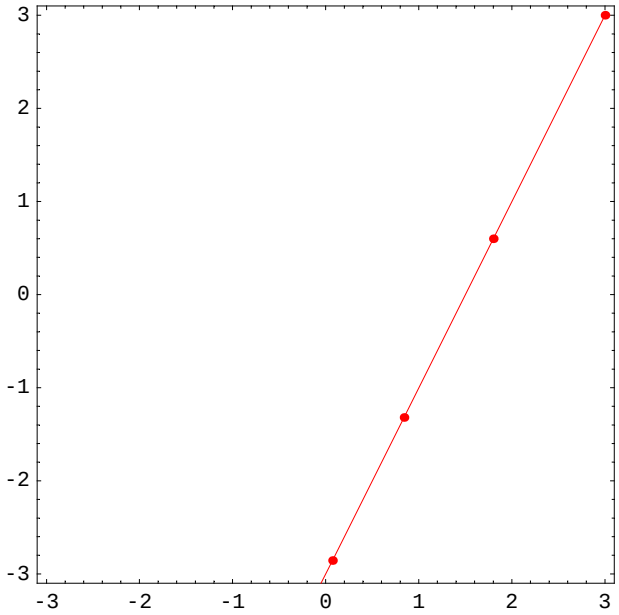
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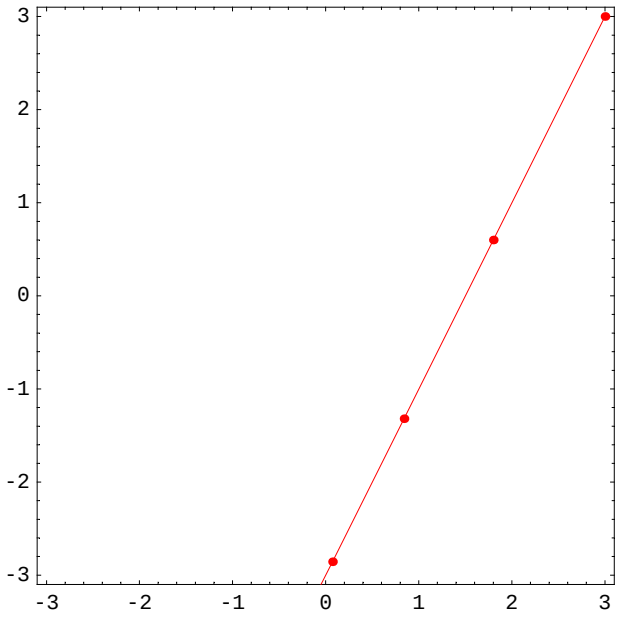
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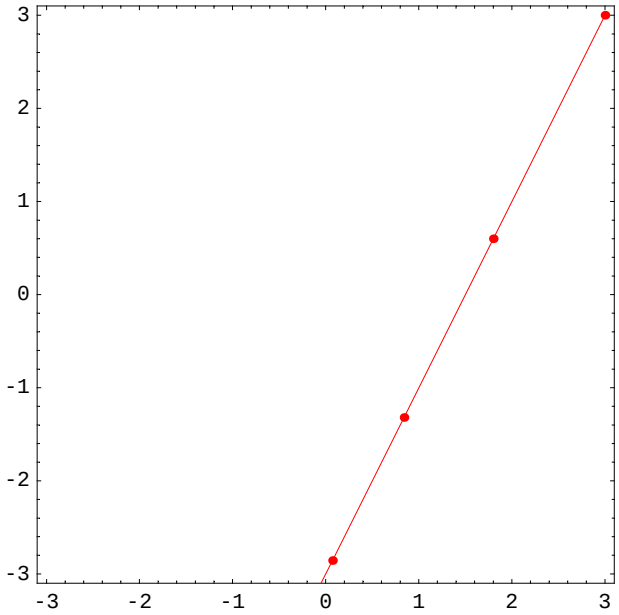
-3
-3 -2 -1 0 1 2 3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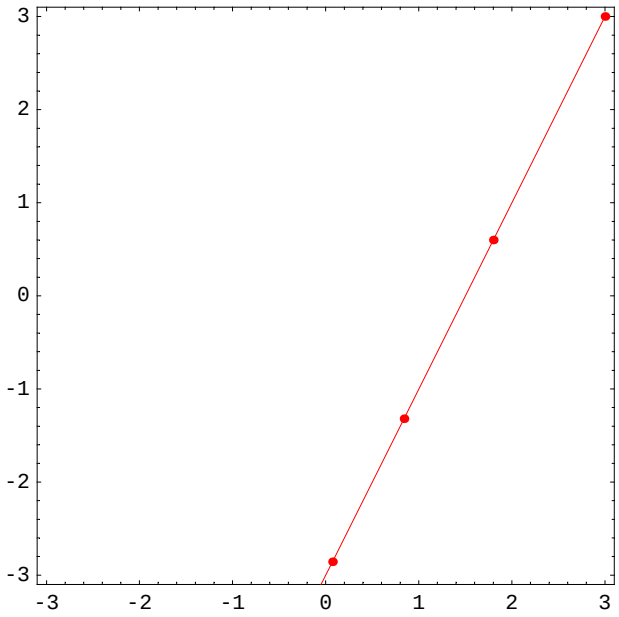
-1
-2
-3
-3 -2 -1 0 1 2 3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-1
-2
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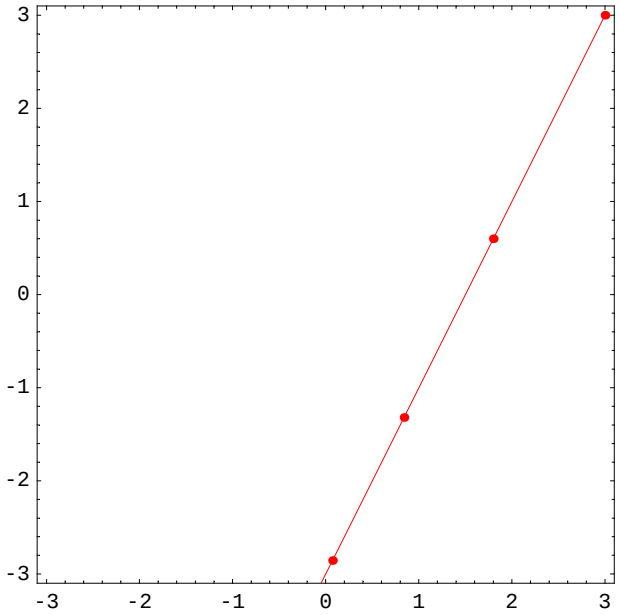
-3 -2 -1 0 1 2 3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-2
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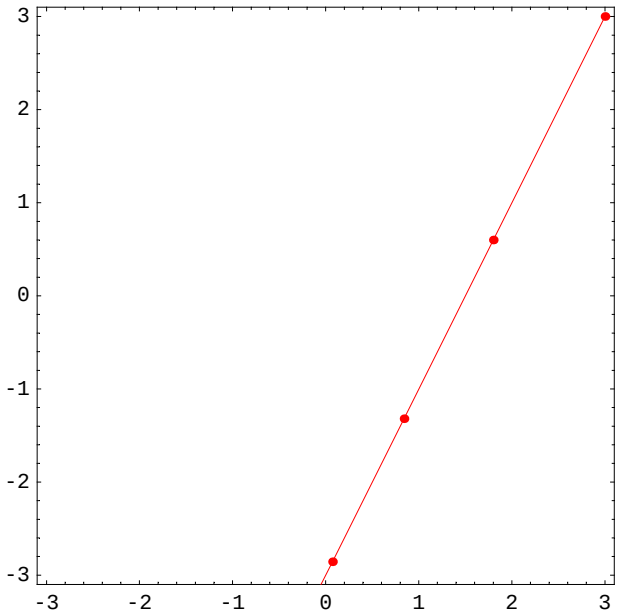
-3 -2 -1 0 1 2 3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2
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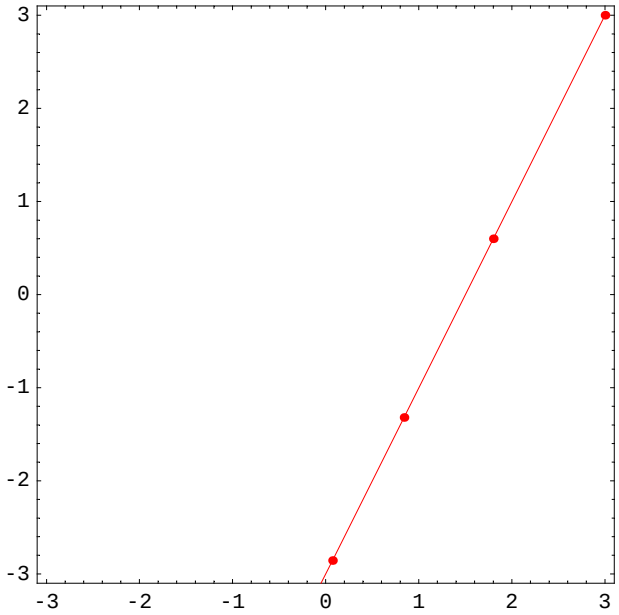
-1
-2
-3
-3 -2 -1 0 1 2 3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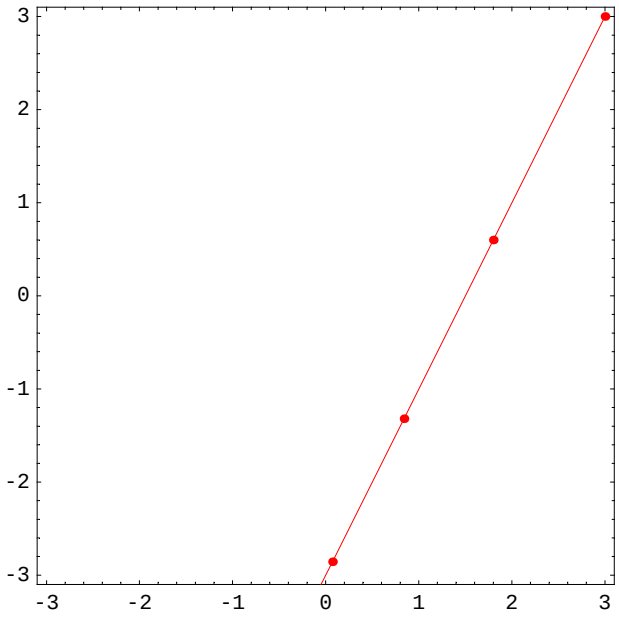
-1
-2
-3
-3 -2 -1 0 1 2 3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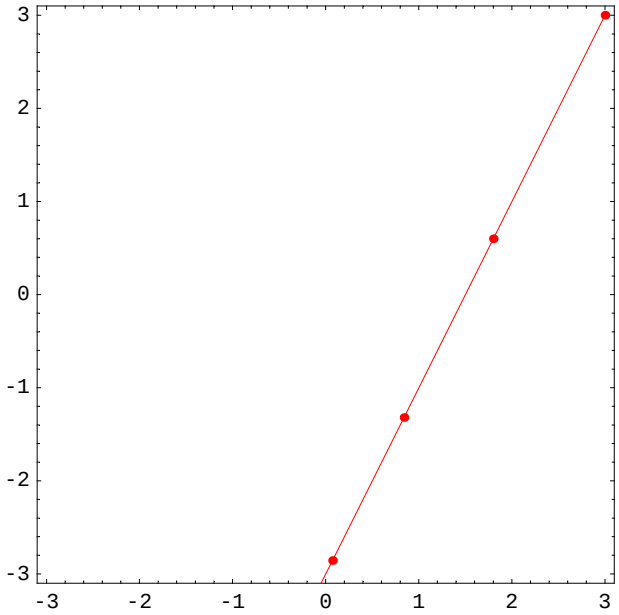
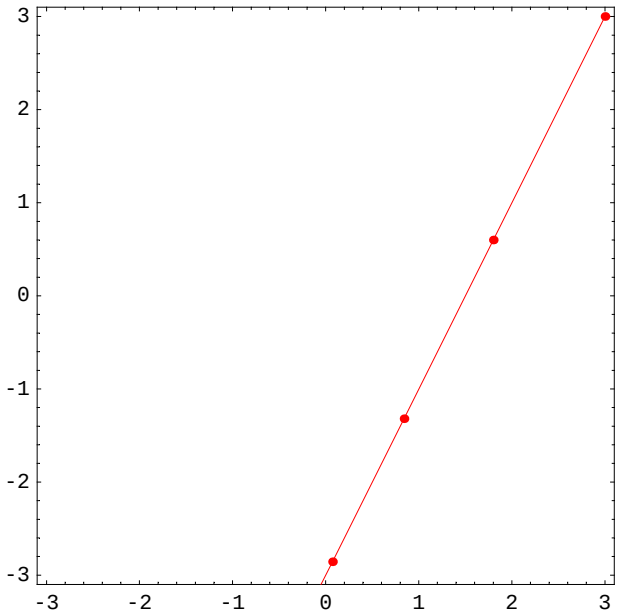
-1
-2
-3
-3
-2
-1
0
1
2
3


3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3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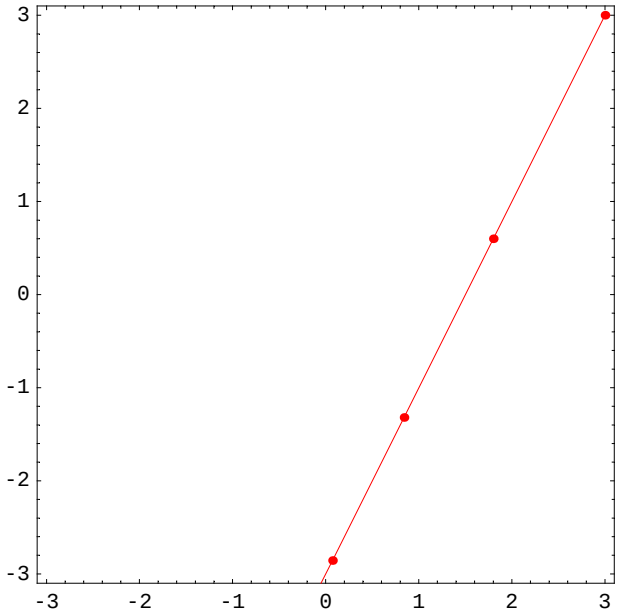
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3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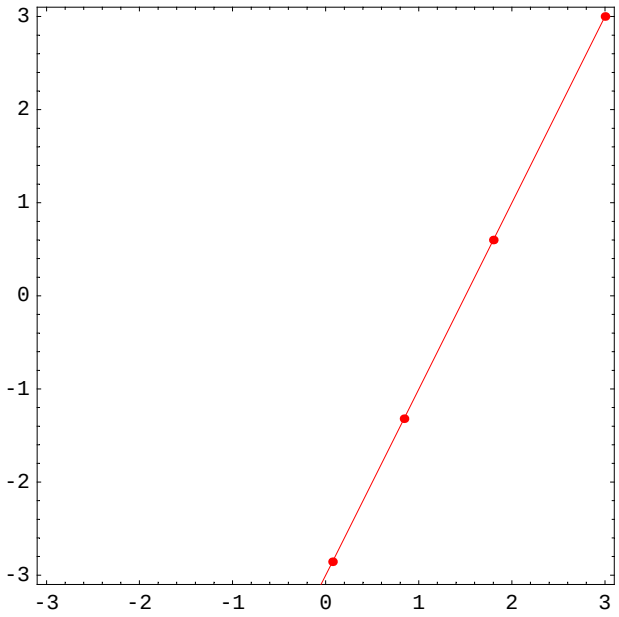
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3



Matrix 2

In[23]:= **mA = {{1.44, 0}, {0, 1.2}}**

Out[23]= {{1.44, 0}, {0, 1.2}}

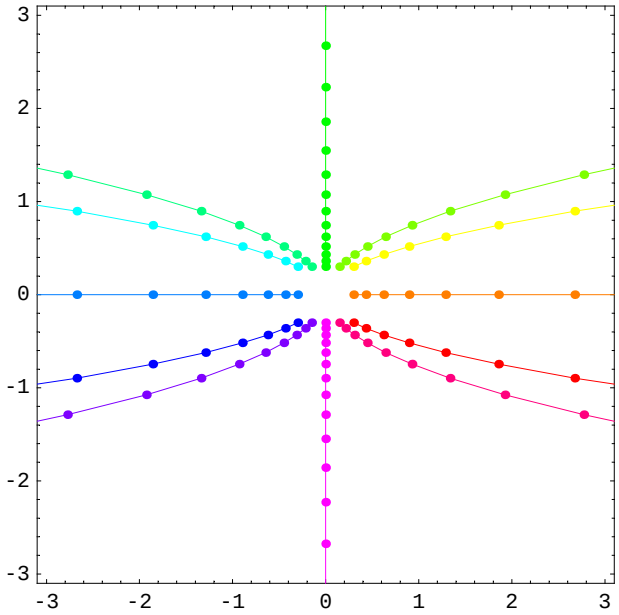
In[24]:= **MatrixPower[mA, 0].{3, 3}**

Out[24]= {3, 3}

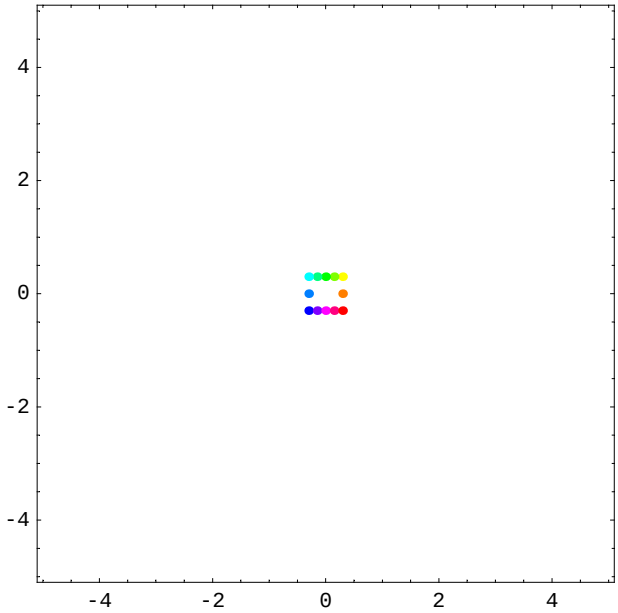
In[25]:=
```
steps = 20;
```

```
ips = {{3, 0}, {3, 3}, {1.5, 3}, {0, 3}, {-1.5, 3}, {-3, 3},
    {-3, 0}, {-3, -3}, {-1.5, -3}, {0, -3}, {1.5, -3}, {3, -3}} / 10;
```

```
Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]] / Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]] / Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → Automatic, Frame → True
 ];
```



In[28]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]] / Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
         {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]] / Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
         {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → {{-5.1, 5.1}, {-5.1, 5.1}}, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```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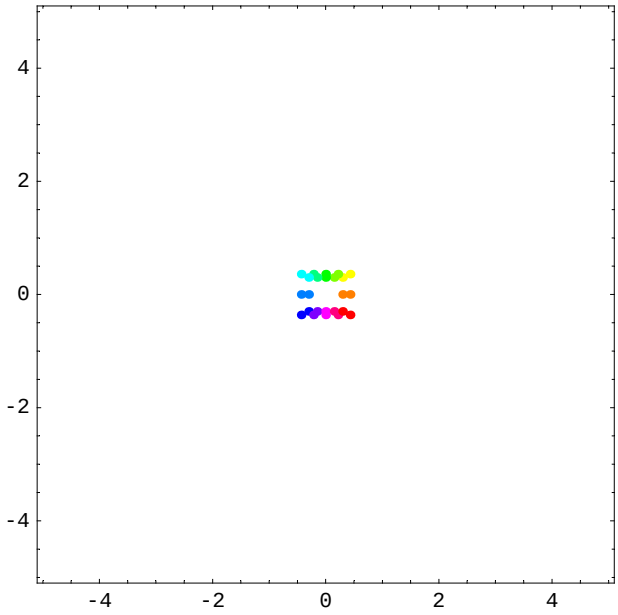
2
0
-2
-4
-4
-2
0
2
4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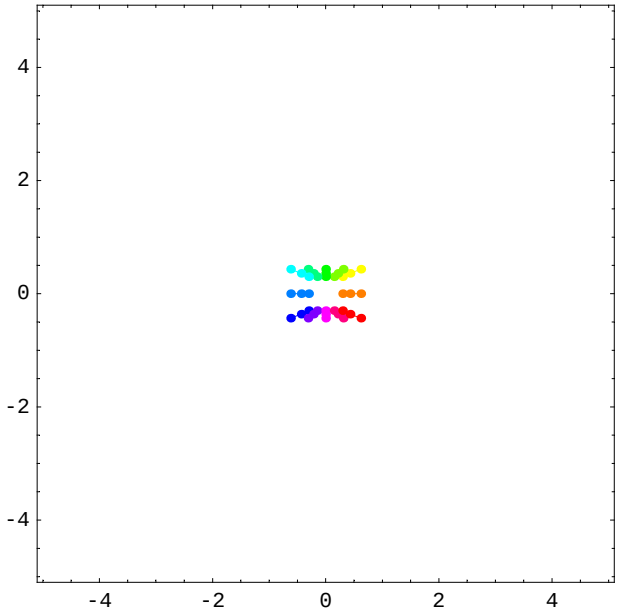
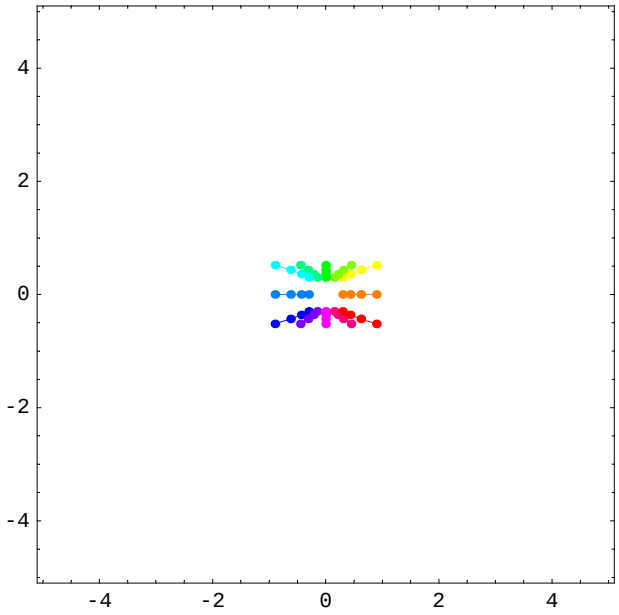
2
0
-2
-4
-4
-2
0
2
4


4
2
0
-2
-4
-4
-2
0
2
4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2
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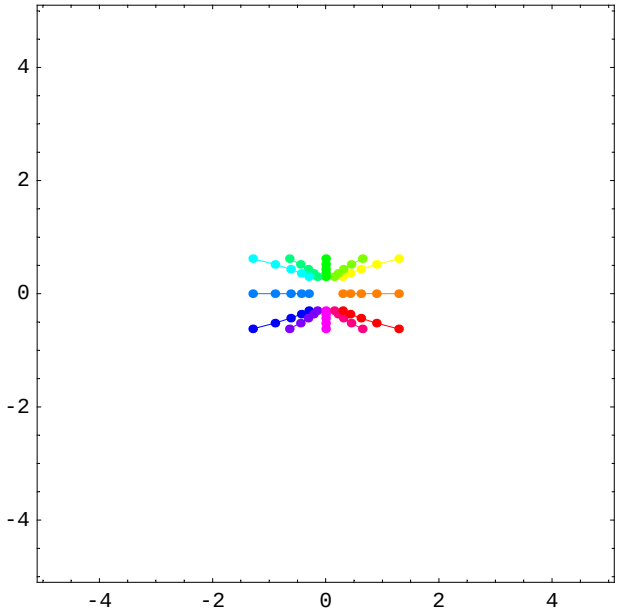
-2
-4
-4
-2
0
2
4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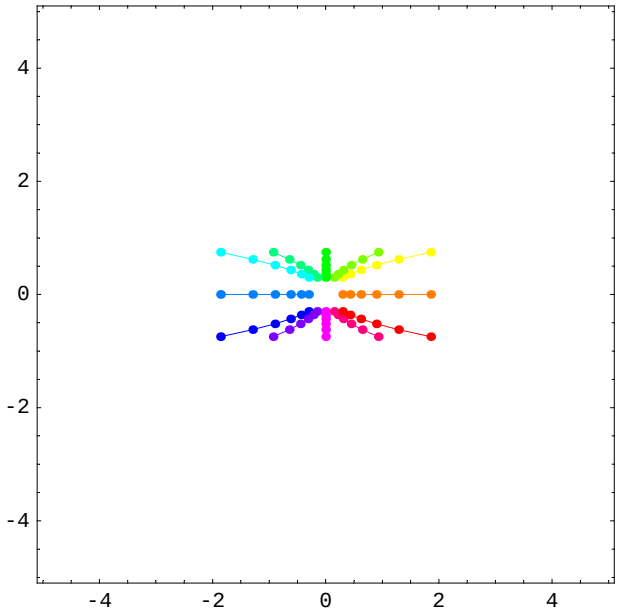
2
0
-2
-4
-4
-2
0
2
4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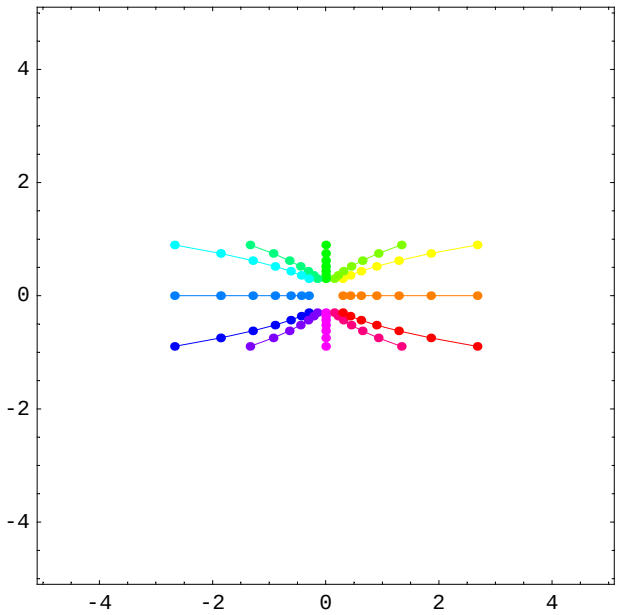
-2
0
2
4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-2
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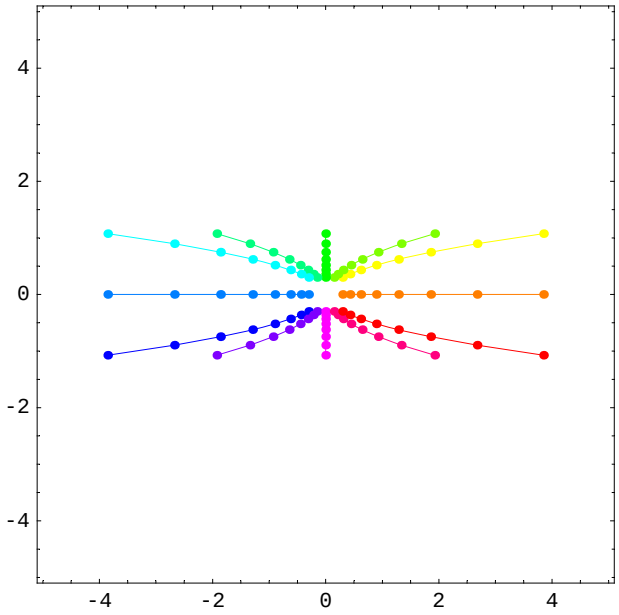
-4
-2
0
2
4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-4
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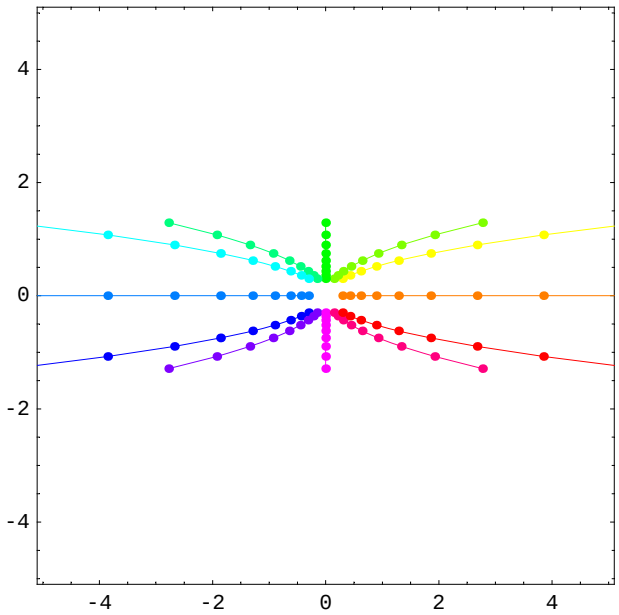
-2
0
2
4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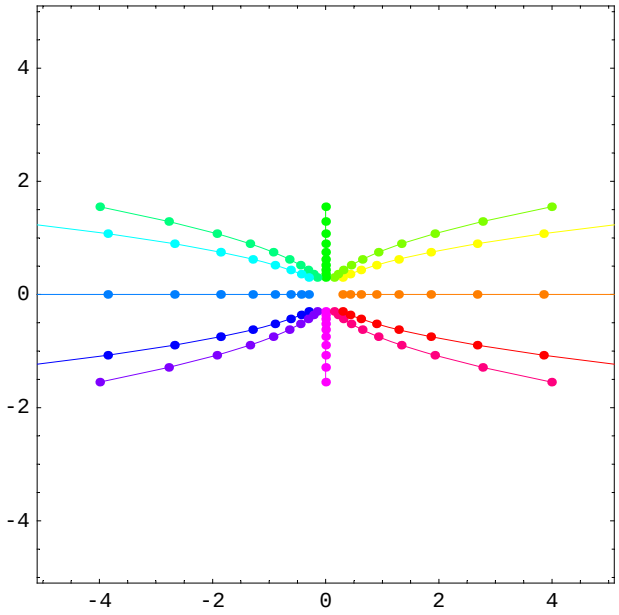
0
2
4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-2
0
2
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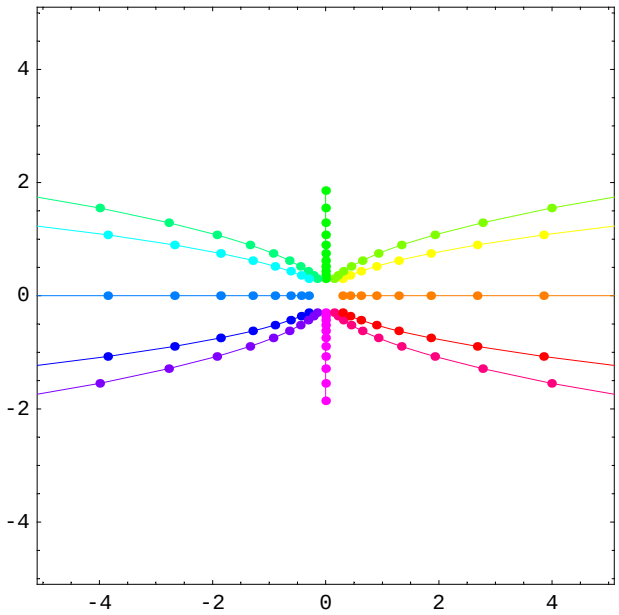
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2
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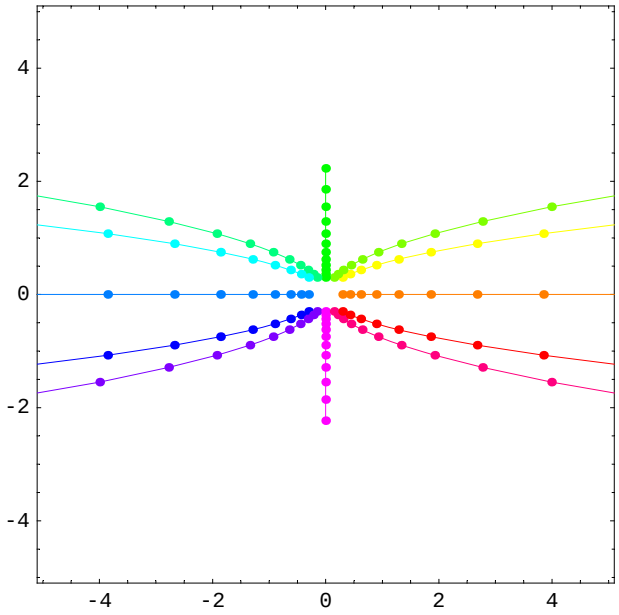
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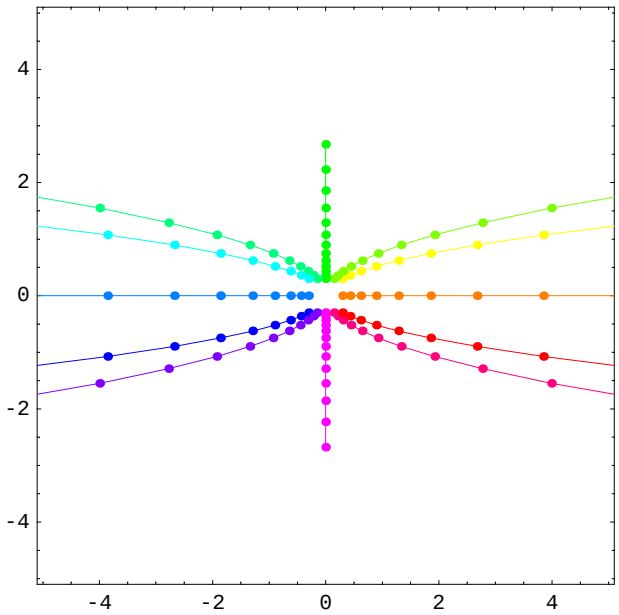
0
2
4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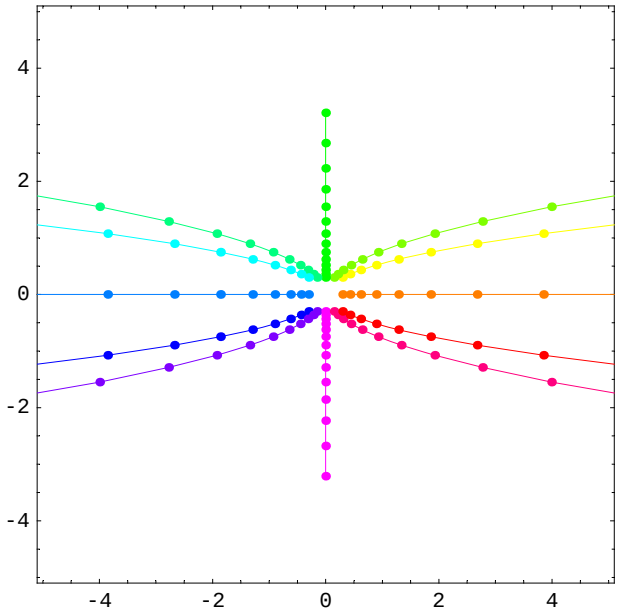
-4
-4
-2
0
2
4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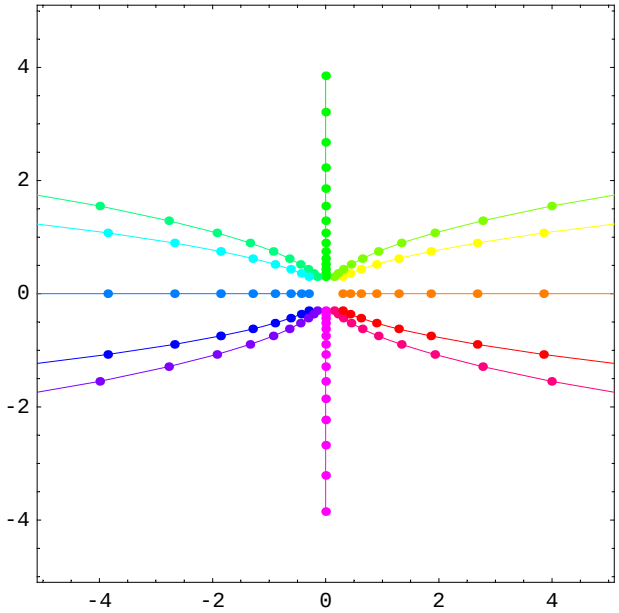
2
4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-2
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2
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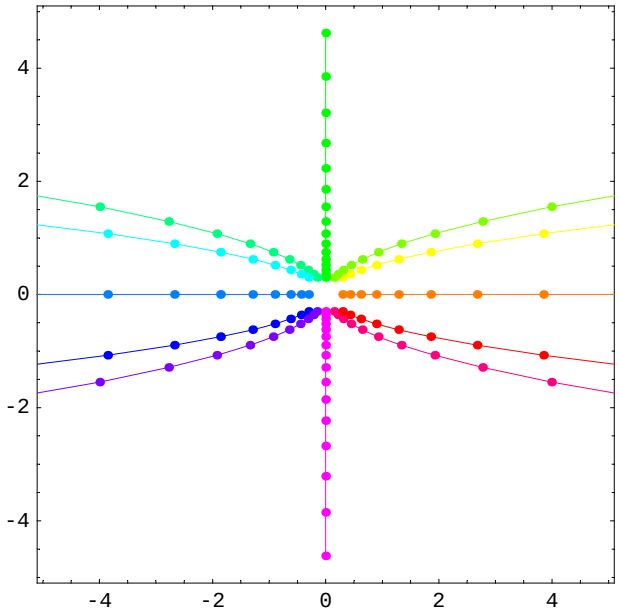
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-4
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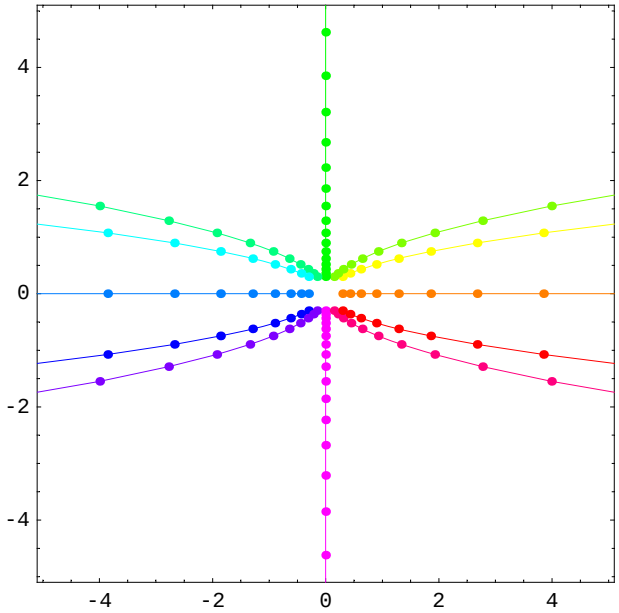
-2
0
2
4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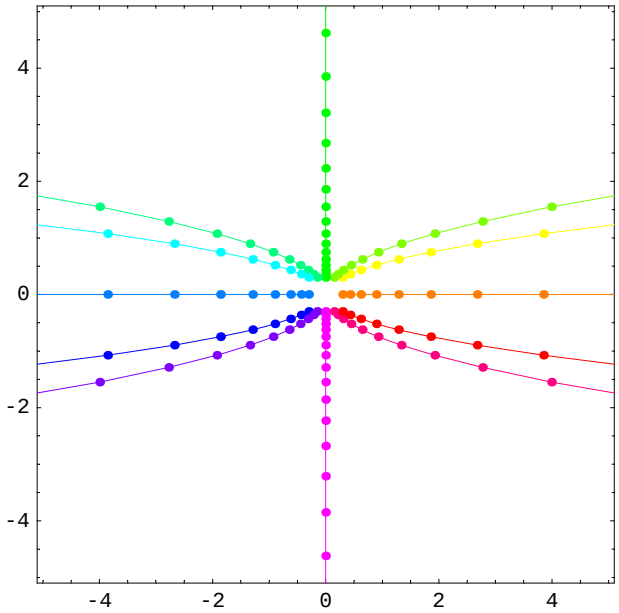
-2
-4
-4
-2
0
2
4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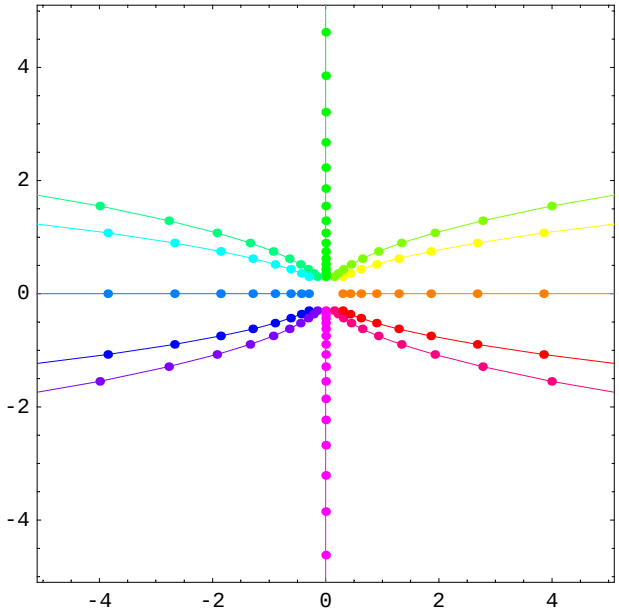
-2
-4
-4
-2
0
2
4


4
2
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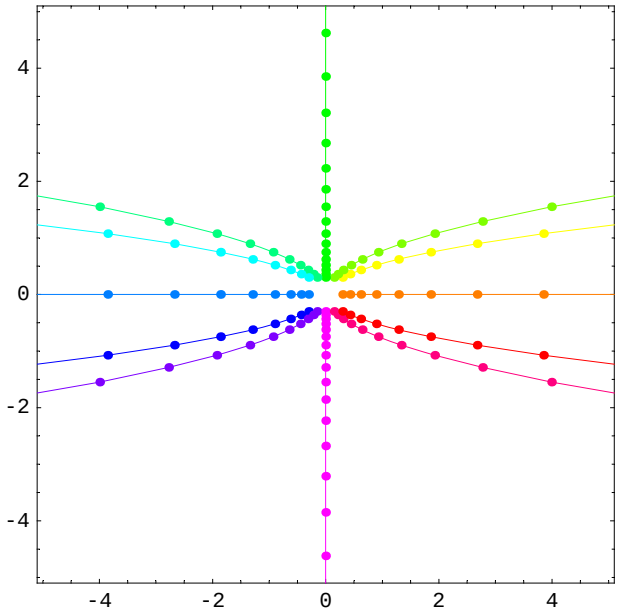
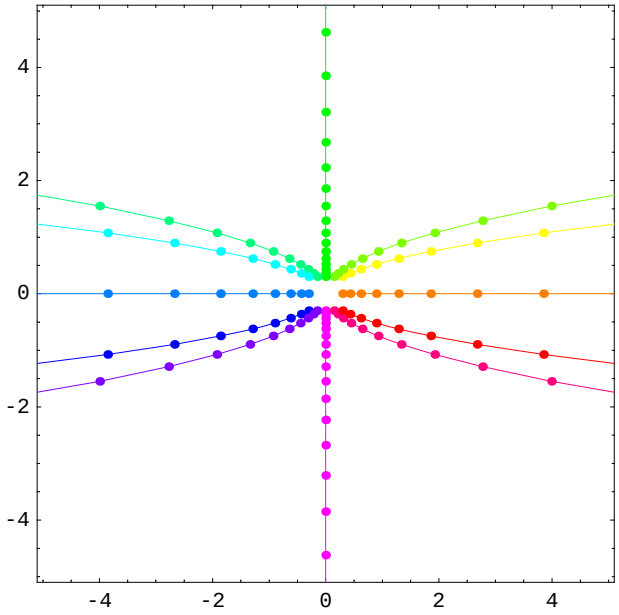
-2
-4
-4
-2
0
2
4


4
2
0
-2
-4
-4
-2
0
2
4



Matrix 3

In[29]:= **mA = {{1.8, 0}, {0, .64}}**

Out[29]= {{1.8, 0}, {0, 0.64}}

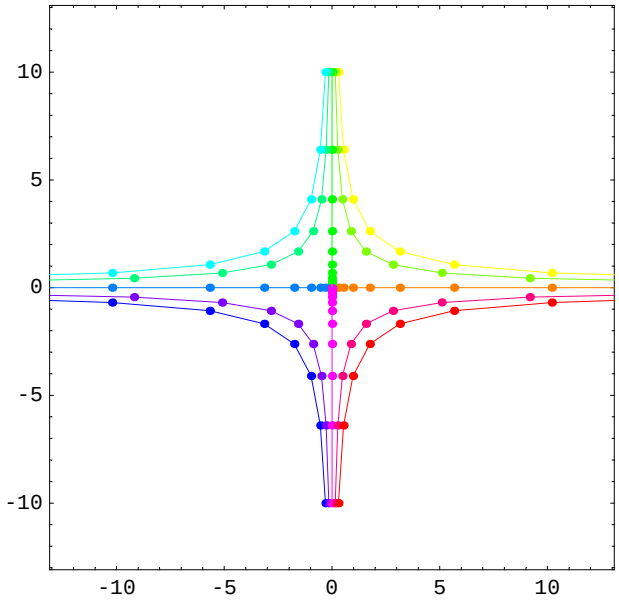
In[30]:= **MatrixPower[mA, 0].{3, 3}**

Out[30]= {3, 3}

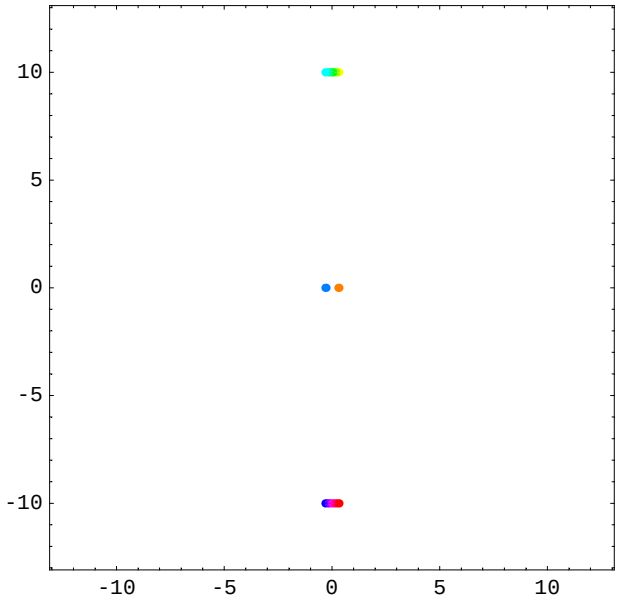
In[31]:=
```
steps = 20;
```

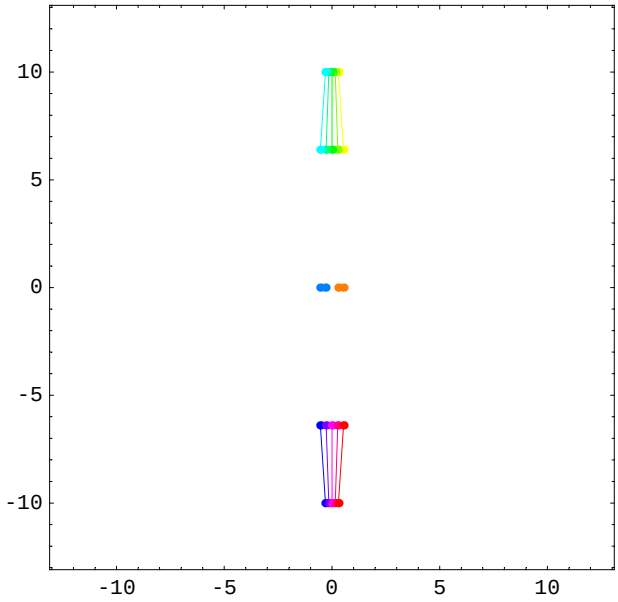
```
ips = {{.3, 0}, {.3, 10}, {0.15, 10}, {0, 10}, {-.15, 10}, {-.3, 10},
   {-.3, 0}, {-.3, -10}, {-.15, -10}, {0, -10}, {.15, -10}, {.3, -10}};
```

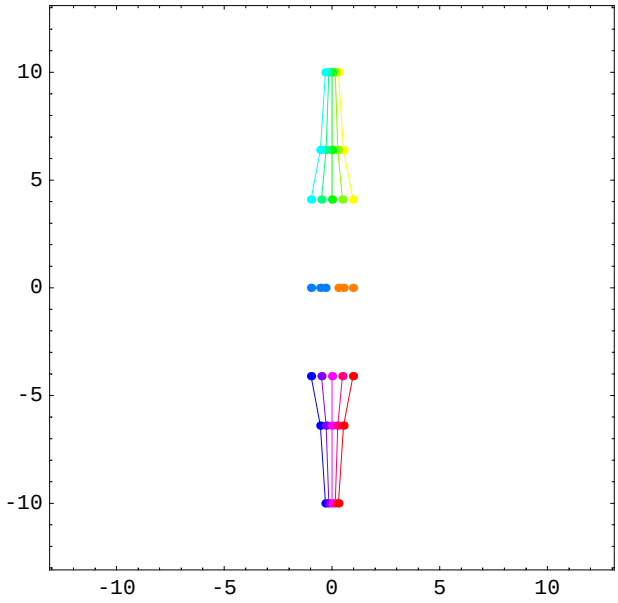
```
Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → {{-13.1, 13.1}, {-13.1, 13.1}}, AspectRatio → Automatic, Frame → True
 ];
```



In[34]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
         {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
         {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → {{-13.1, 13.1}, {-13.1, 13.1}}, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```


10
5
0
-5
-10
-10
-5
0
5
10


10
5
0
-5
-10
-10
-5
0
5
10


10
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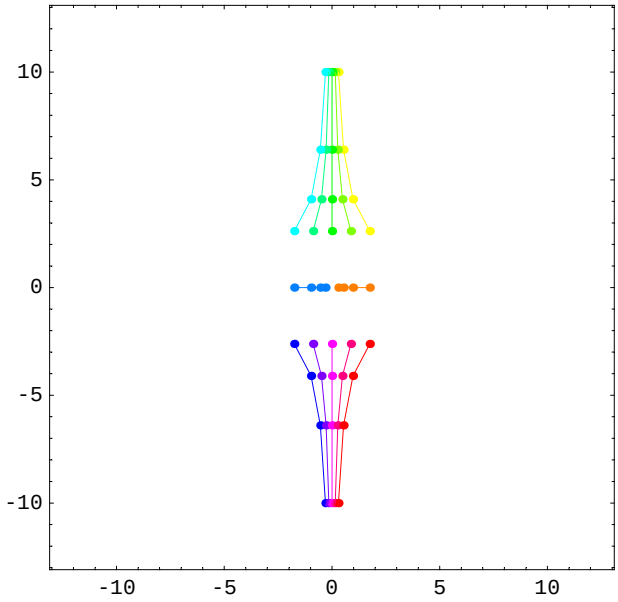
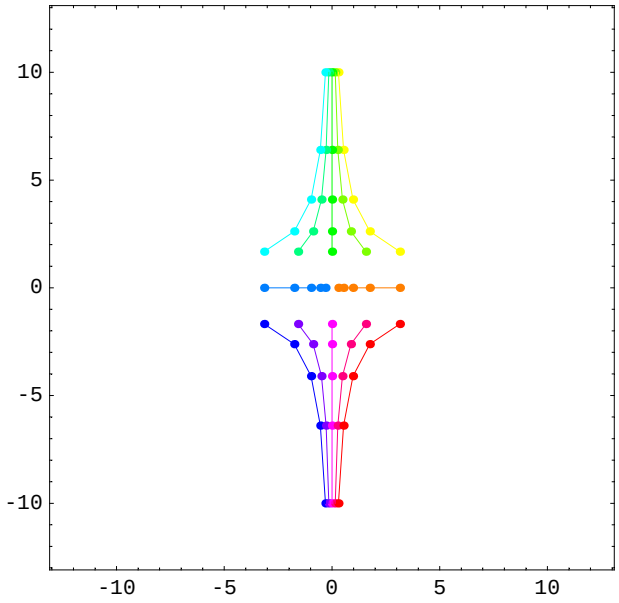
0
-5
-10
-10
-5
0
5
10


10
5
0
-5
-10
-10
-5
0
5
10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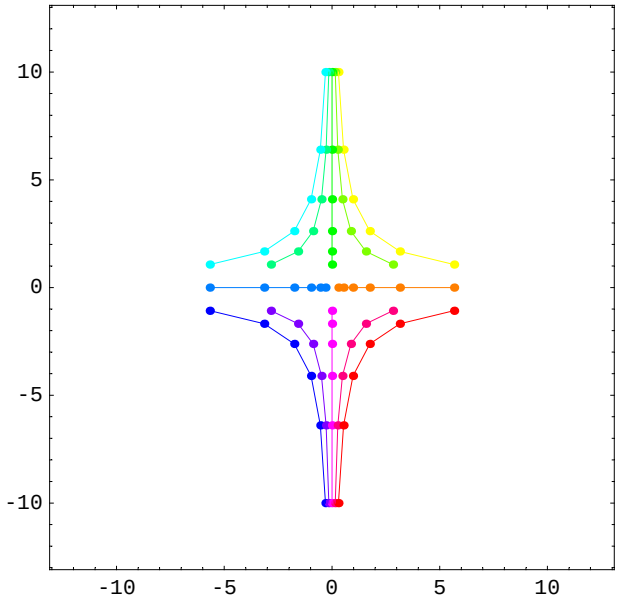
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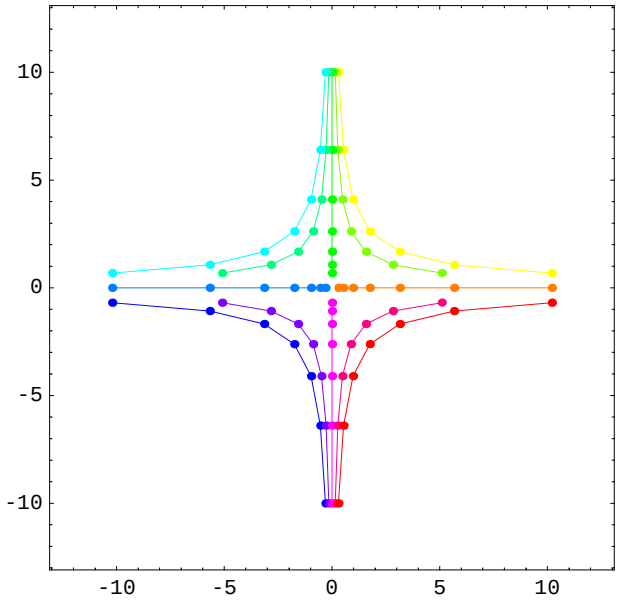
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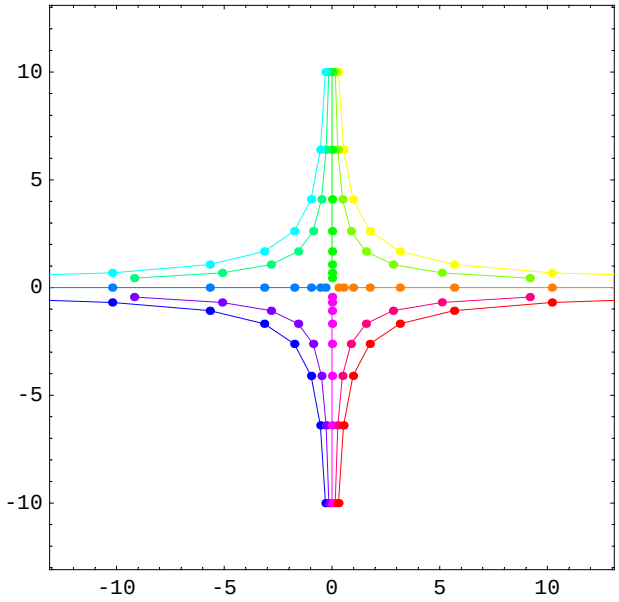
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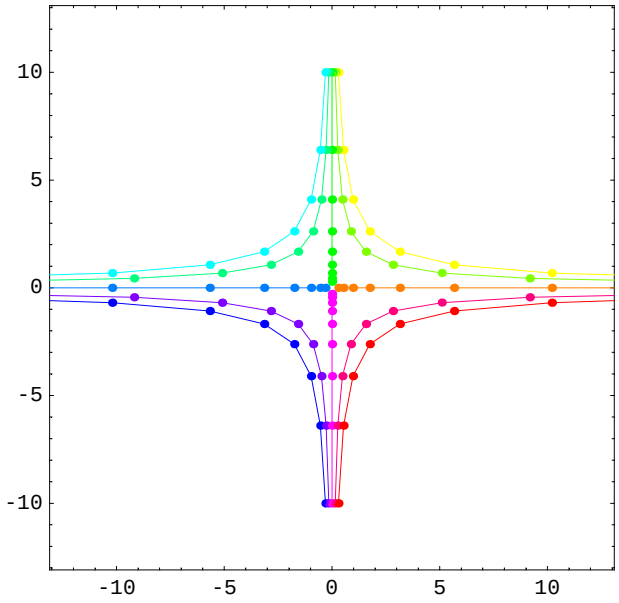
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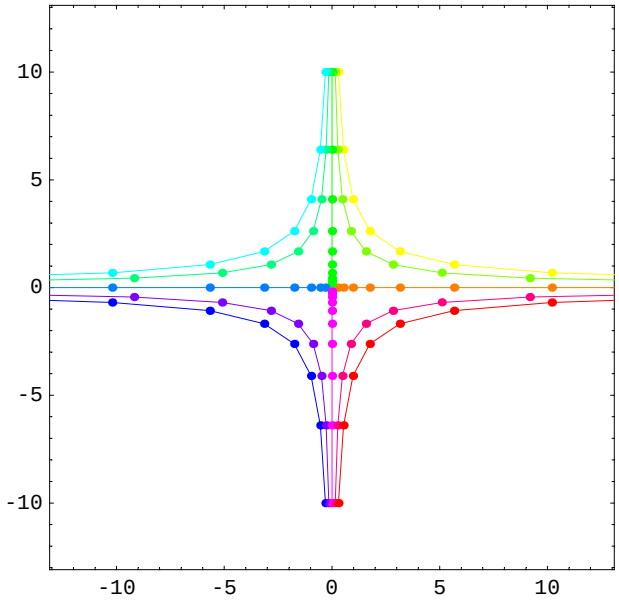
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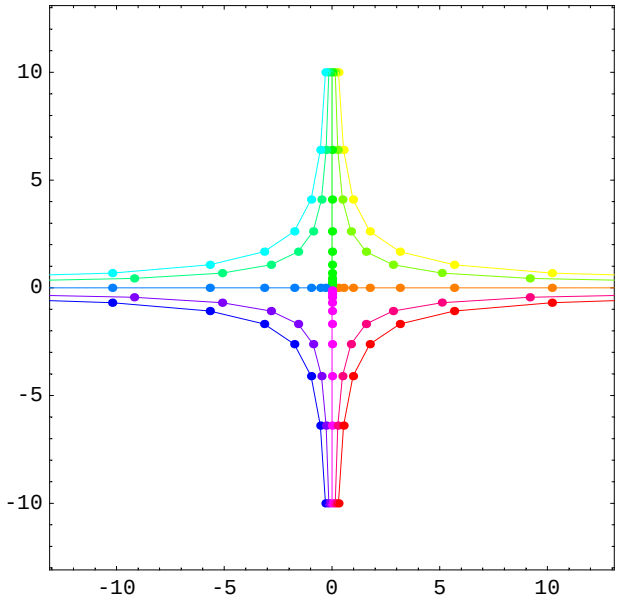
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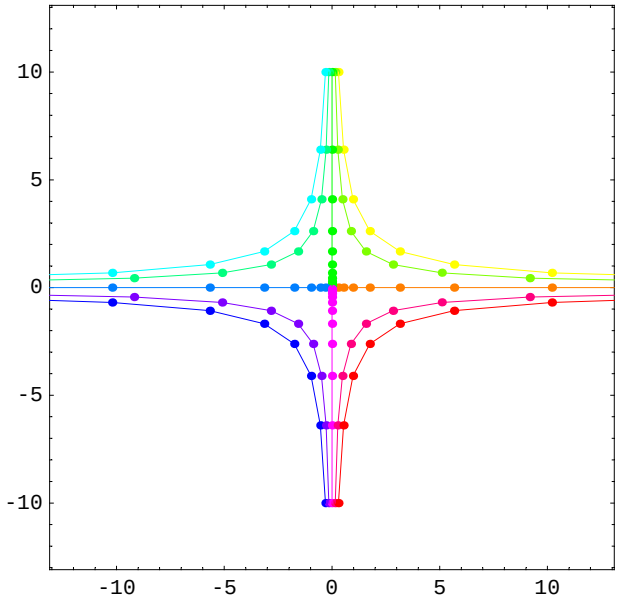
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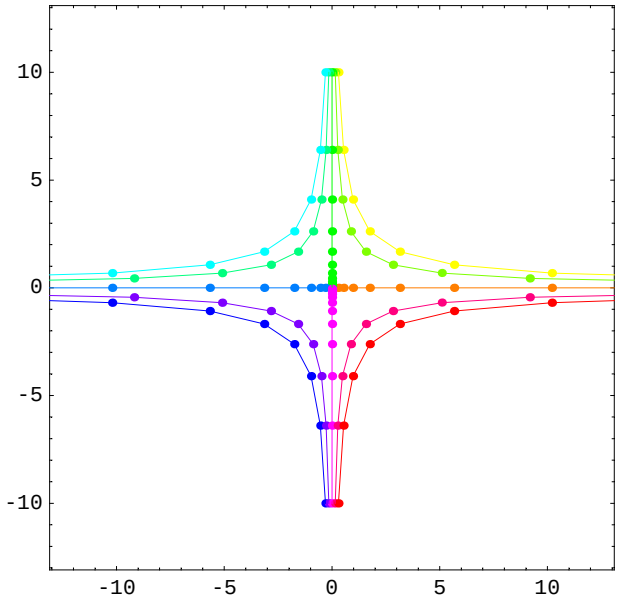
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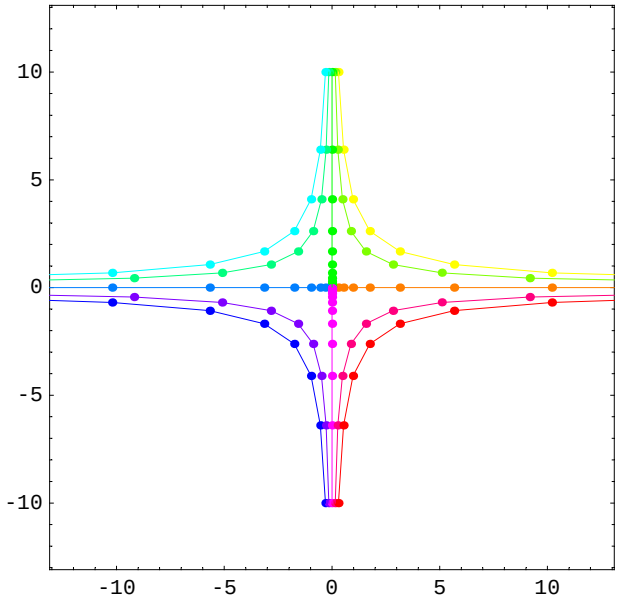
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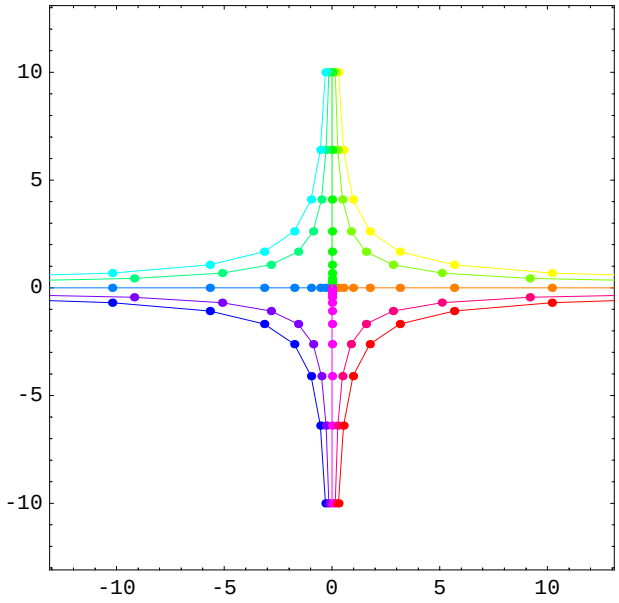
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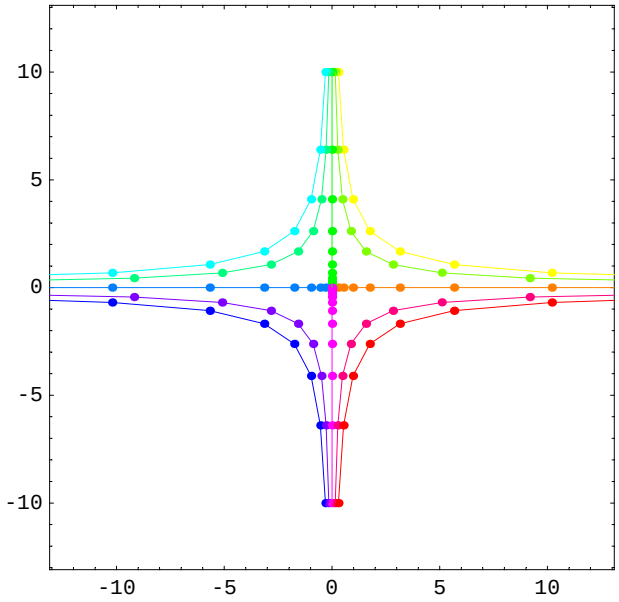
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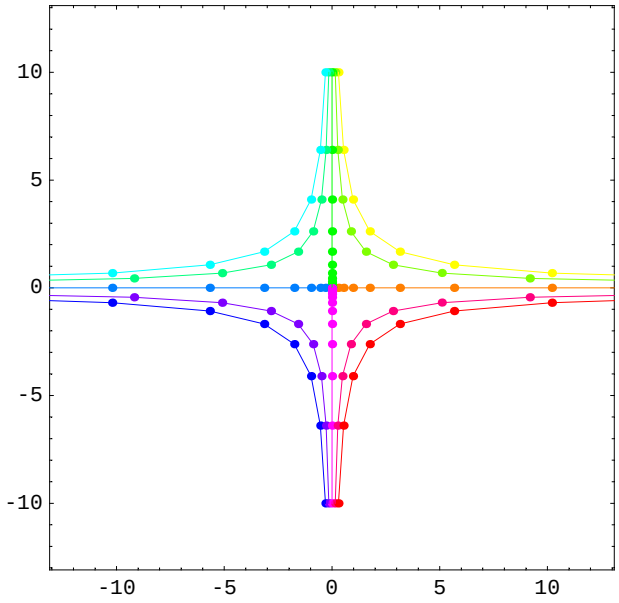
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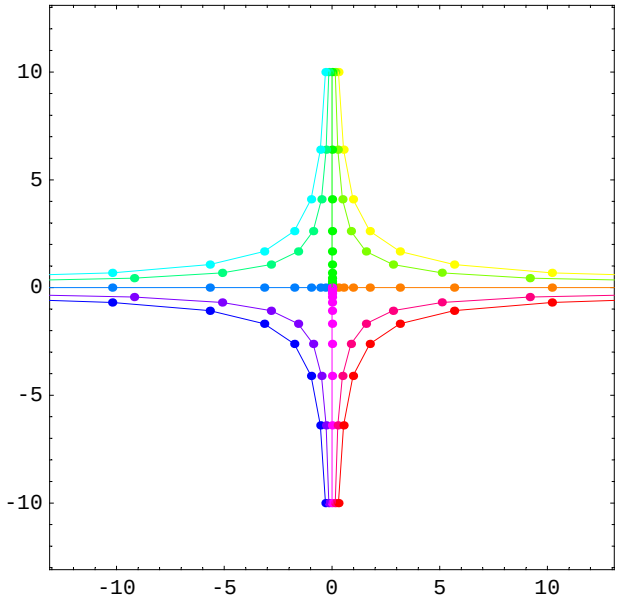
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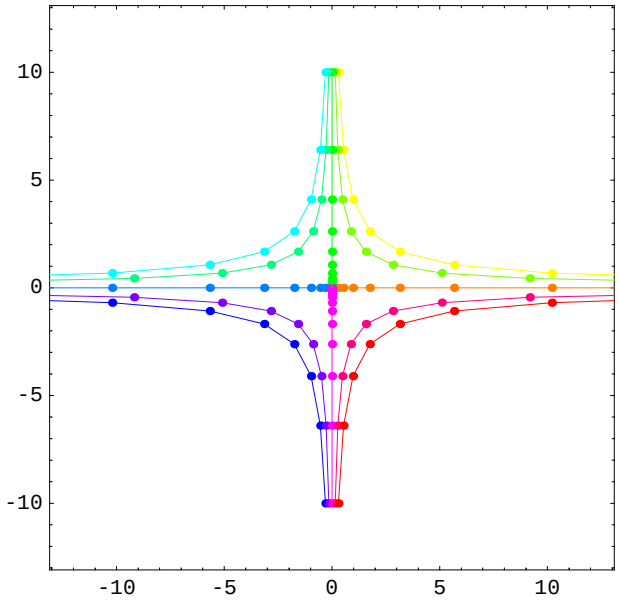
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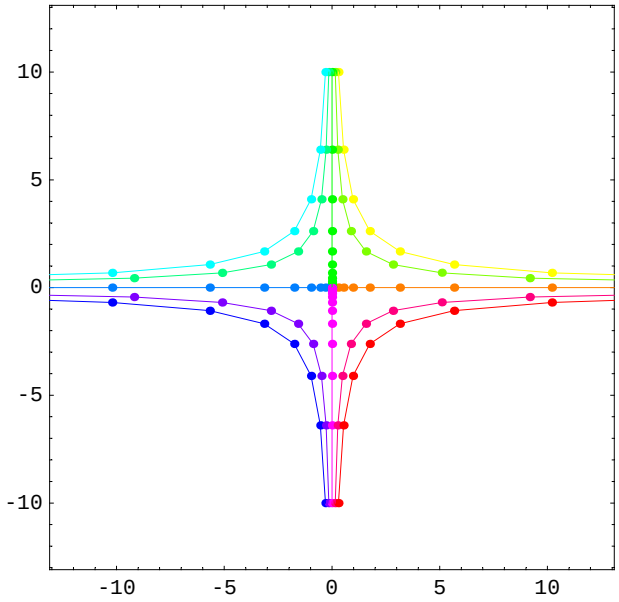
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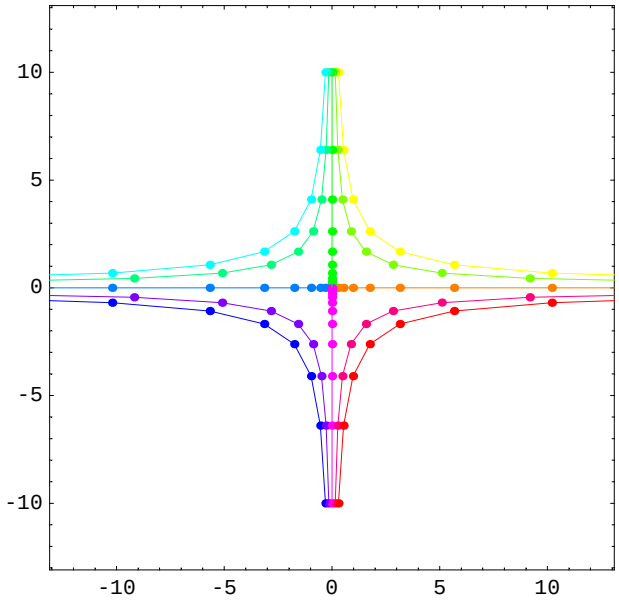


Matrix 4

In[35]:=
```
mA = {{.8, 0.5}, {-0.1, 1}}
```

Out[35]=
```
{{0.8, 0.5}, {-0.1, 1}}
```

In[36]:=
```
MatrixPower[mA, 0].{3, 3}
```

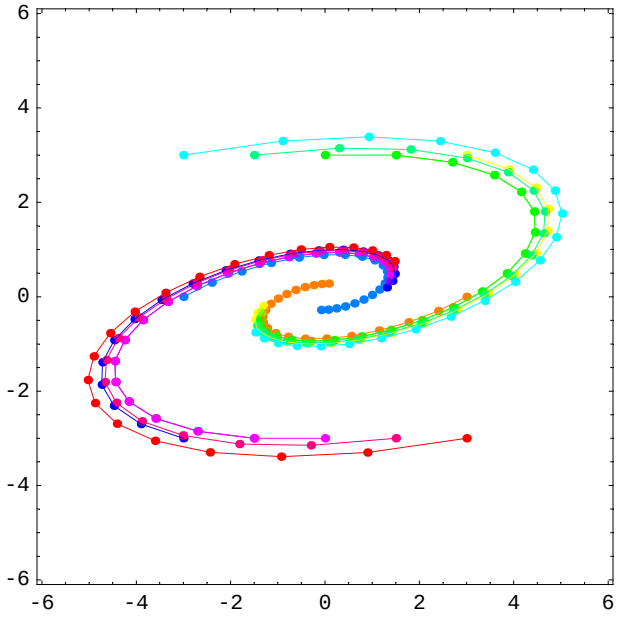
Out[36]=
```
{3, 3}
```

In[37]:=
```
steps = 20;

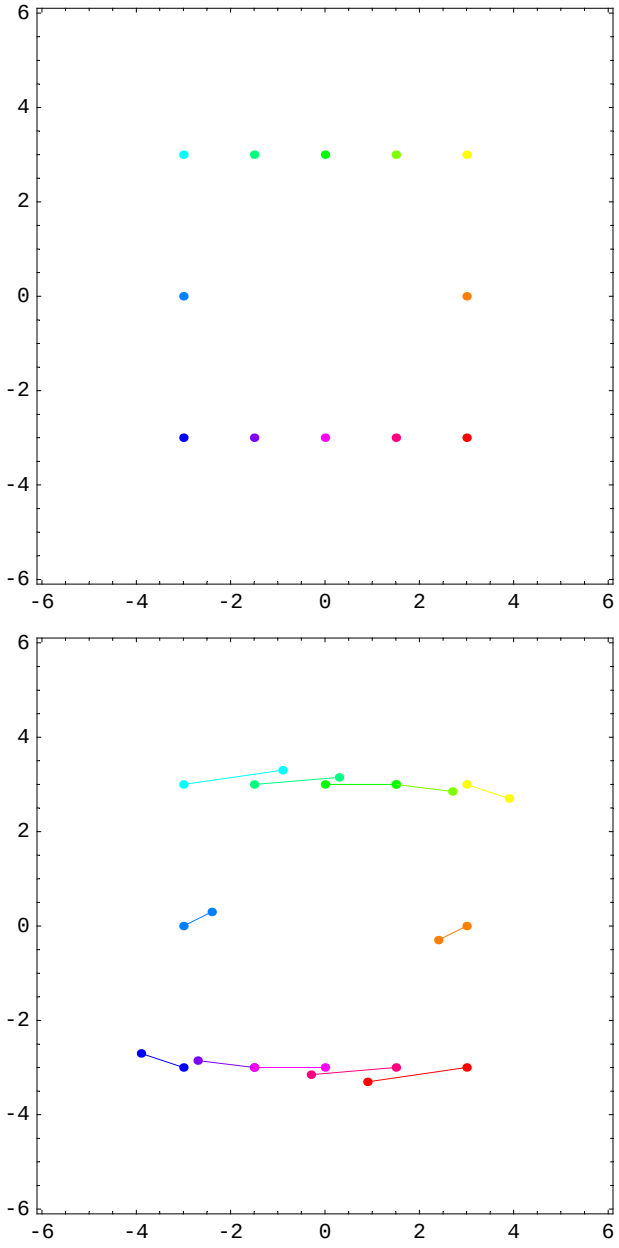
ips = {{3, 0}, {3, 3}, {1.5, 3}, {0, 3}, {-1.5, 3},
   {-3, 3}, {-3, 0}, {-3, -3}, {-1.5, -3}, {0, -3}, {1.5, -3}, {3, -3}};

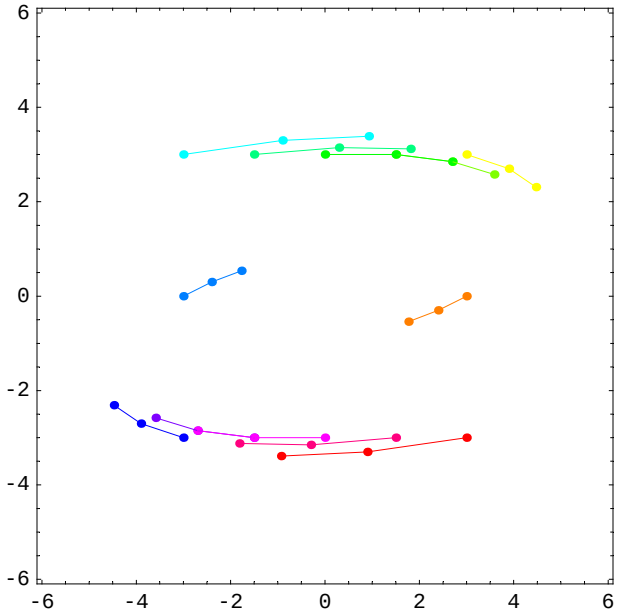
PR = {{-6.1, 6.1}, {-6.1, 6.1}};

Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → PR, AspectRatio → Automatic, Frame → True
 ];
```



In[41]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → PR, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```

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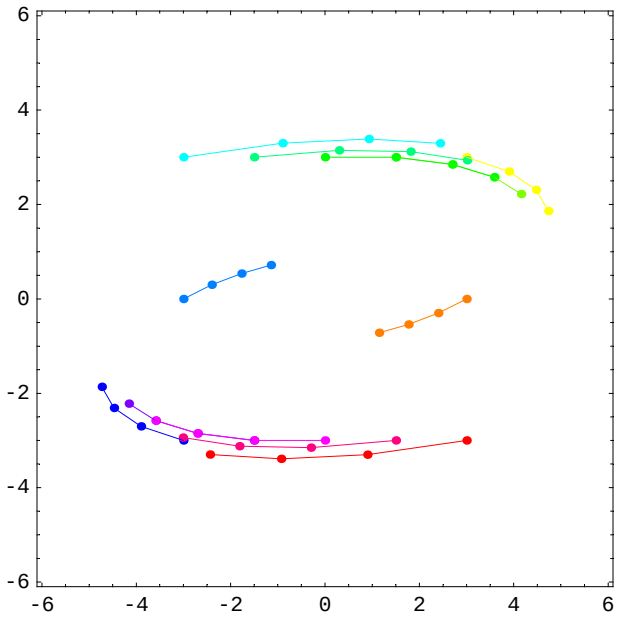
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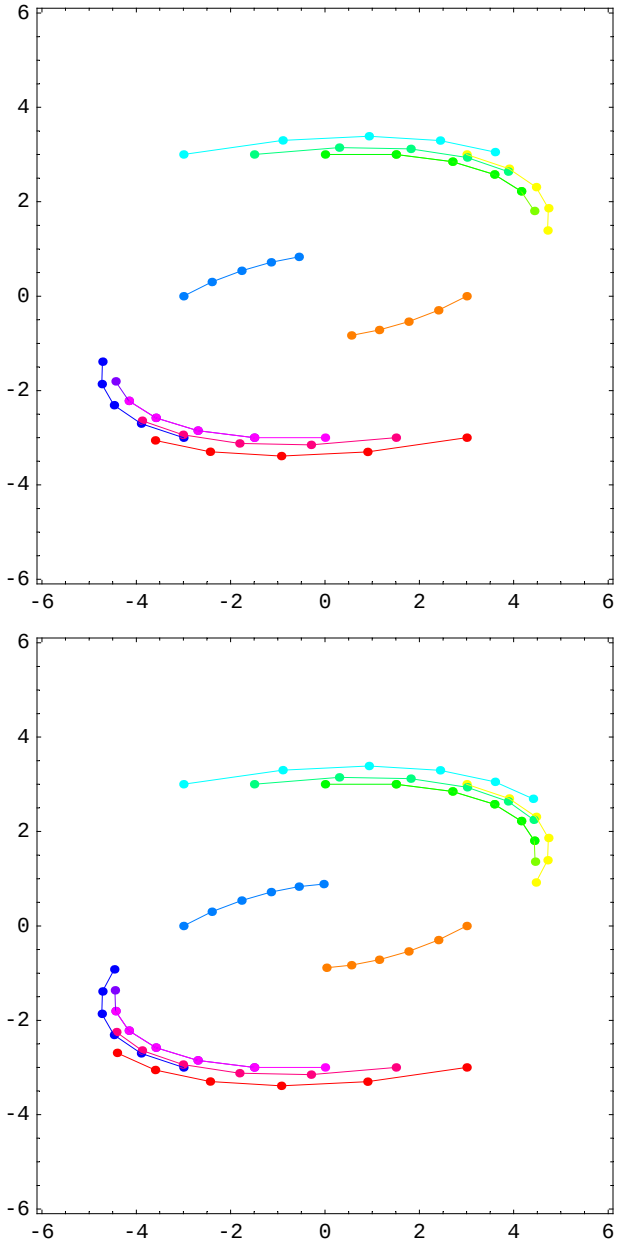
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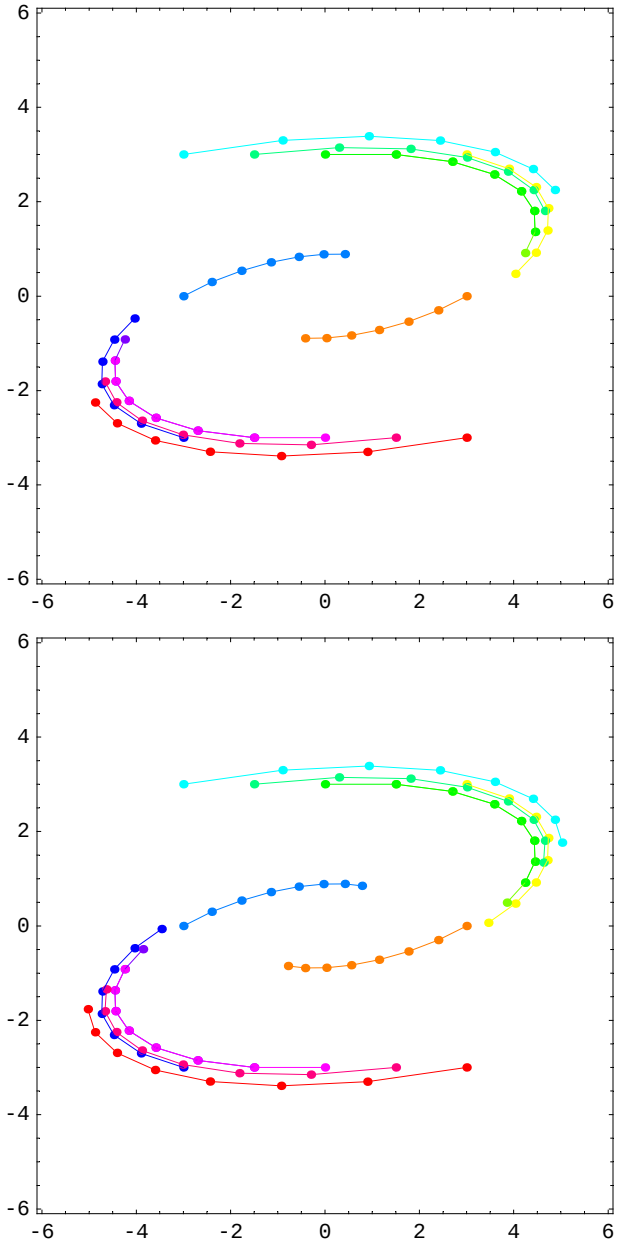
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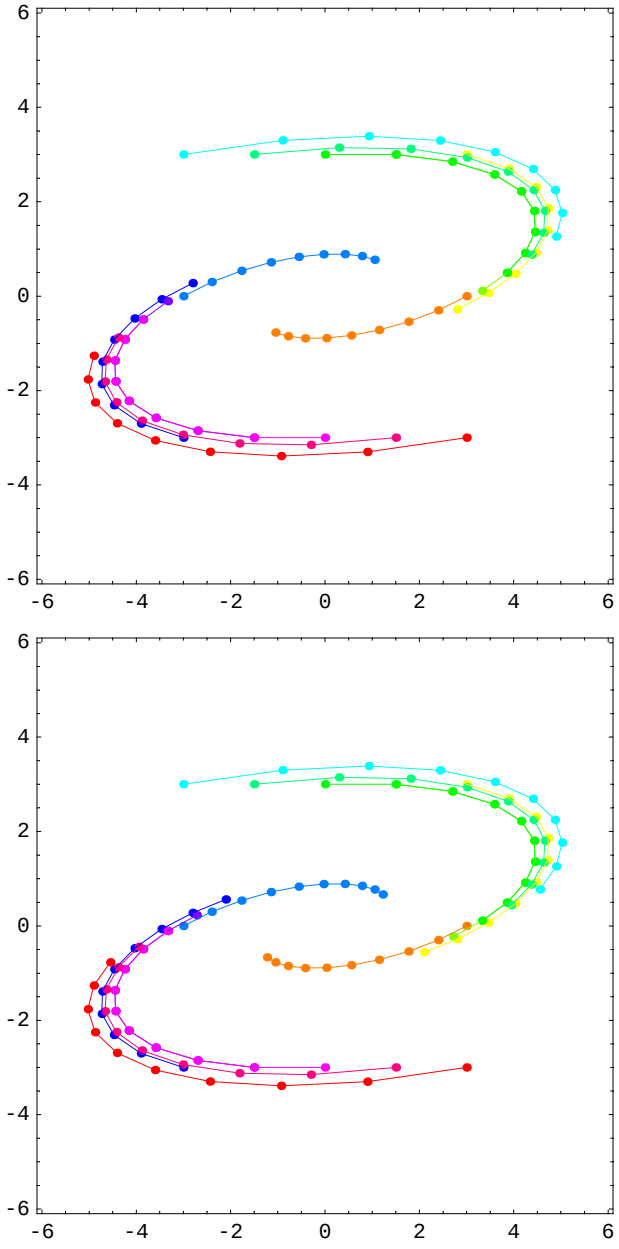
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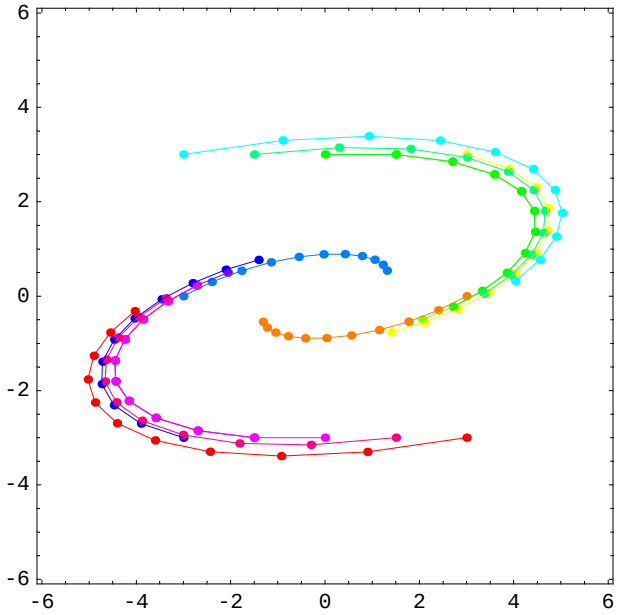
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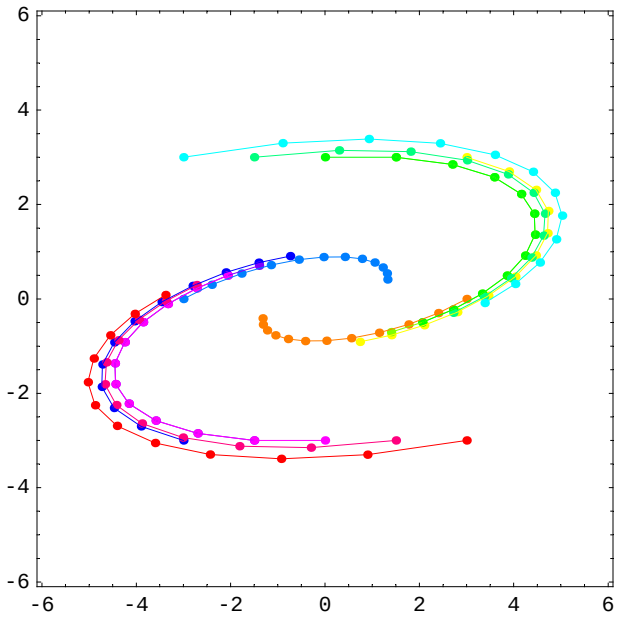
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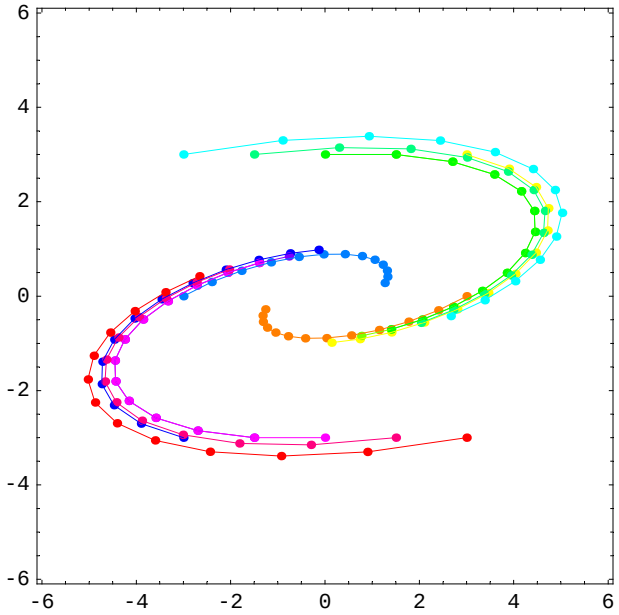
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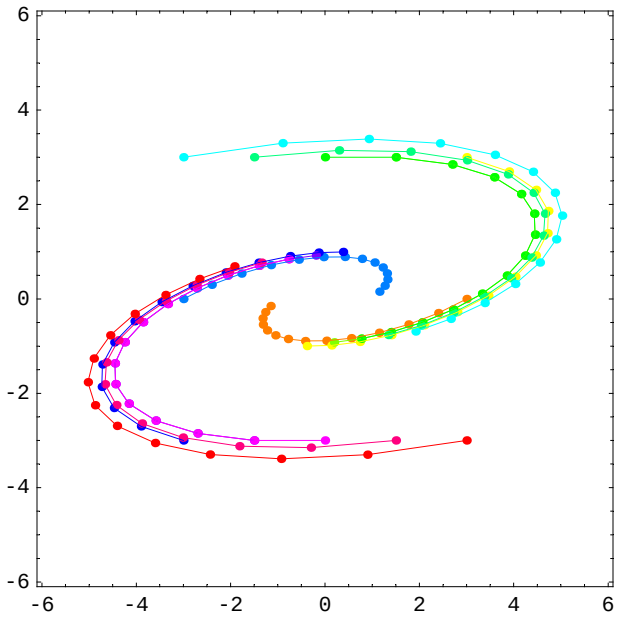
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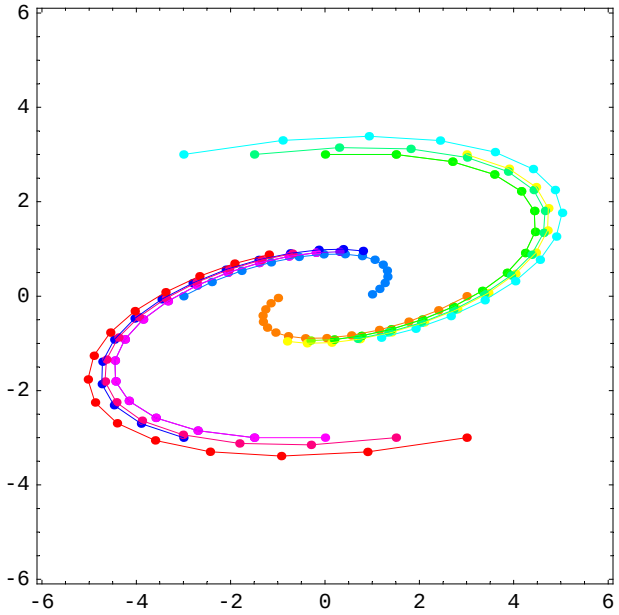
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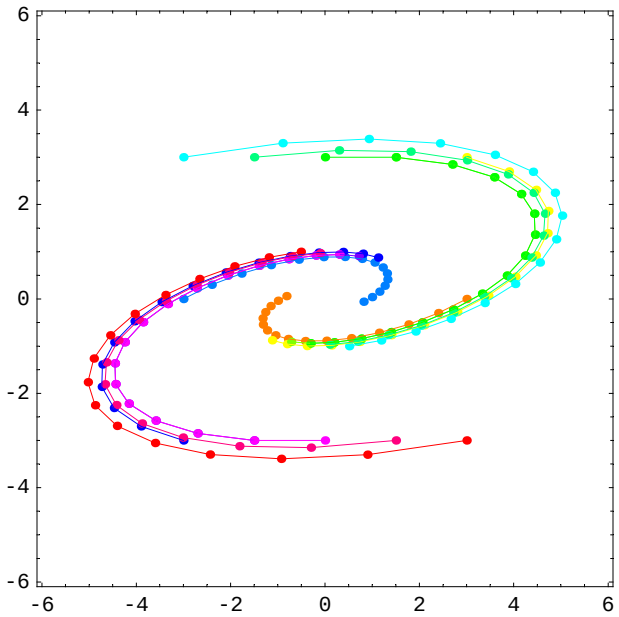
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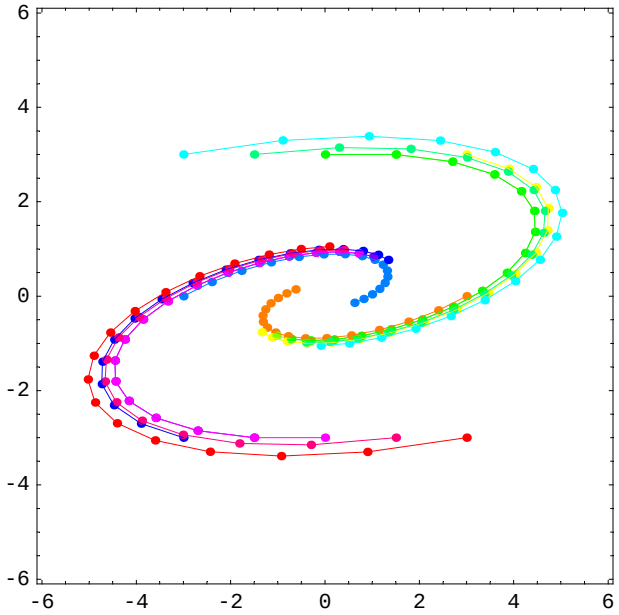
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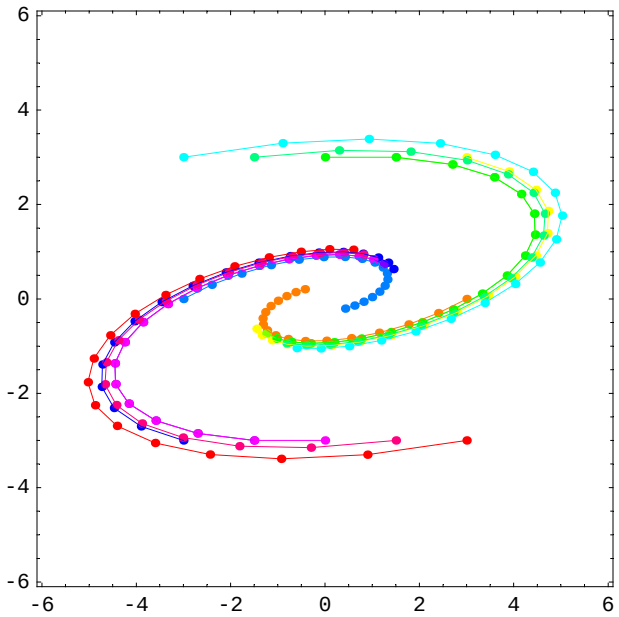
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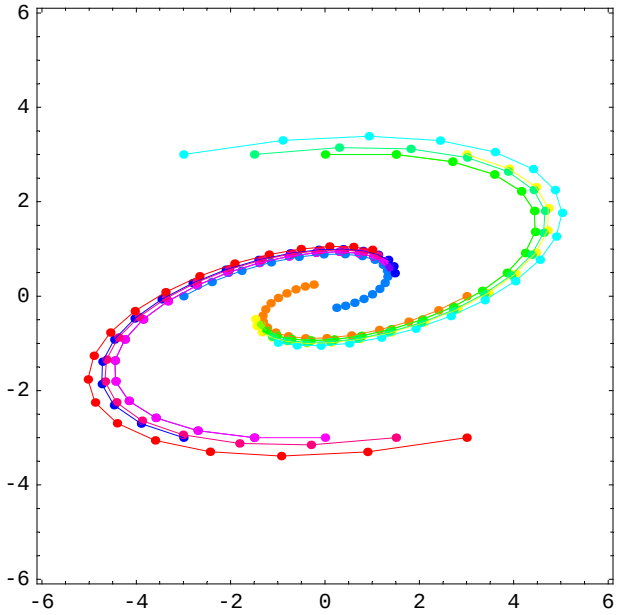
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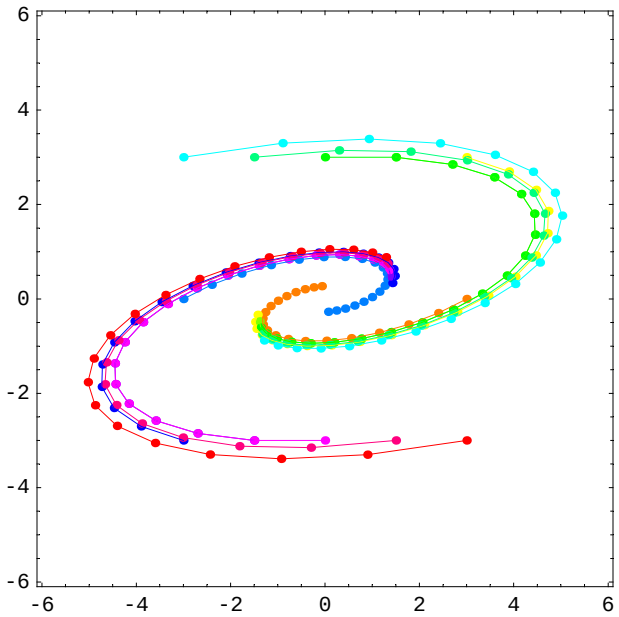
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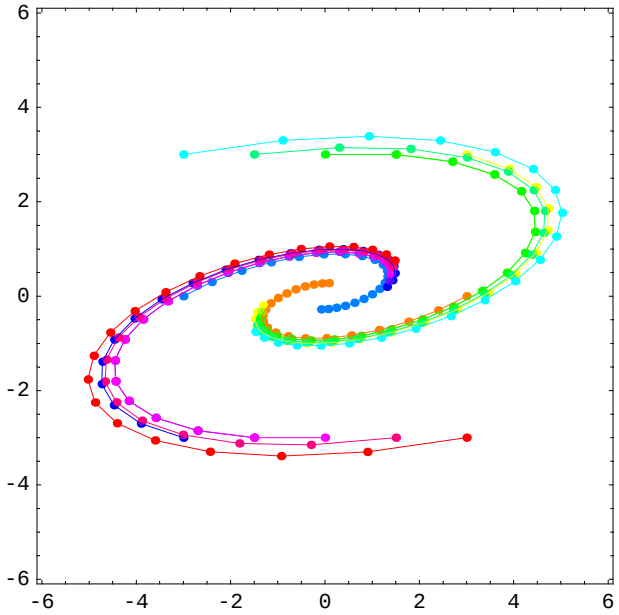


In[42]:= **Eigenvalues[mA]**

Out[42]= {0.9 + 0.2 i, 0.9 - 0.2 i}

In[43]:= **Abs[Eigenvalues[mA]]**

Out[43]= {0.921954, 0.921954}

Matrix 5 (Example 1)

In[44]:= **Eigensystem**$\left[\left\{\left\{\frac{1}{2}, \frac{2}{5}\right\}, \left\{-.125, \frac{11}{10}\right\}\right\}\right]$

Out[44]= {{1., 0.6}, {{-0.624695, -0.780869}, {-0.970143, -0.242536}}}

In[45]:= **Eigensystem**$\left[\left\{\left\{\frac{1}{2}, \frac{2}{5}\right\}, \left\{-\mathbf{p}, \frac{11}{10}\right\}\right\}\right]$

Out[45]= $\left\{\left\{\frac{1}{10}\left(8-\sqrt{9-40\,p}\right), \frac{1}{10}\left(8+\sqrt{9-40\,p}\right)\right\}, \left\{\left\{-\frac{-3-\sqrt{9-40\,p}}{10\,p}, 1\right\}, \left\{-\frac{-3+\sqrt{9-40\,p}}{10\,p}, 1\right\}\right\}\right\}$

■ Example 1

In[46]:= **mA** = $\left\{\left\{\frac{1}{2}, \frac{2}{5}\right\}, \left\{-.104, \frac{11}{10}\right\}\right\}$

Out[46]= $\left\{\left\{\frac{1}{2}, \frac{2}{5}\right\}, \left\{-0.104, \frac{11}{10}\right\}\right\}$

In[47]:= **Eigensystem[mA]**

Out[47]= {{1.02, 0.58}, {{-0.609711, -0.792624}, {-0.980581, -0.196116}}}

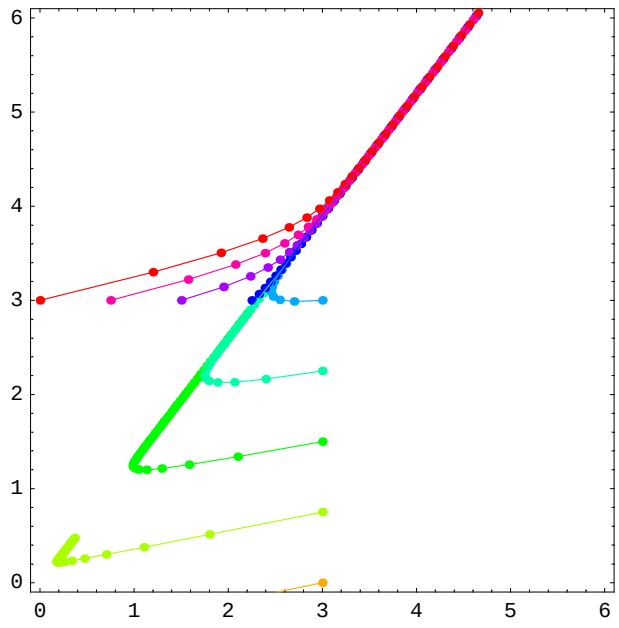
```
In[48]:= MatrixPower[mA, 0].{3, 3}

Out[48]= {3, 3}

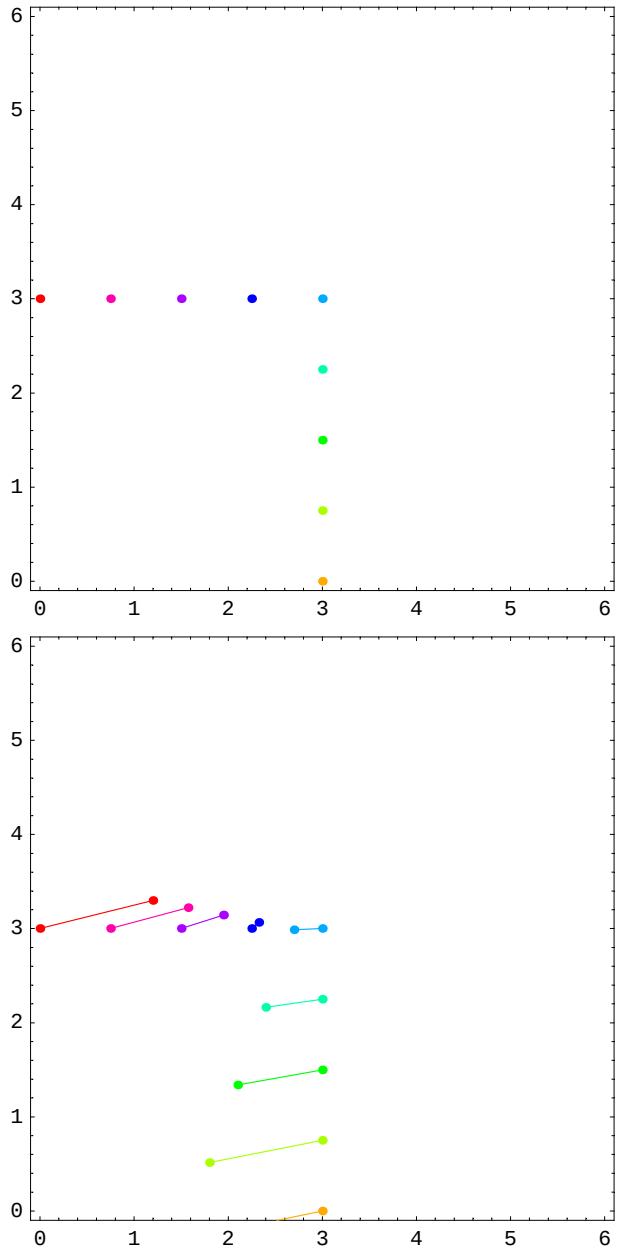
In[49]:= steps = 50;
         ips =
           {{3, 0}, {3, .75}, {3, 1.5}, {3, 2.25}, {3, 3}, {2.25, 3}, {1.5, 3}, {.75, 3}, {0, 3}};

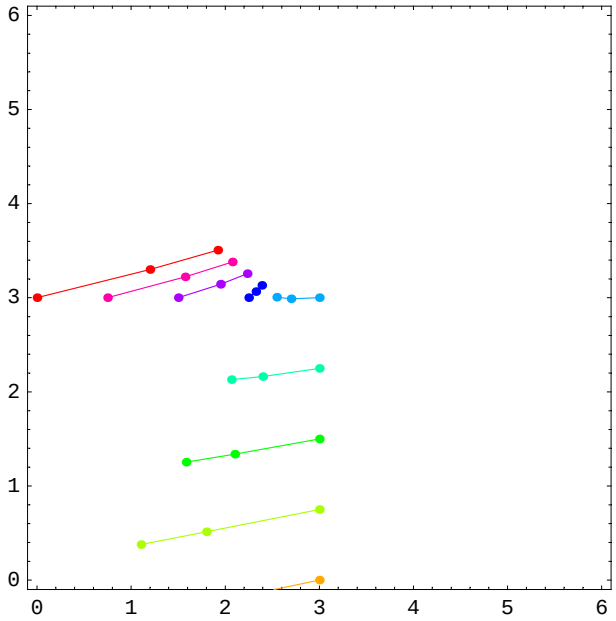
         PR = {{-0.1, 6.1}, {-0.1, 6.1}};

         Show[
           Graphics[{
             {PointSize[0.015], Hue[#[[2]] / Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
                 {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
             {Thickness[0.002], Hue[#[[2]] / Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
                 {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
            }],
           PlotRange → PR, AspectRatio → Automatic, Frame → True
          ];
```



In[53]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → PR, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```

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2
3
4
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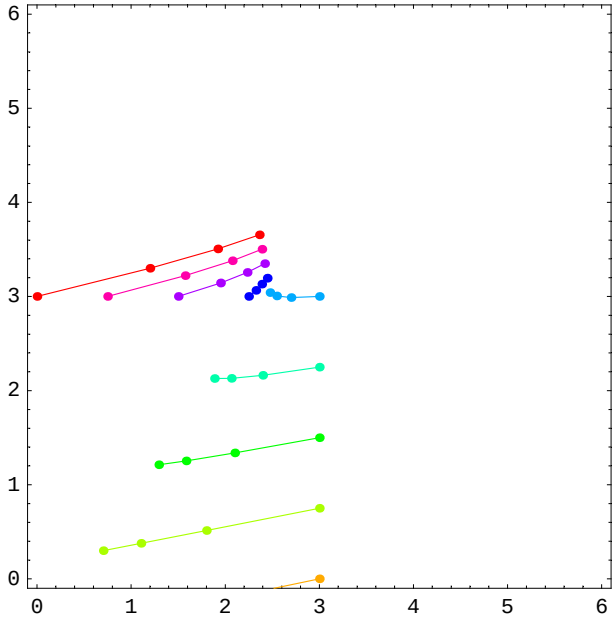
6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2
1
0
0
1
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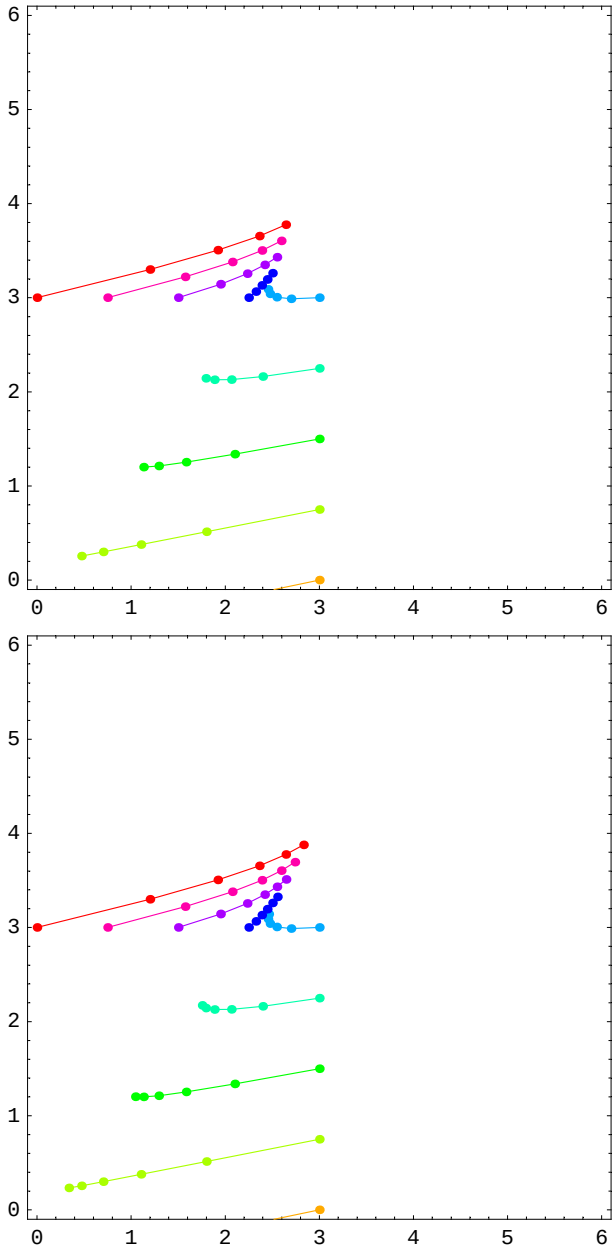
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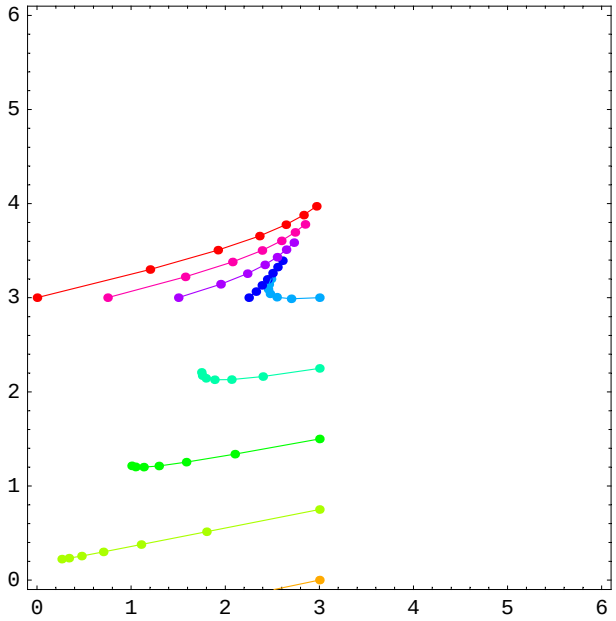
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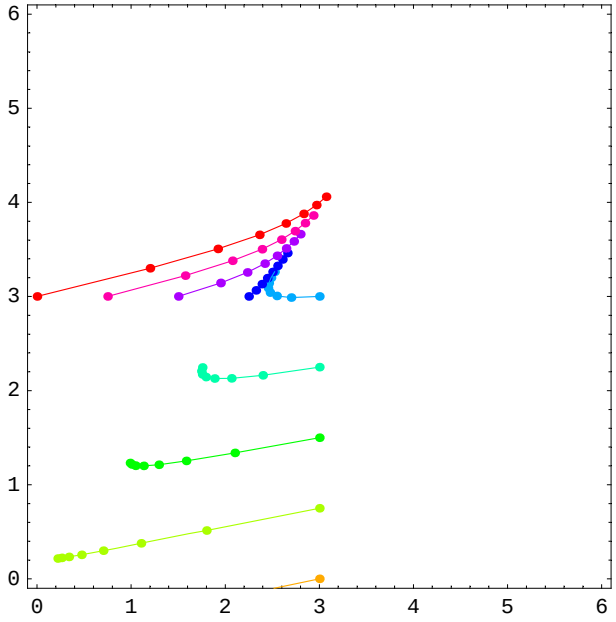
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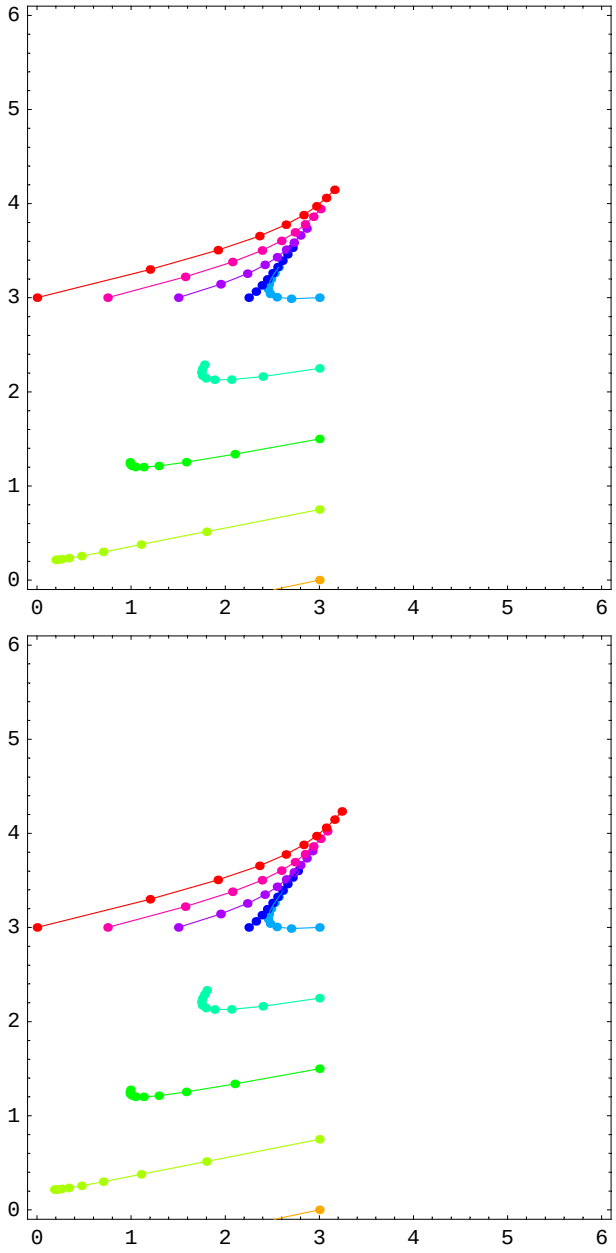
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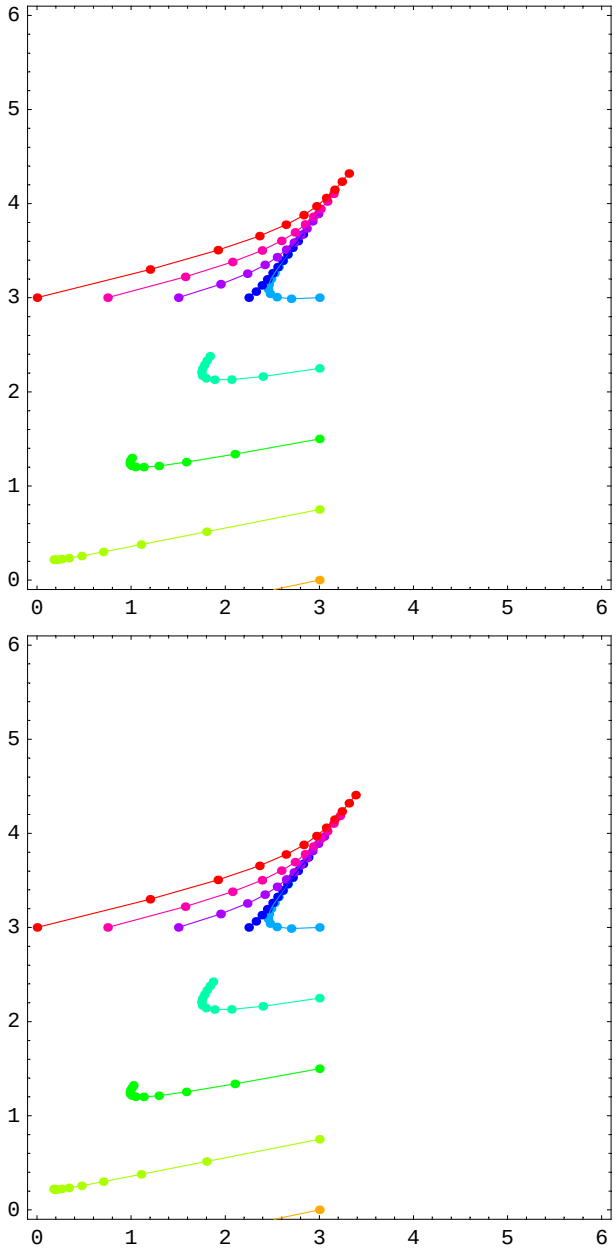
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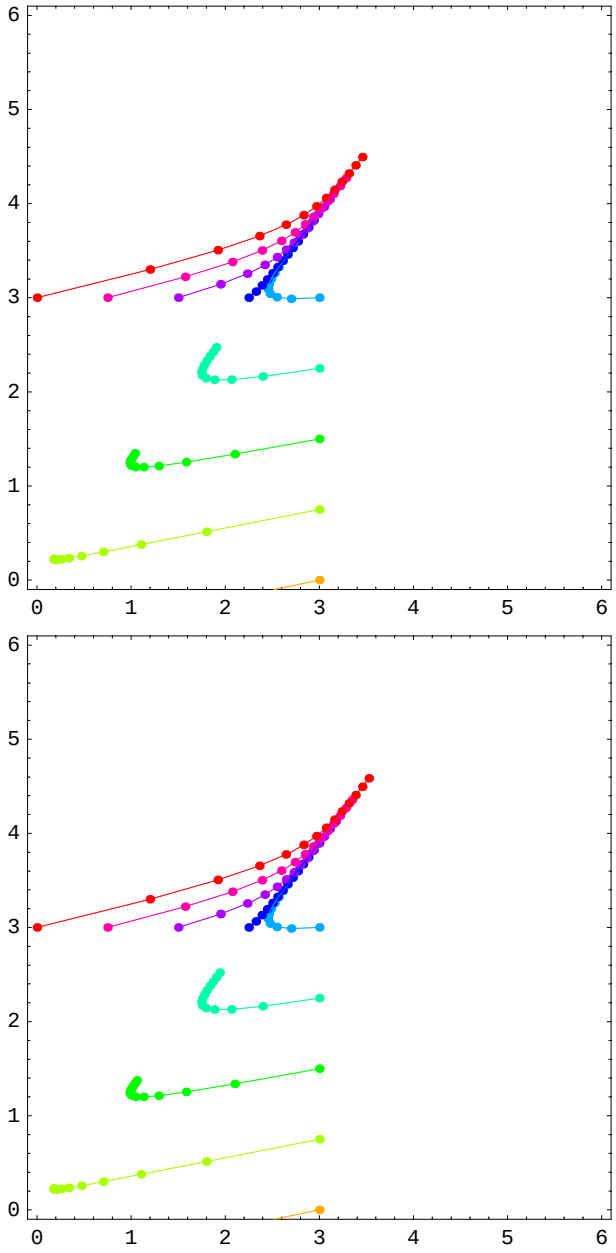
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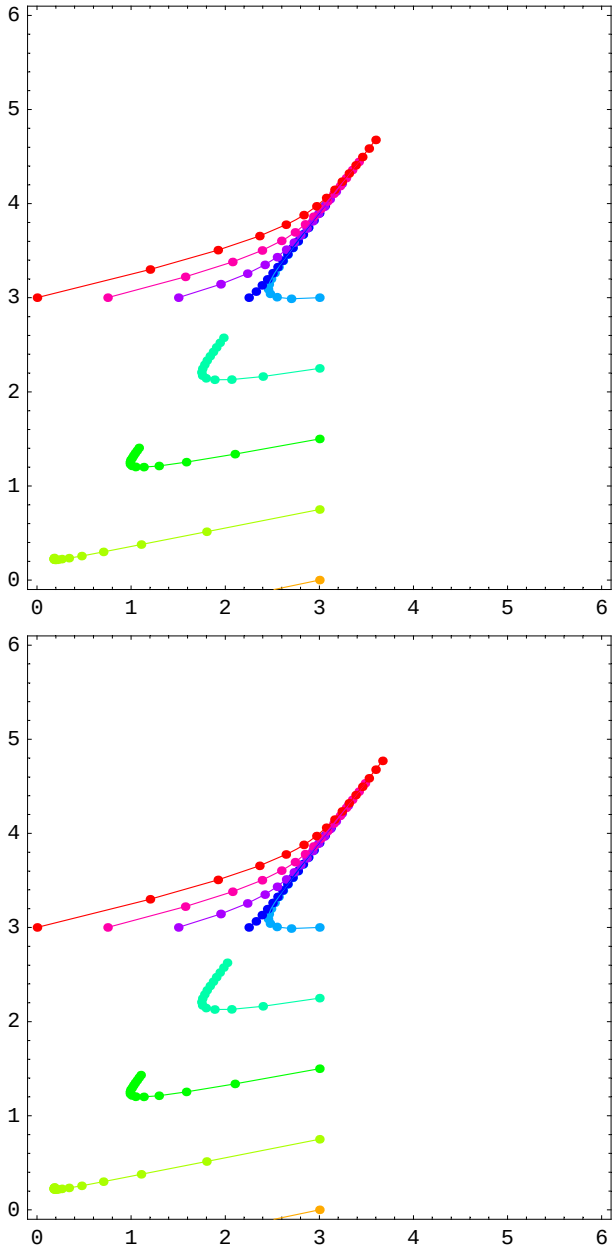
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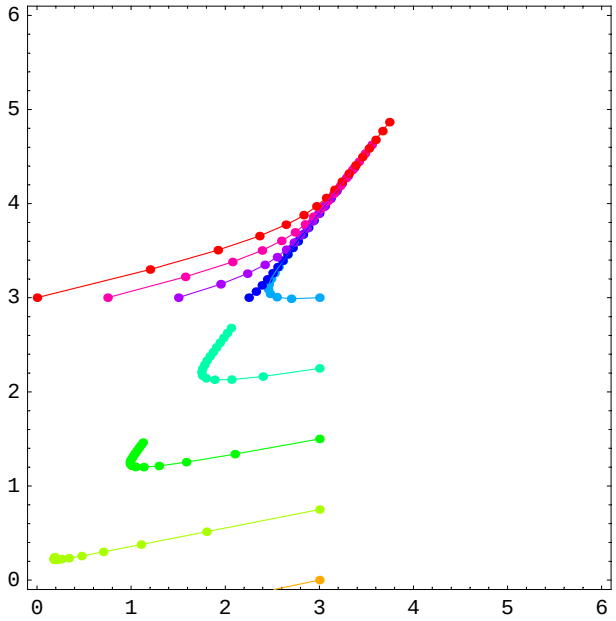
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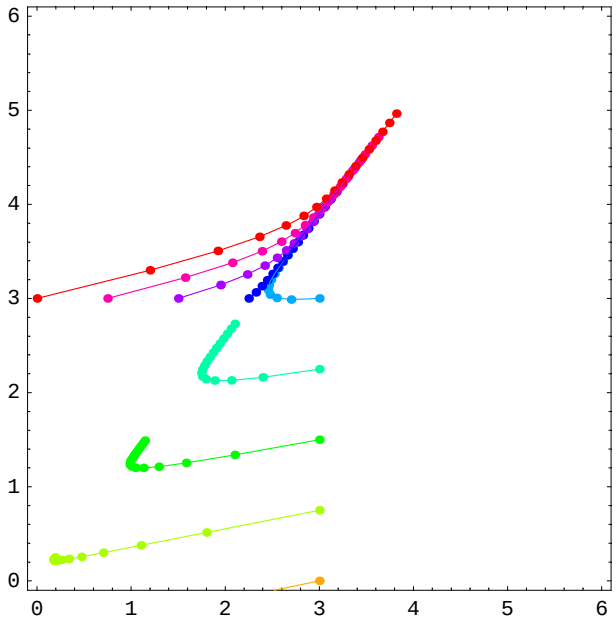
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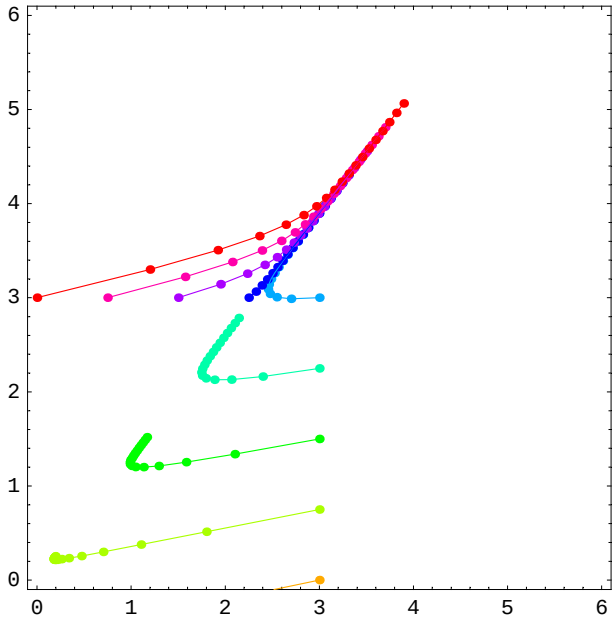
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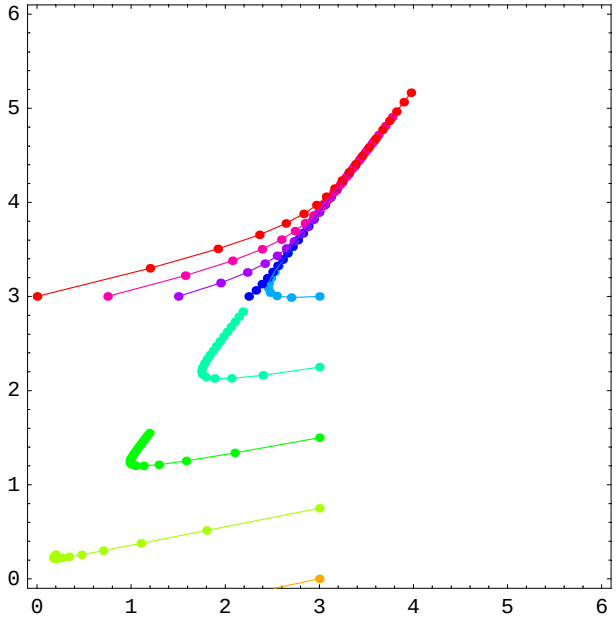
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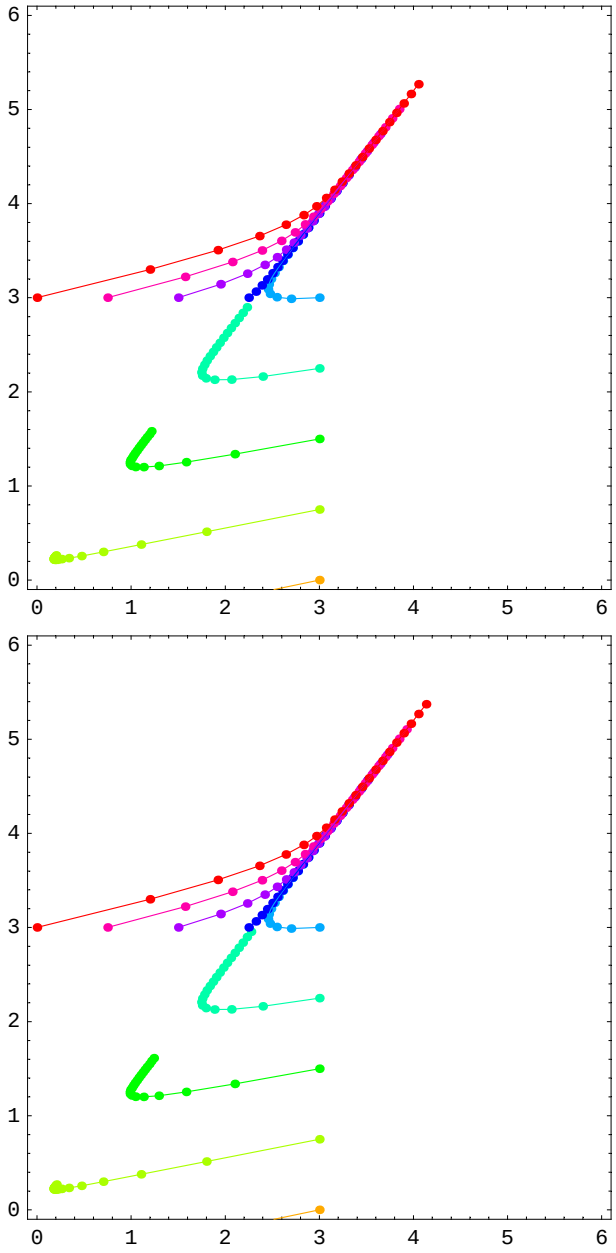
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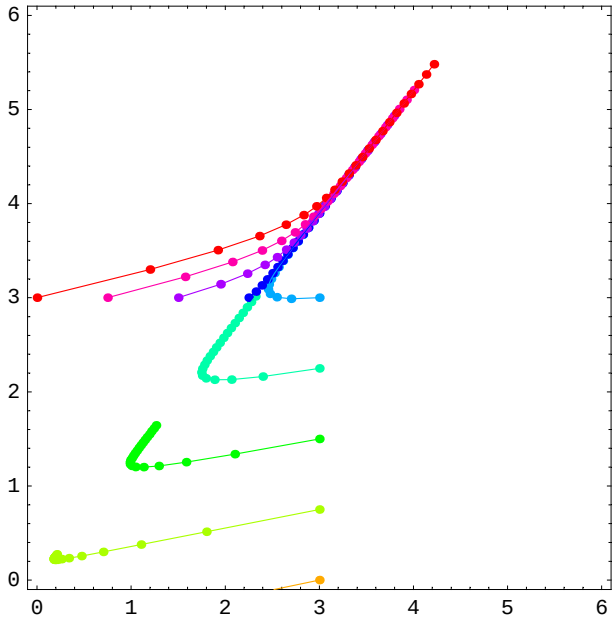
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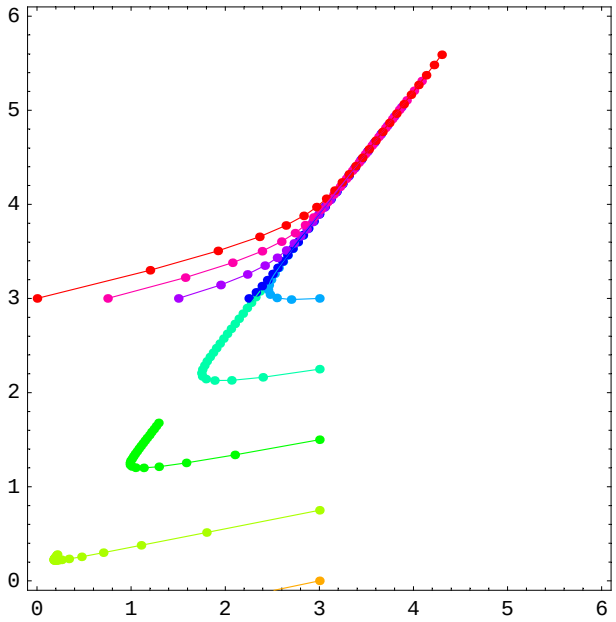
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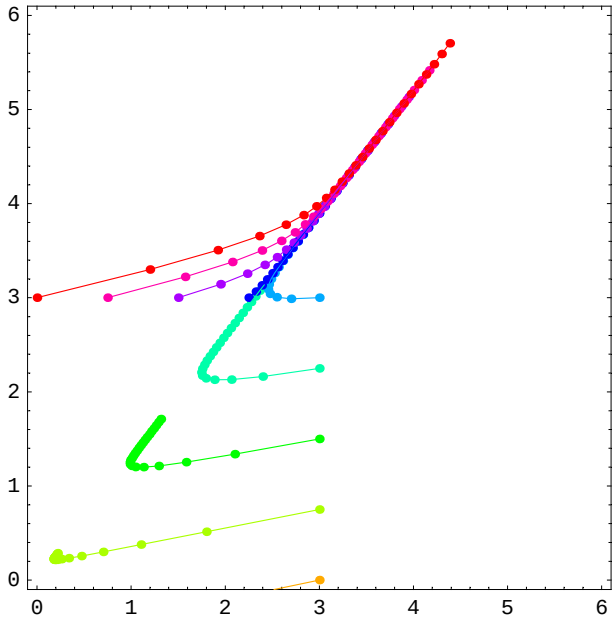
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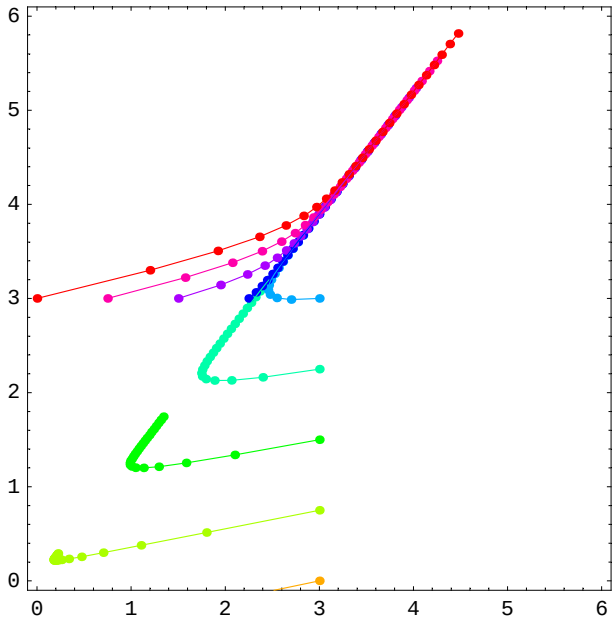
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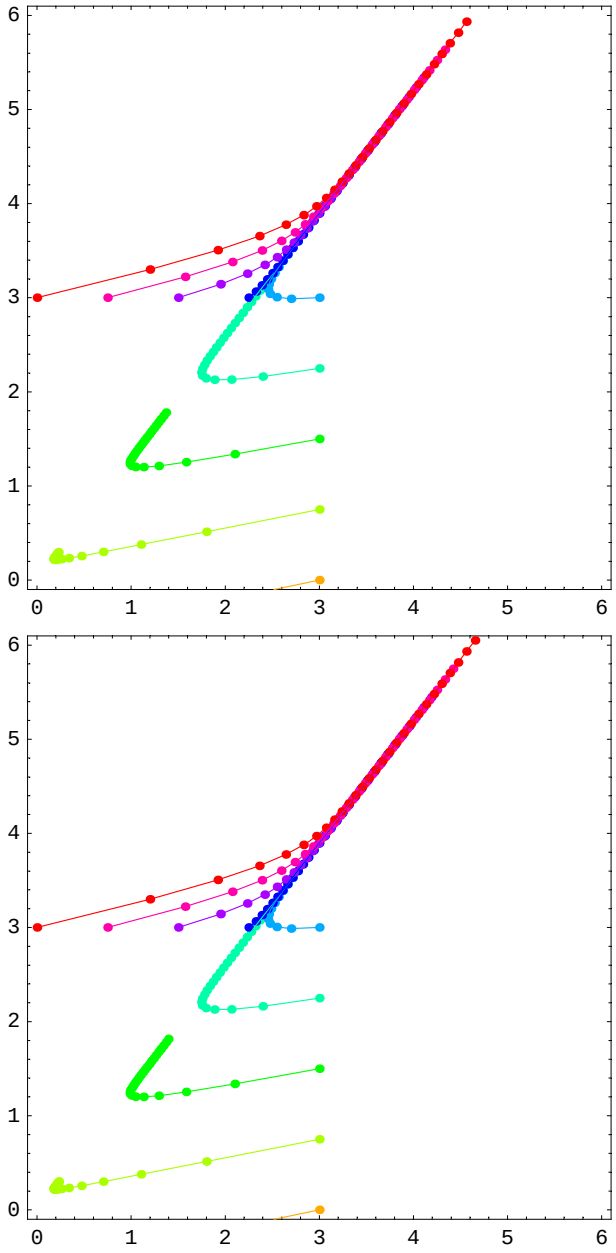
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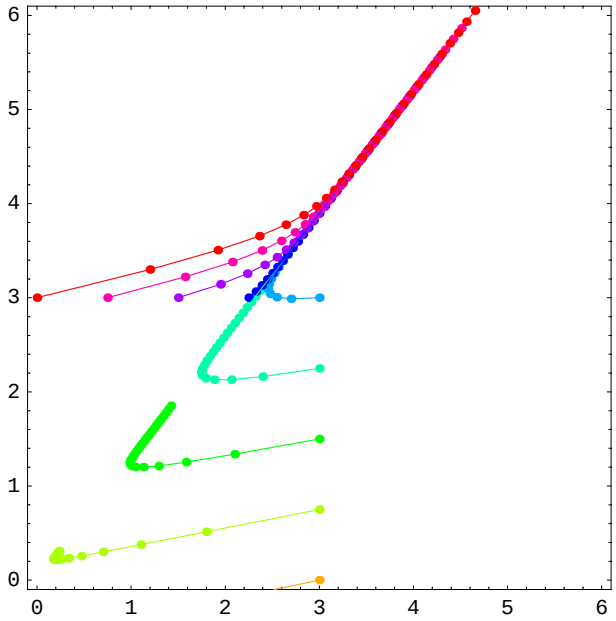
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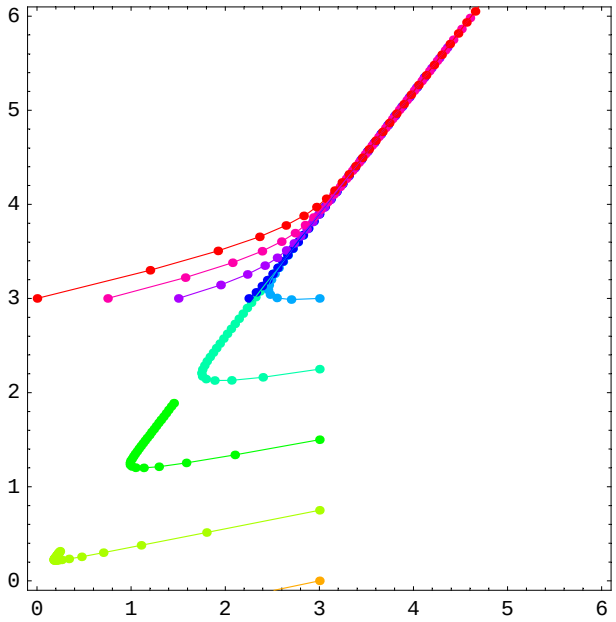
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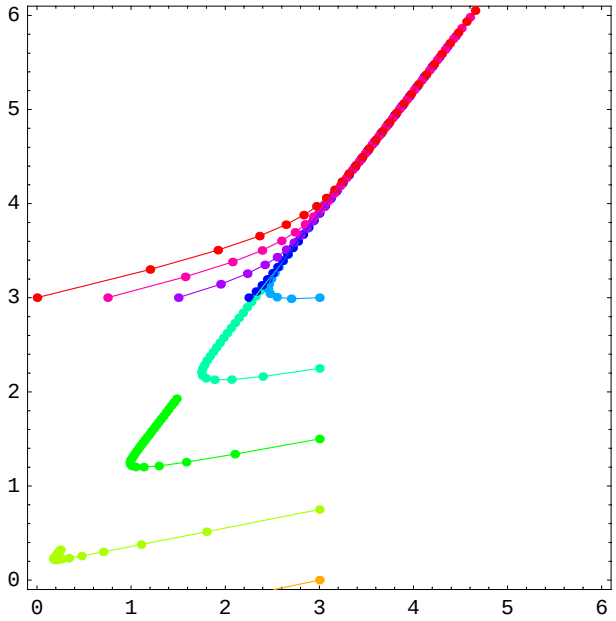
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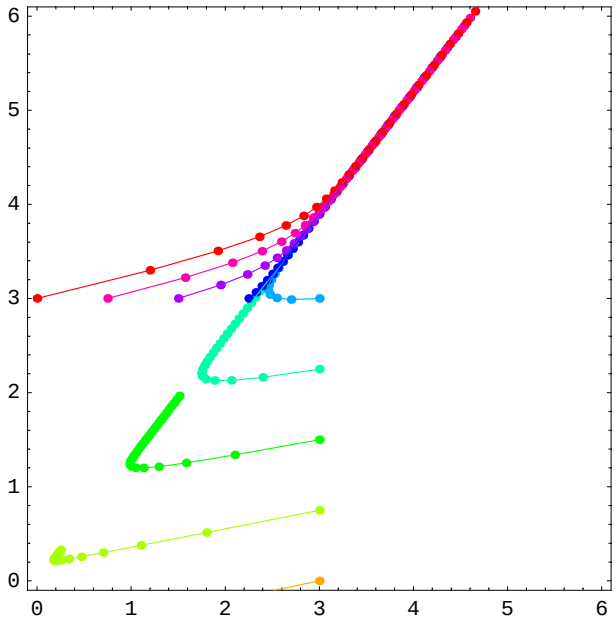
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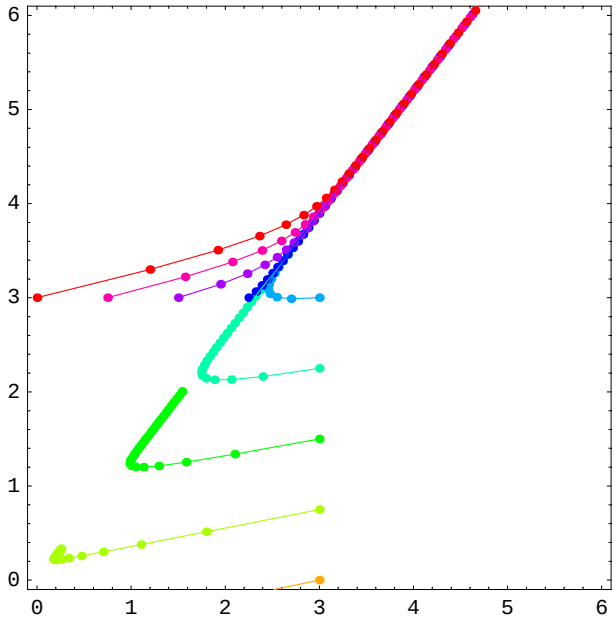
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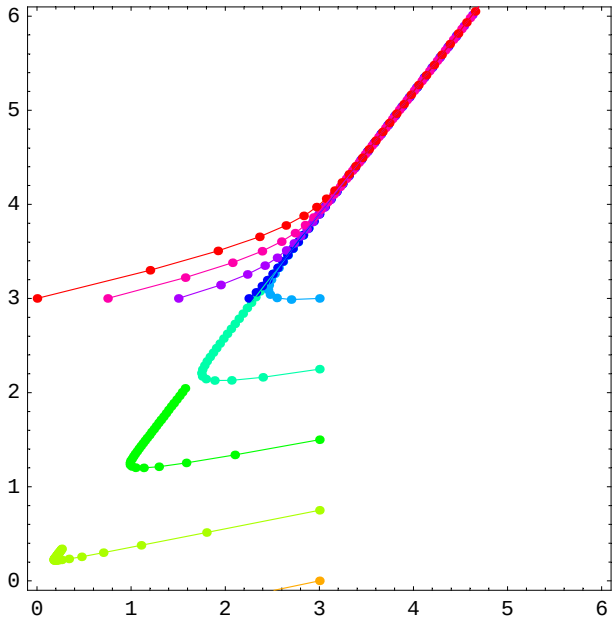
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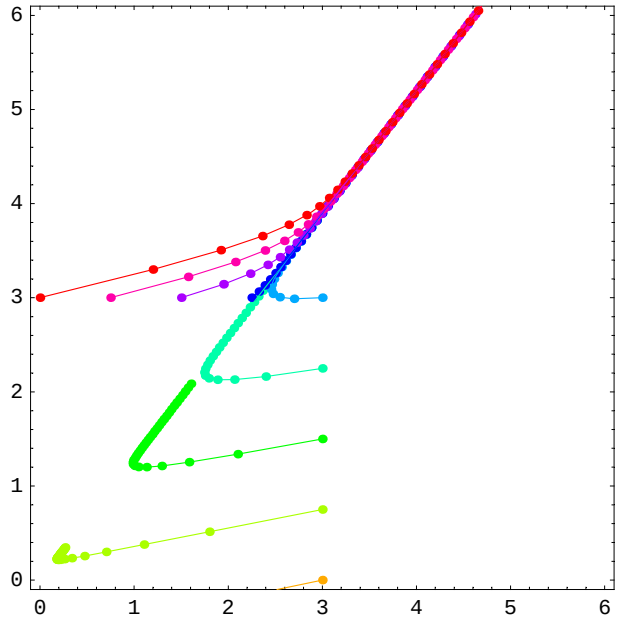
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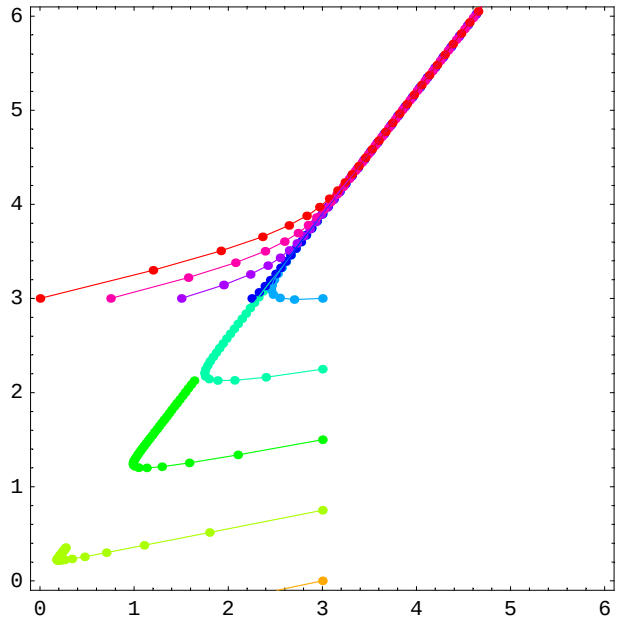
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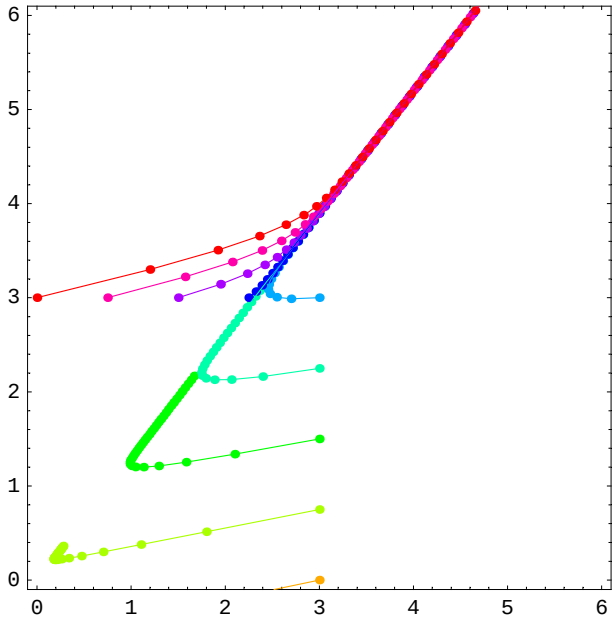
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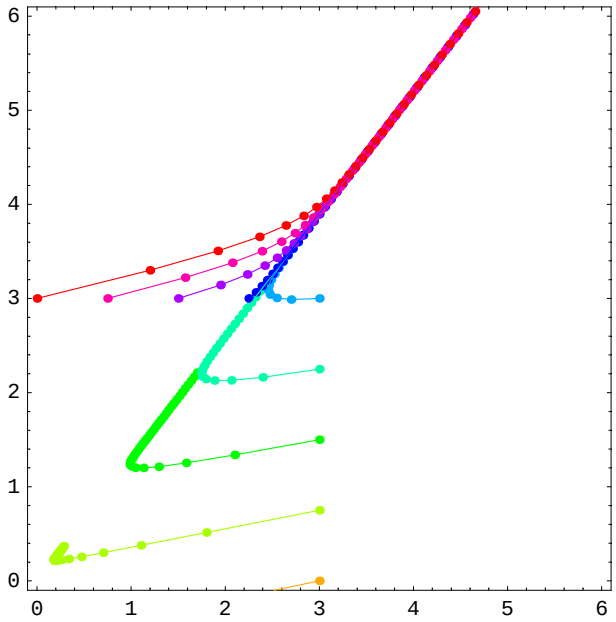
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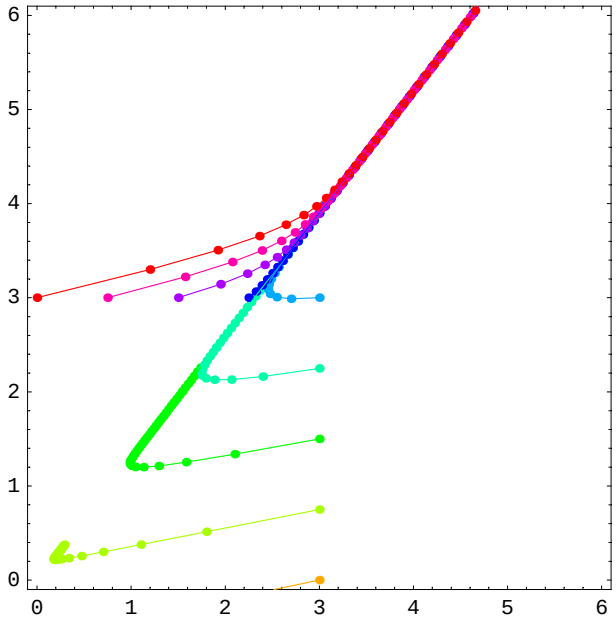
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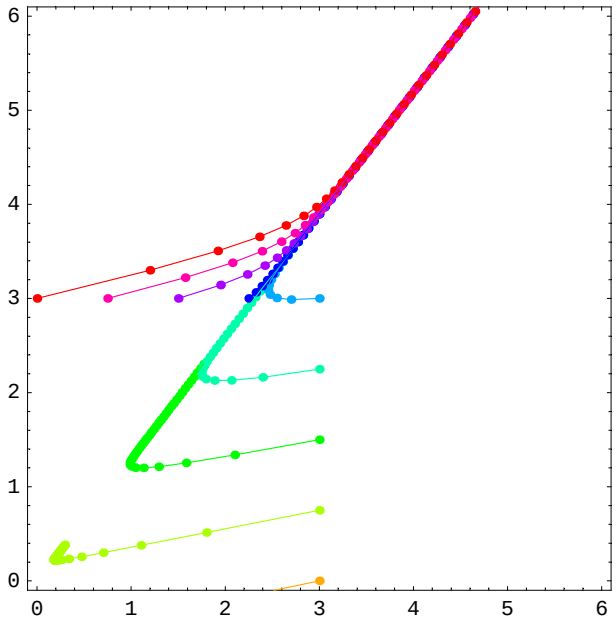
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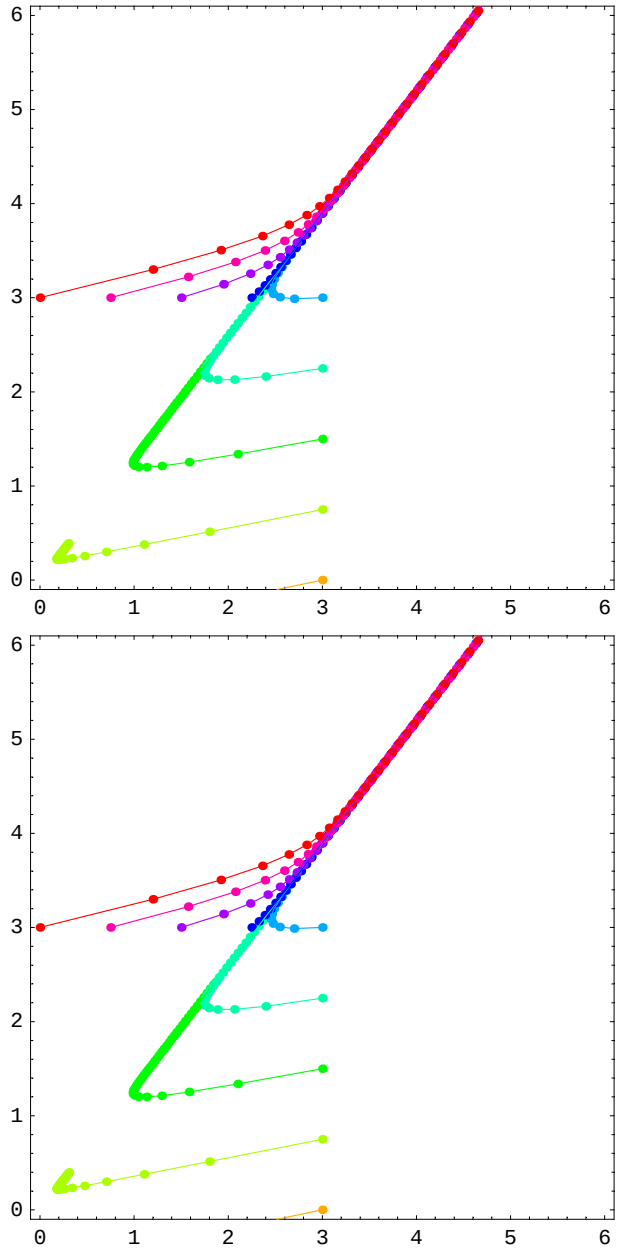
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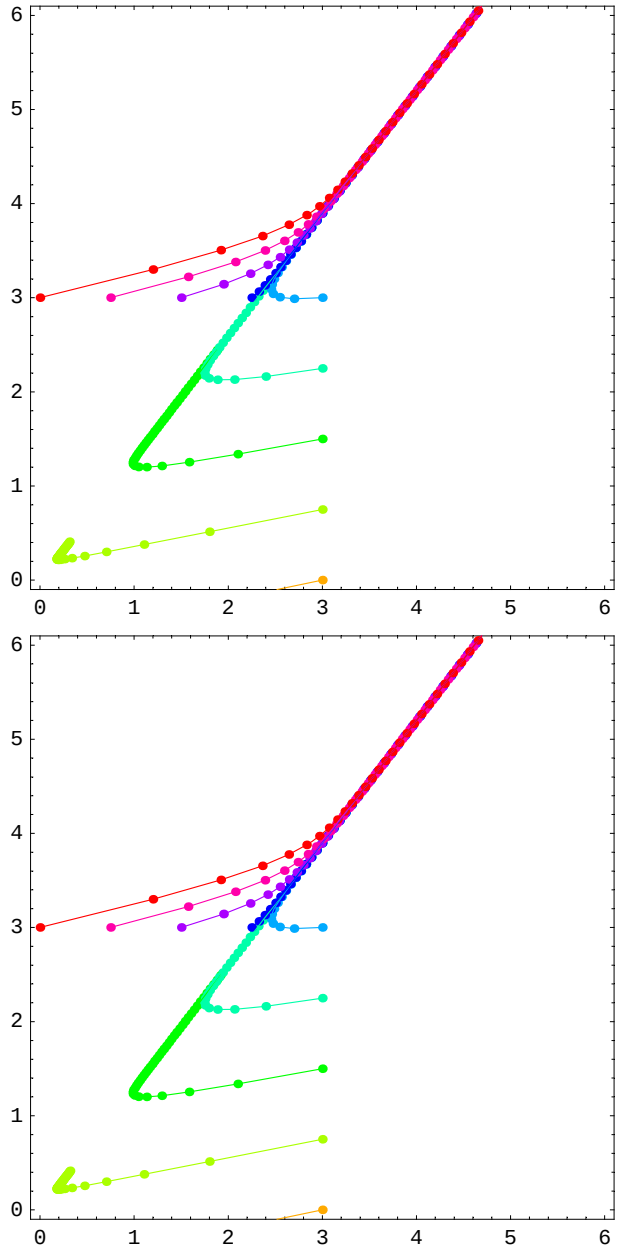
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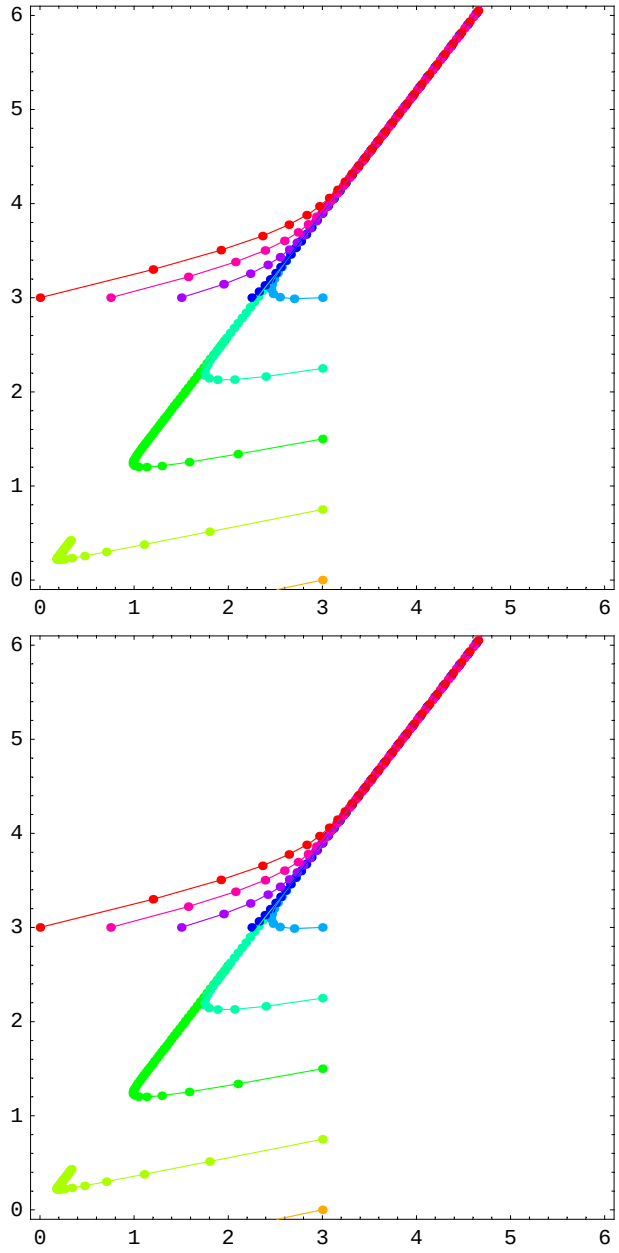
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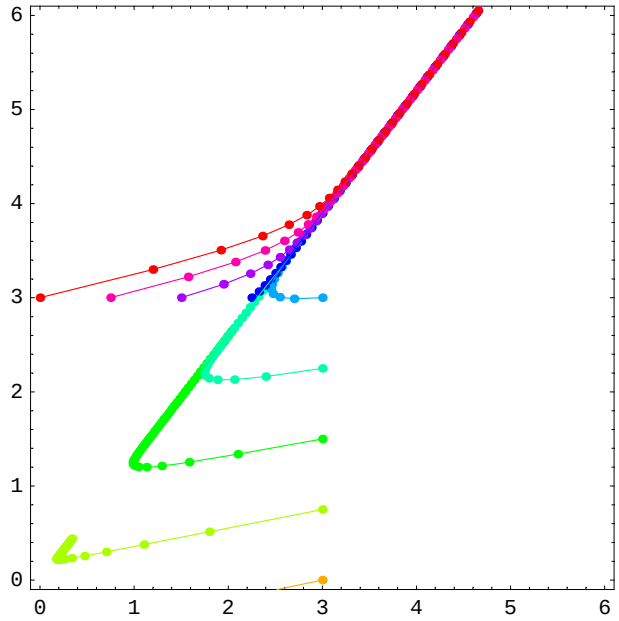
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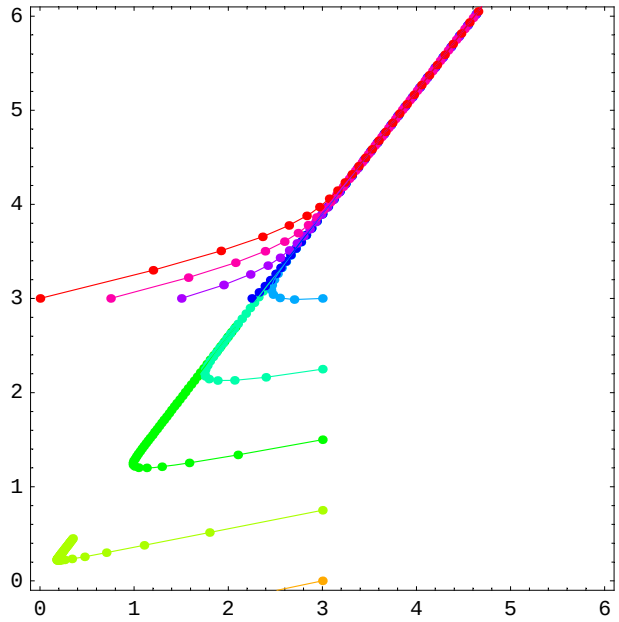
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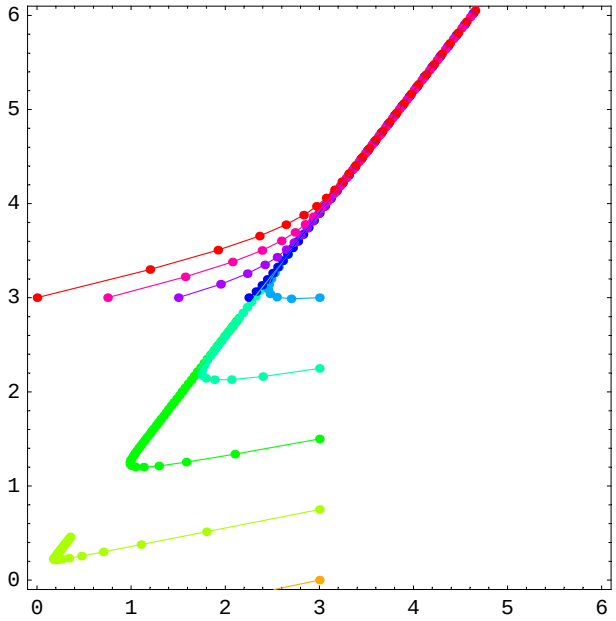
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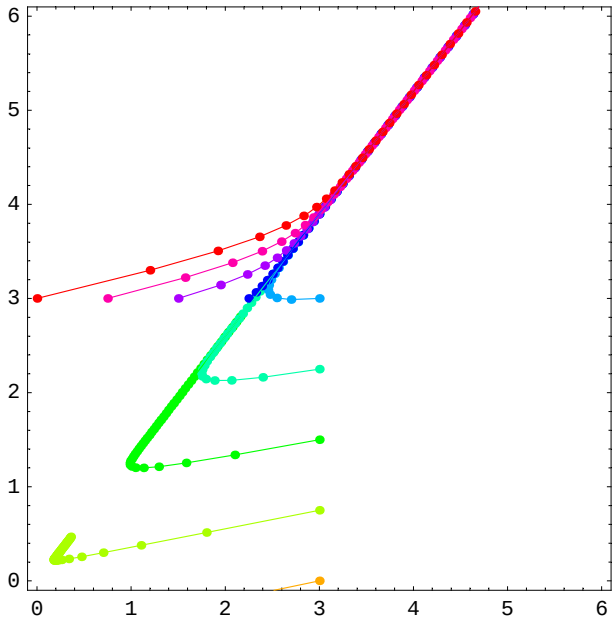
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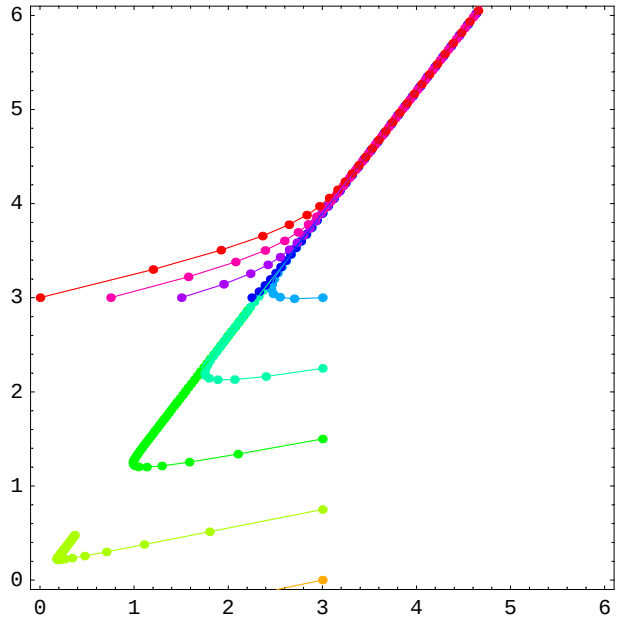


■ Example 1, different p

In[54]:=
```
mA = {{1/2, 2/5}, {p, 11/10}}
```

Out[54]=
$$\left\{\left\{\frac{1}{2}, \frac{2}{5}\right\}, \left\{p, \frac{11}{10}\right\}\right\}$$

In[55]:=
```
Eigensystem[mA]
```

Out[55]=
$$\left\{\left\{\frac{1}{10}\left(8-\sqrt{9+40\,p}\right), \frac{1}{10}\left(8+\sqrt{9+40\,p}\right)\right\}, \left\{\left\{-\frac{3+\sqrt{9+40\,p}}{10\,p}, 1\right\}, \left\{-\frac{3-\sqrt{9+40\,p}}{10\,p}, 1\right\}\right\}\right\}$$

In[56]:=
```
mA = {{1/2, 2/5}, {-1., 11/10}}
```

Out[56]=
$$\left\{\left\{\frac{1}{2}, \frac{2}{5}\right\}, \left\{-1., \frac{11}{10}\right\}\right\}$$

In[57]:=
```
Eigensystem[mA]
```

Out[57]=
```
{{0.8 + 0.556776 i, 0.8 - 0.556776 i},
 {{0.253546 - 0.470562 i, 0.845154 + 0. i}, {0.253546 + 0.470562 i, 0.845154 + 0. i}}}
```

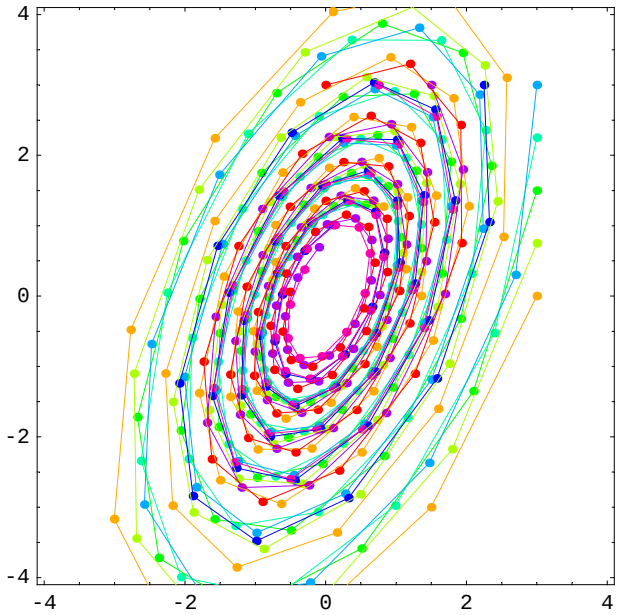
In[58]:=
```
Abs[Eigensystem[mA][[1, 1]]]
```

Out[58]=
```
0.974679
```

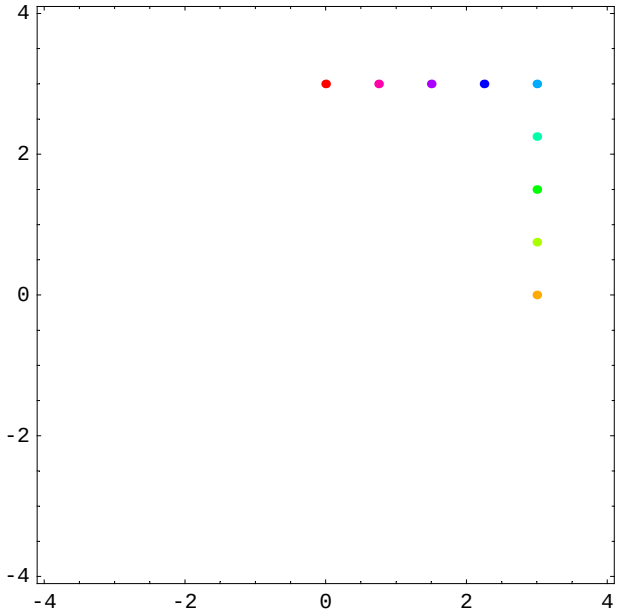
In[59]:=
```
steps = 50;
ips =
  {{3, 0}, {3, .75}, {3, 1.5}, {3, 2.25}, {3, 3}, {2.25, 3}, {1.5, 3}, {.75, 3}, {0, 3}};

PR = {{-4.1, 4.1}, {-4.1, 4.1}};

Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → PR, AspectRatio → Automatic, Frame → True
 ];
```



In[63]:=
```
Table[Show[
   Graphics[{
     {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
         {k, 0, st}]} & /@ Transpose[{ips, Range[Length[ips]]}],
     {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
         {k, 0, st}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
    }],
   PlotRange → PR, AspectRatio → Automatic, Frame → True
  ], {st, 0, steps}];
```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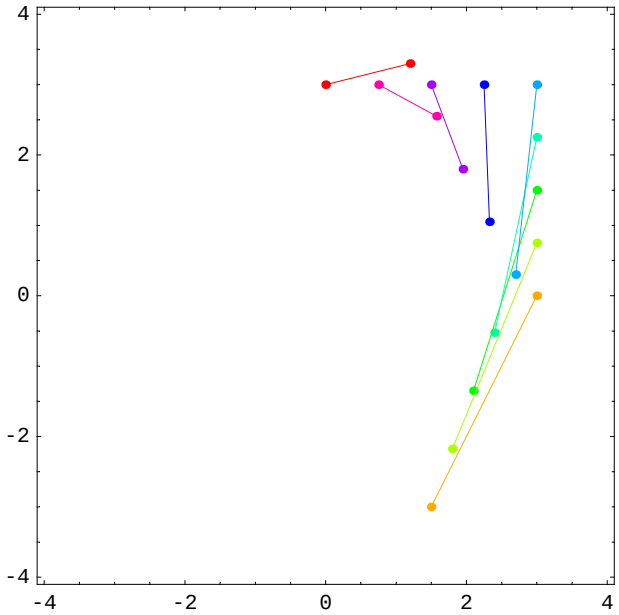
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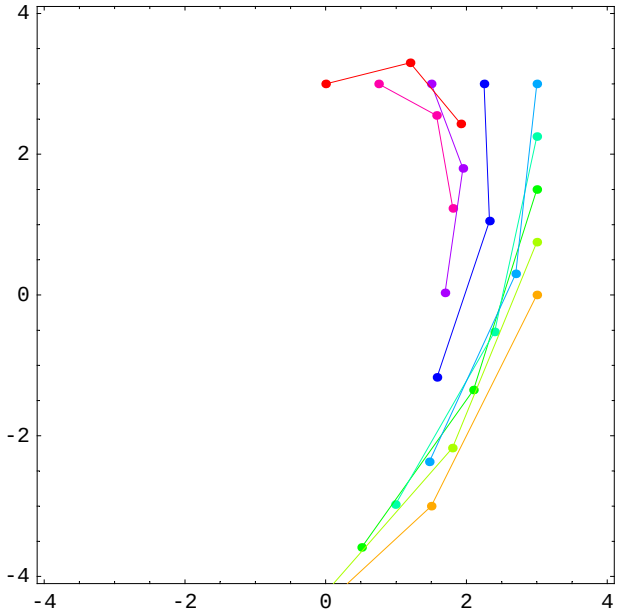
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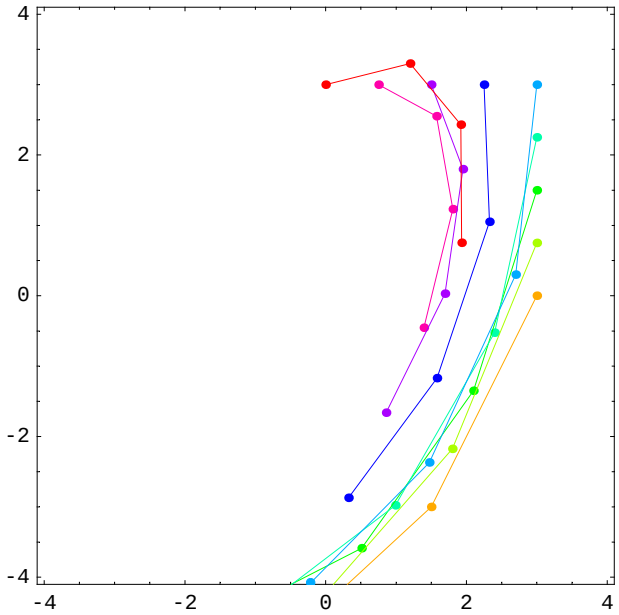
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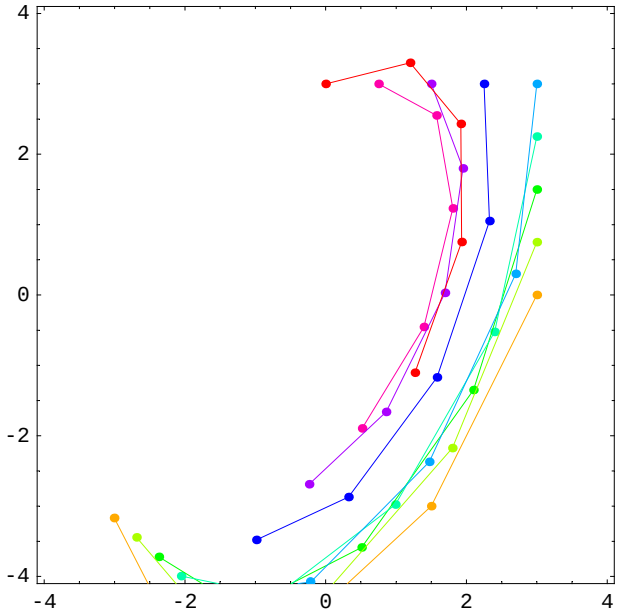
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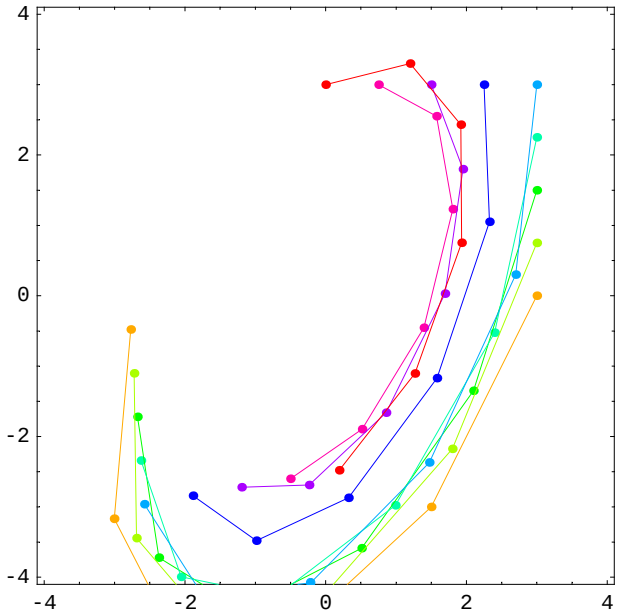
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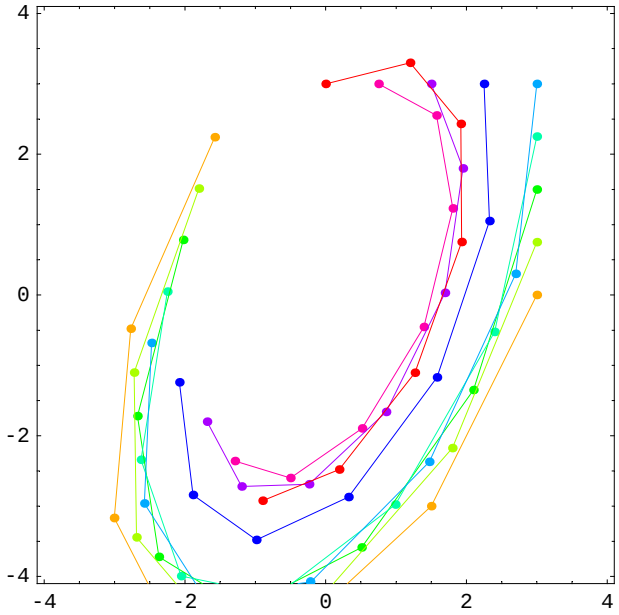
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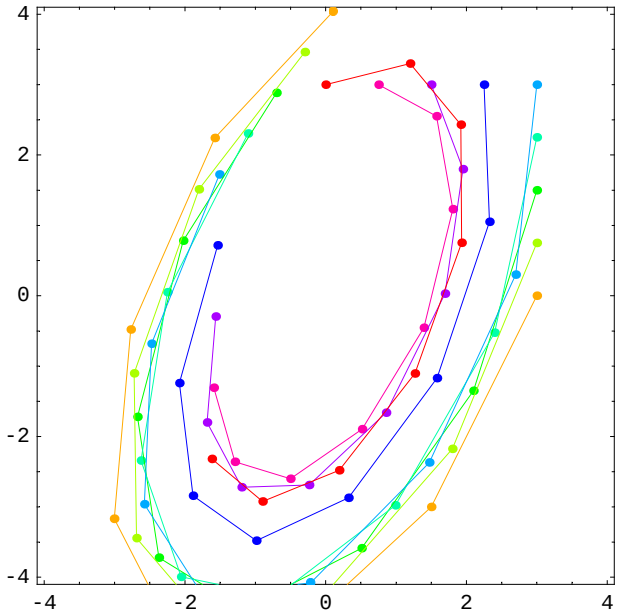
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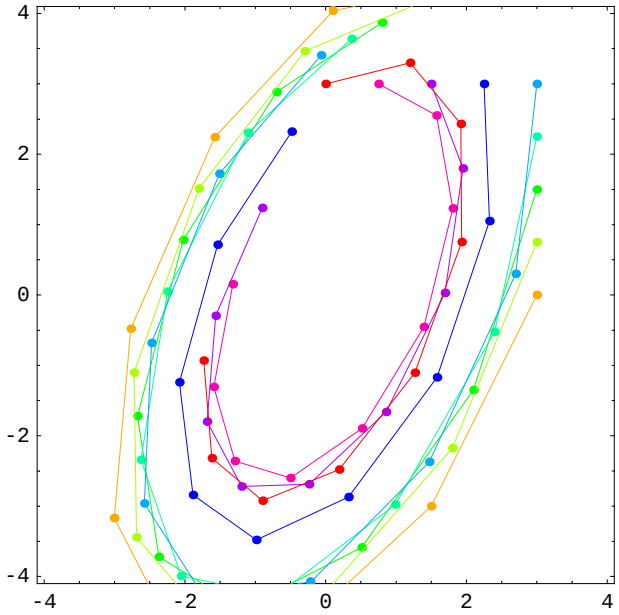
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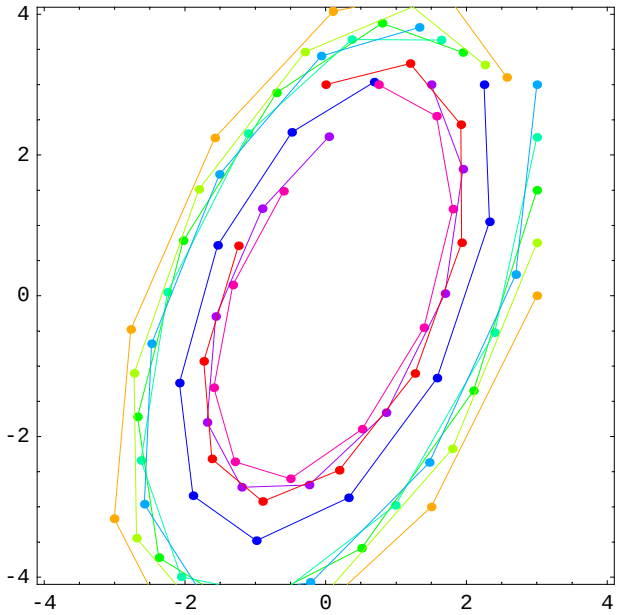
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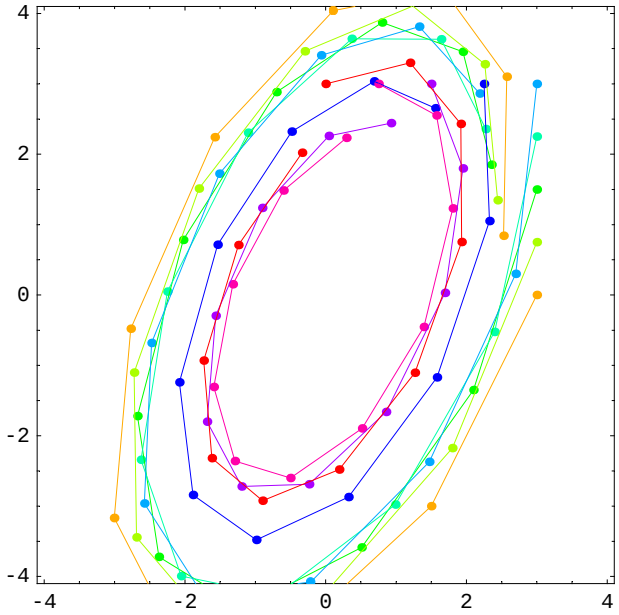
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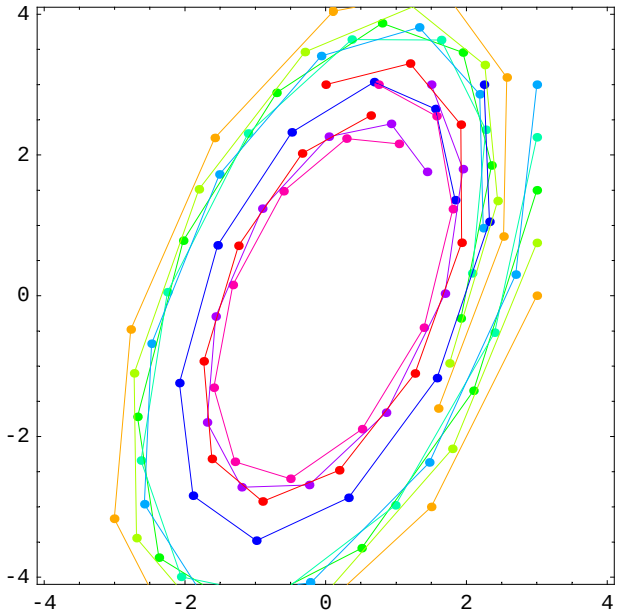
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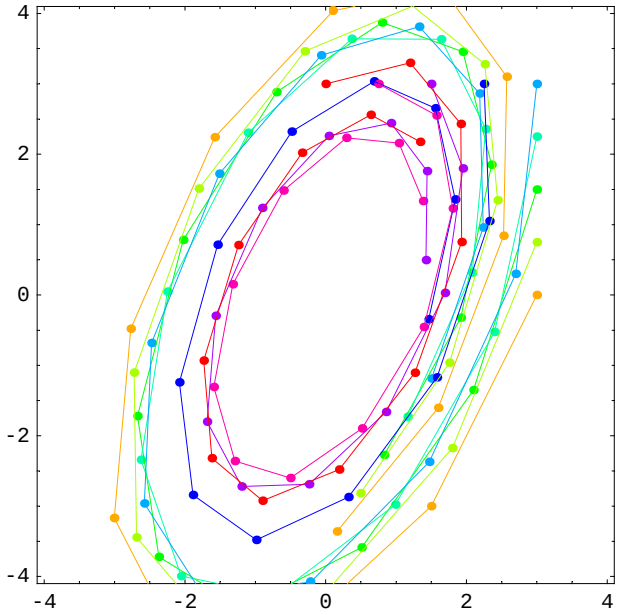
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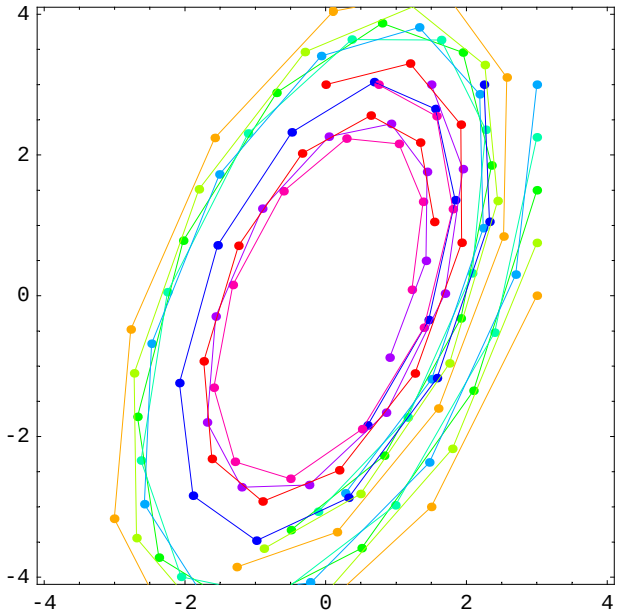
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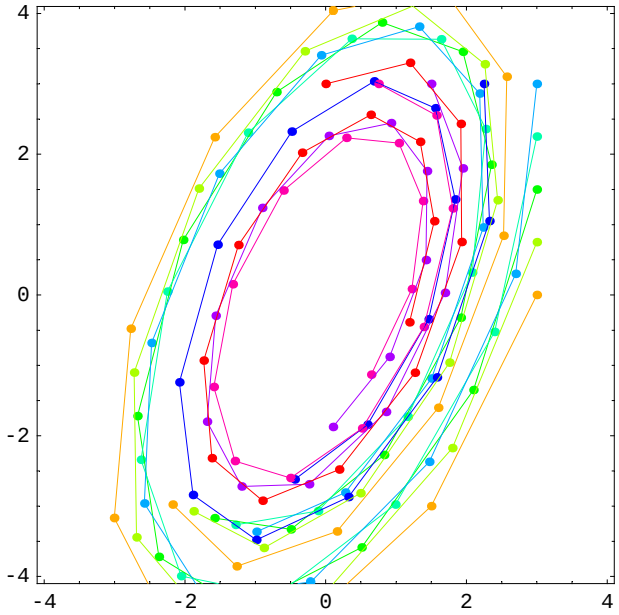
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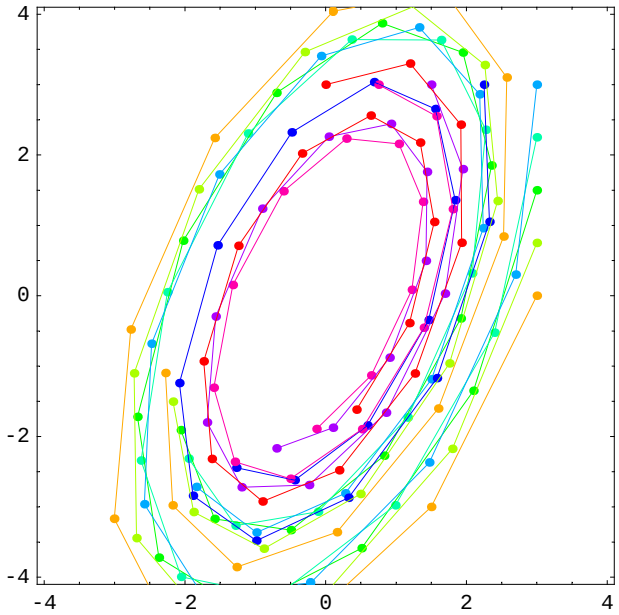
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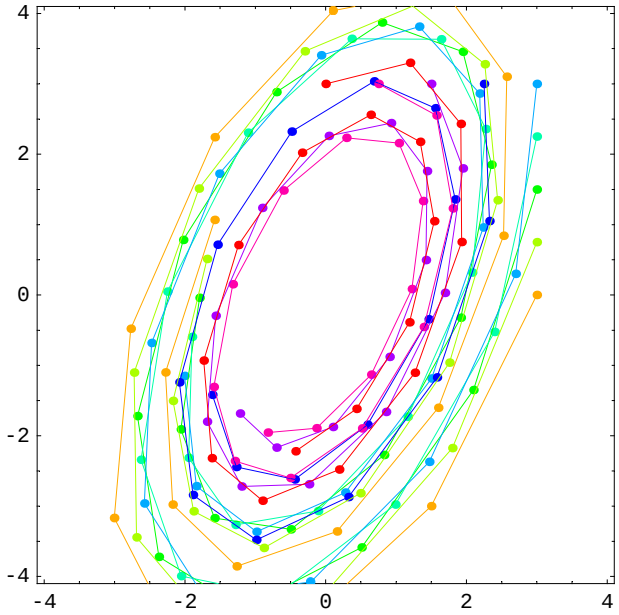
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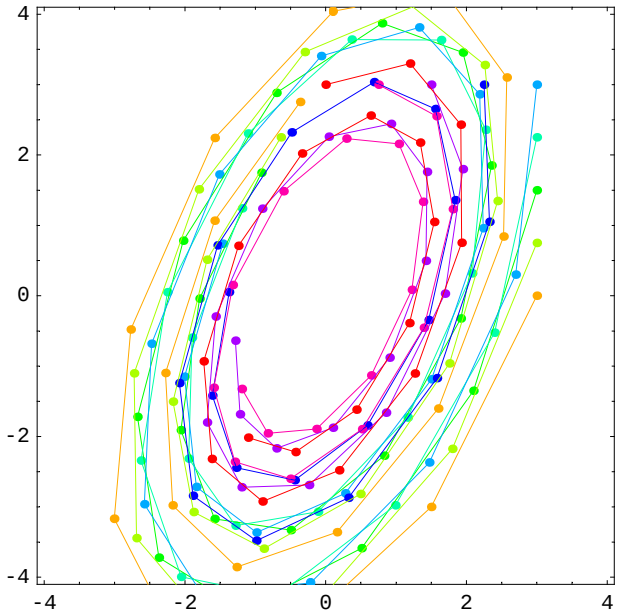
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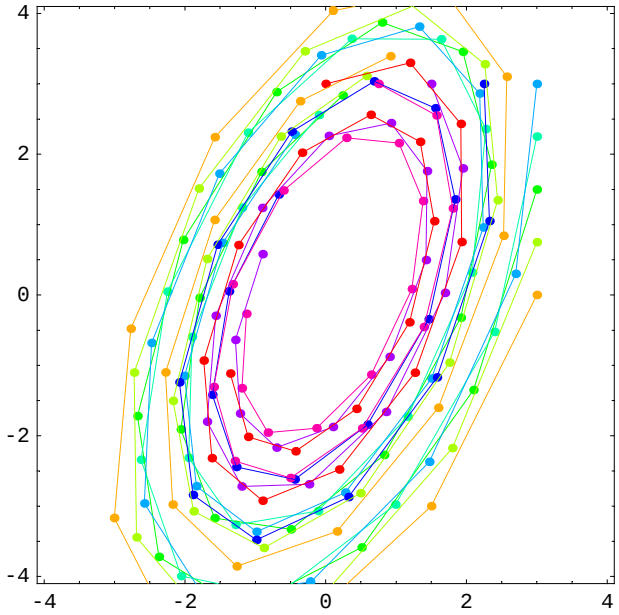
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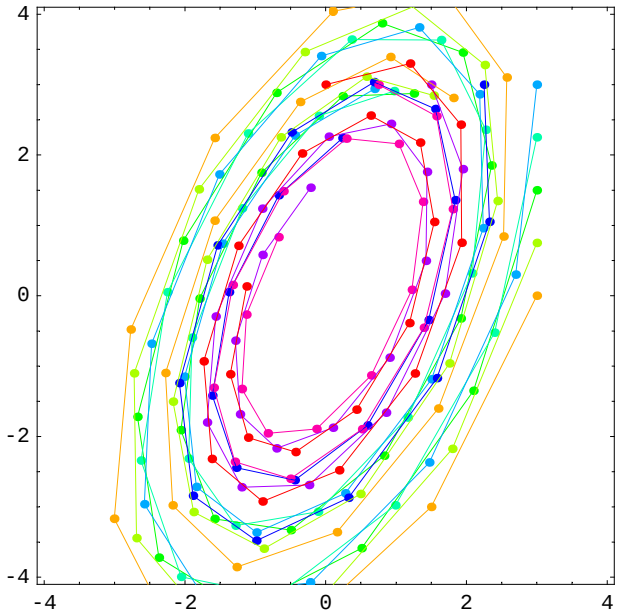
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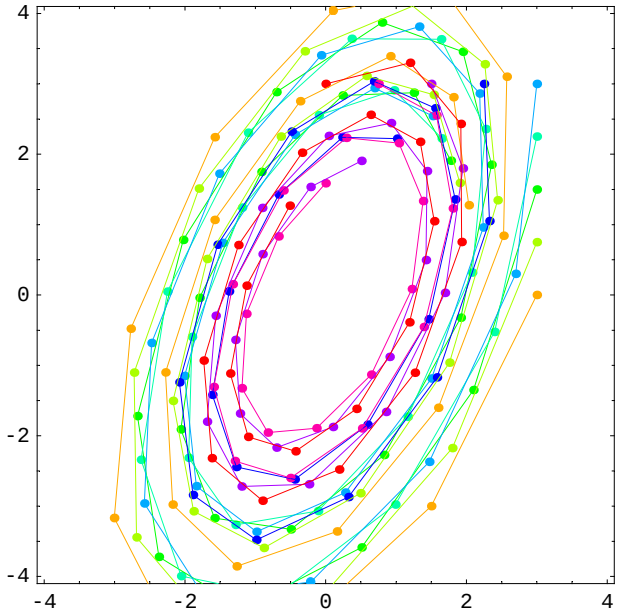
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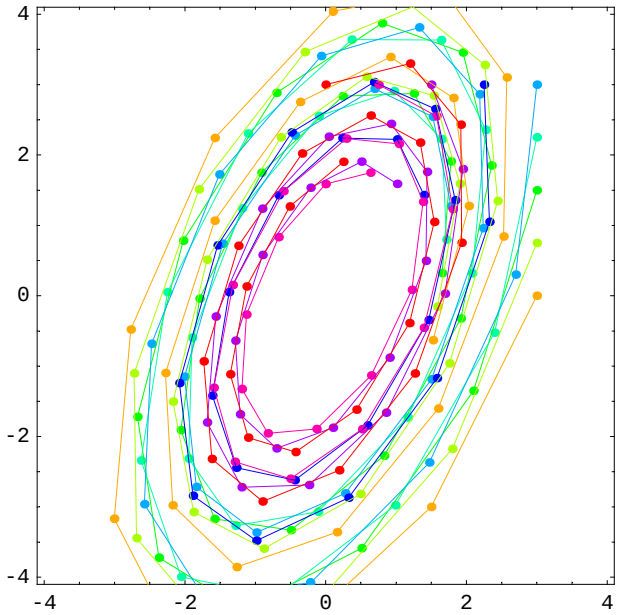
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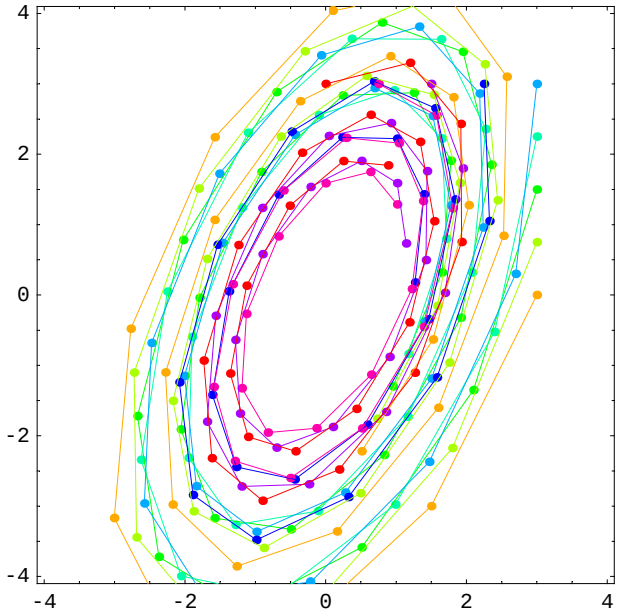
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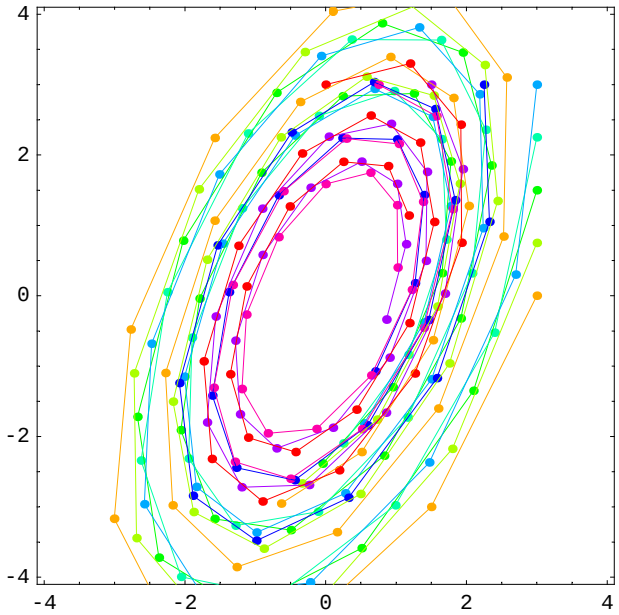
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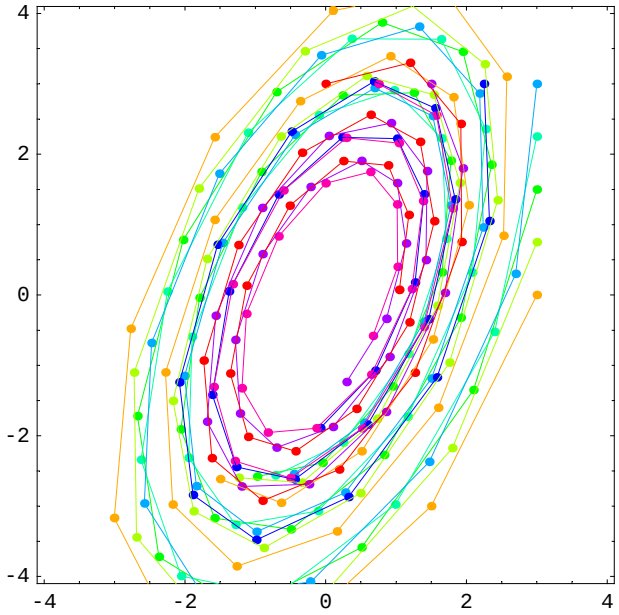
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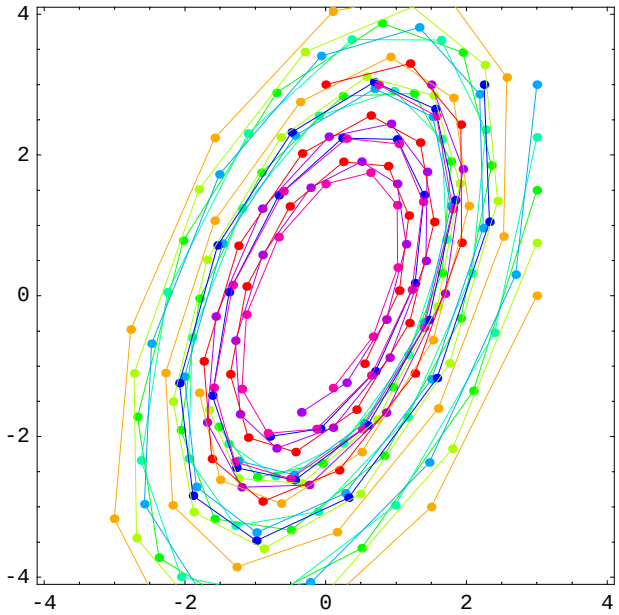
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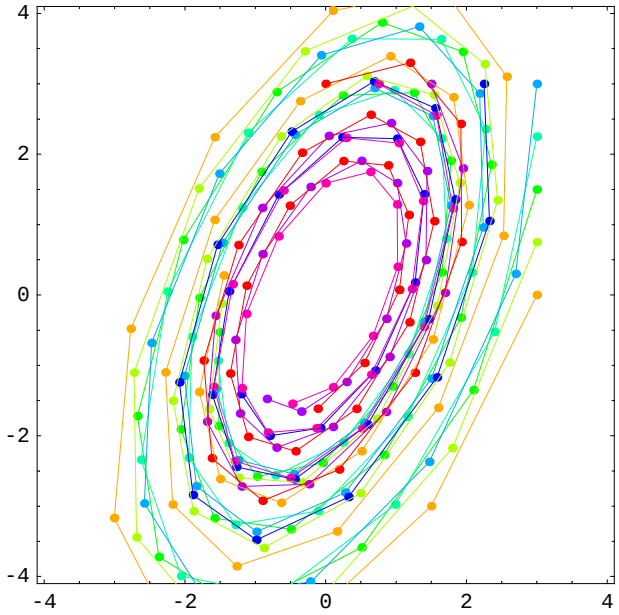
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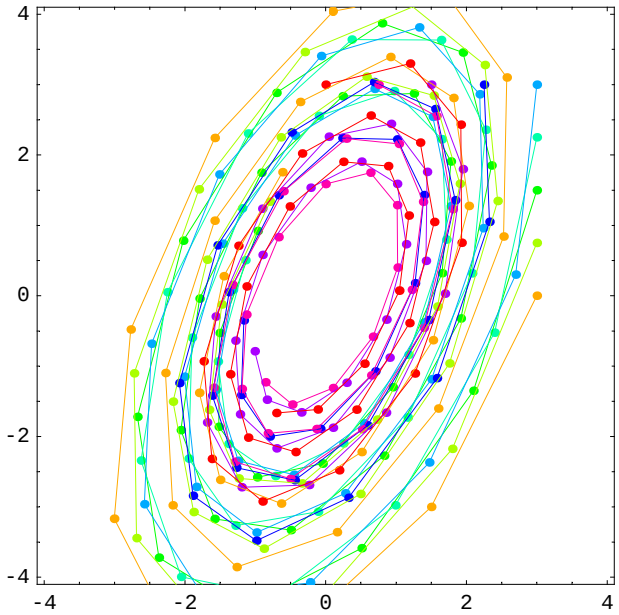
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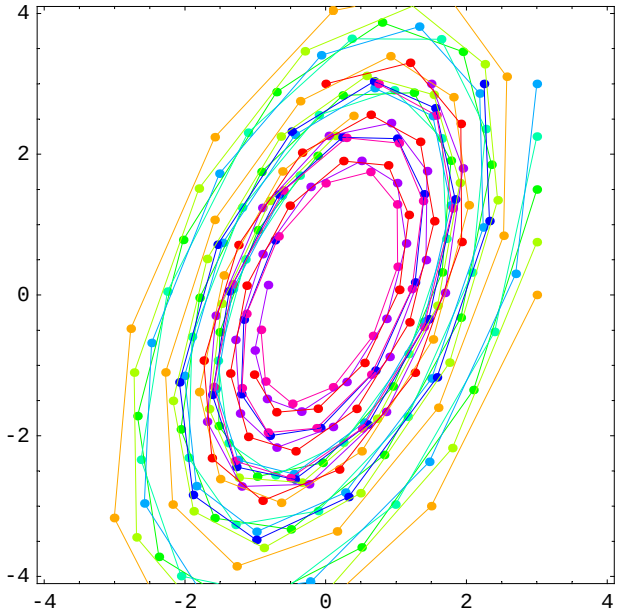
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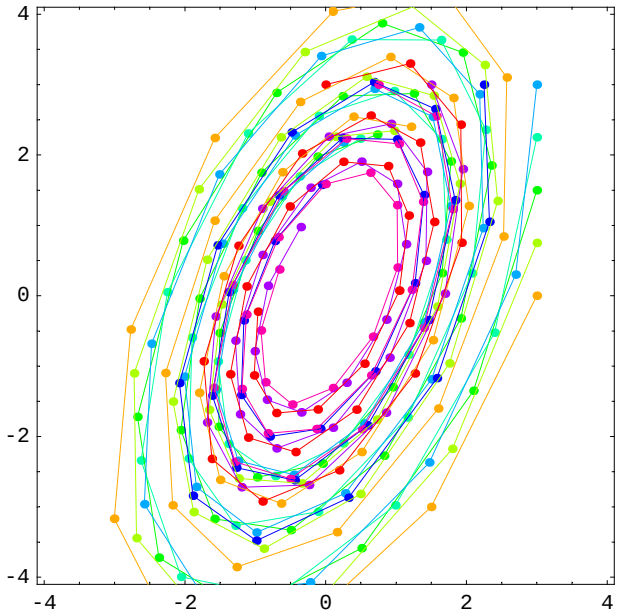
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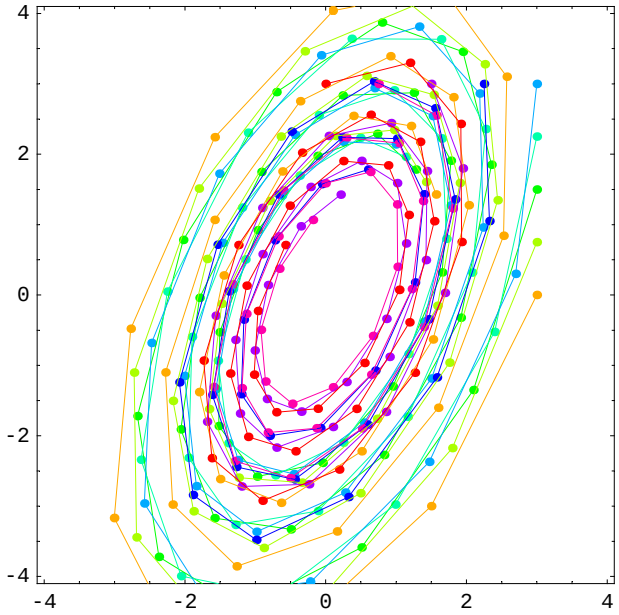
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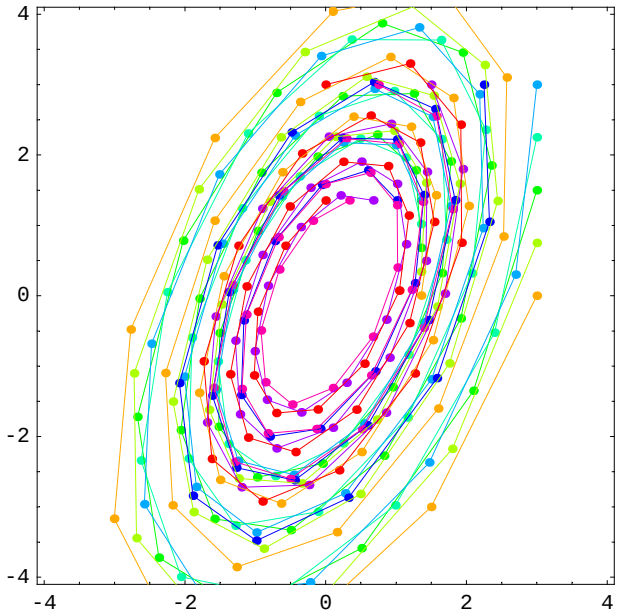
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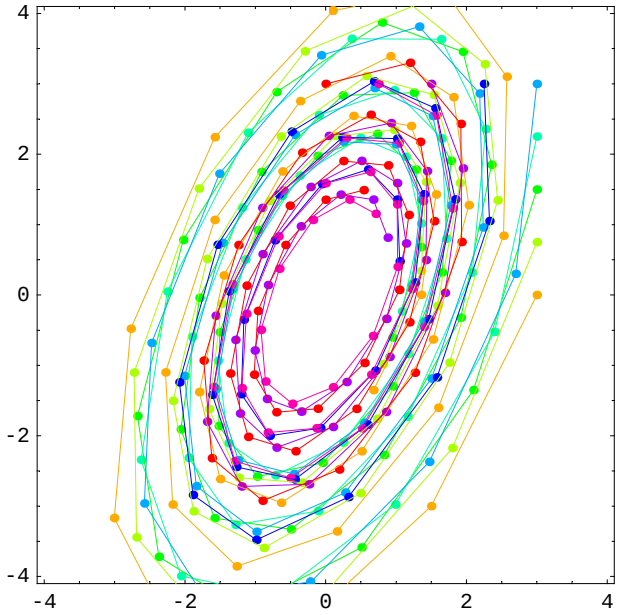
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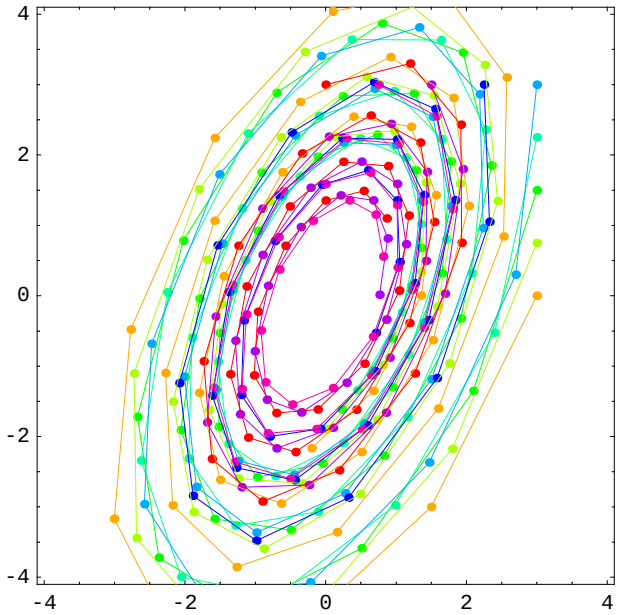
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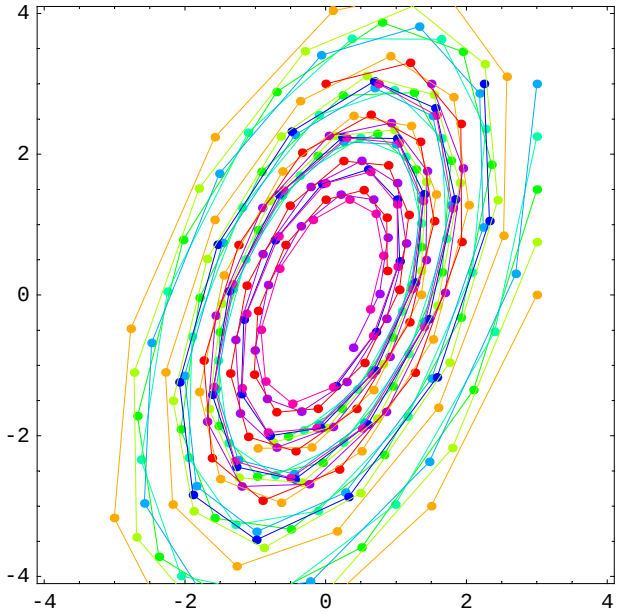
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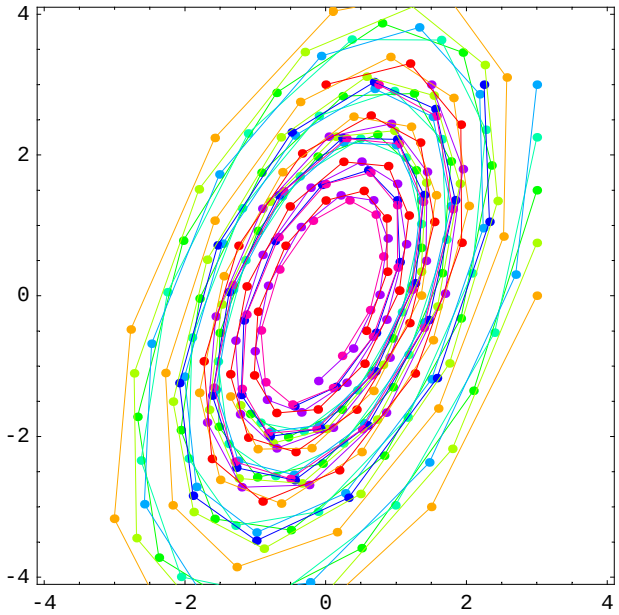
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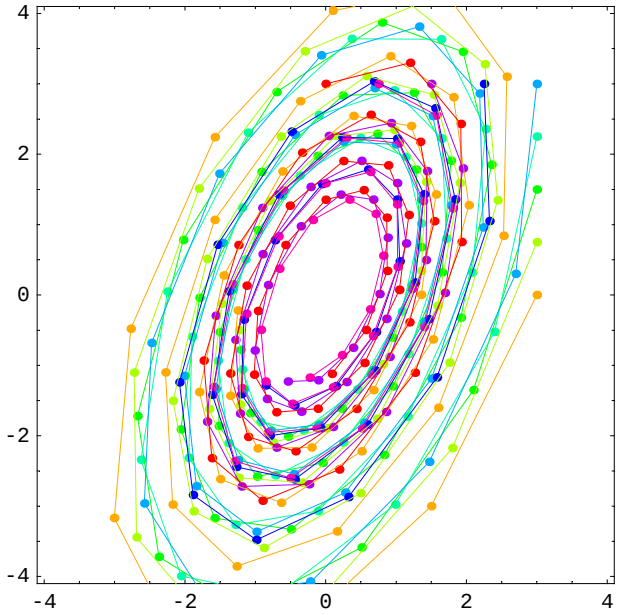
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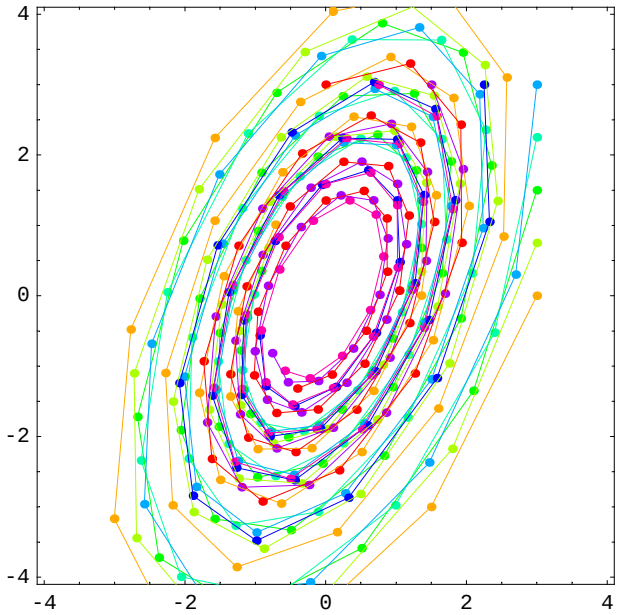
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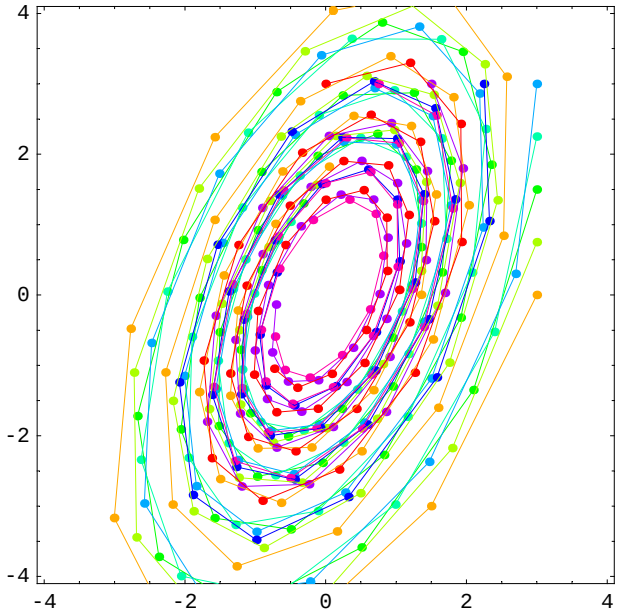
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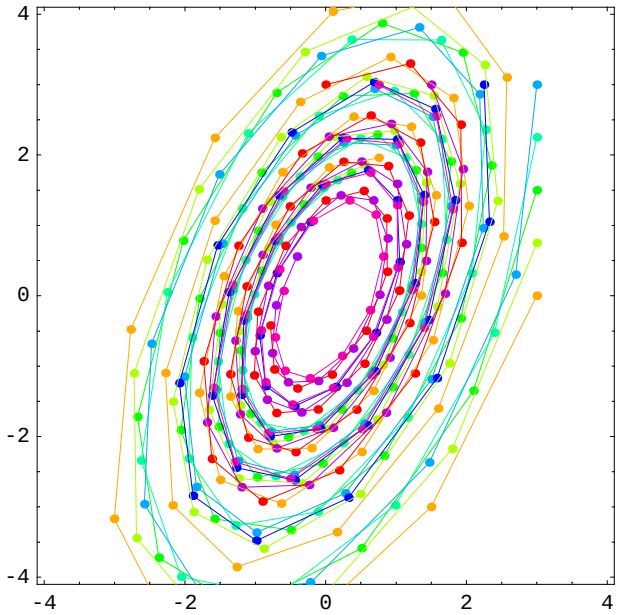
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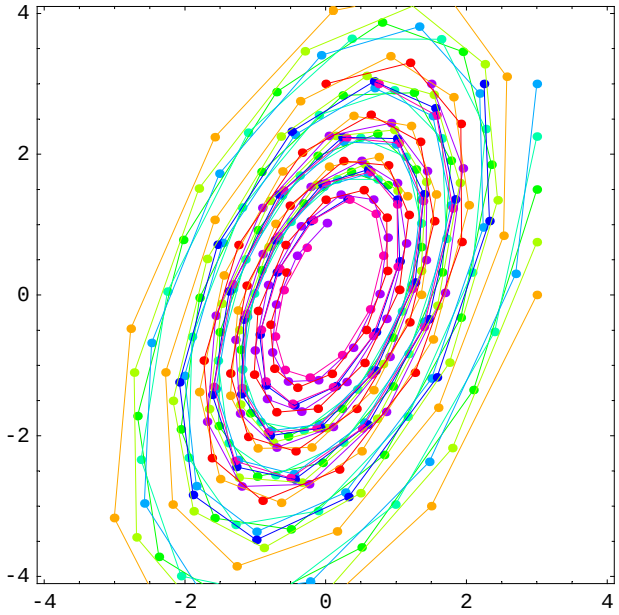
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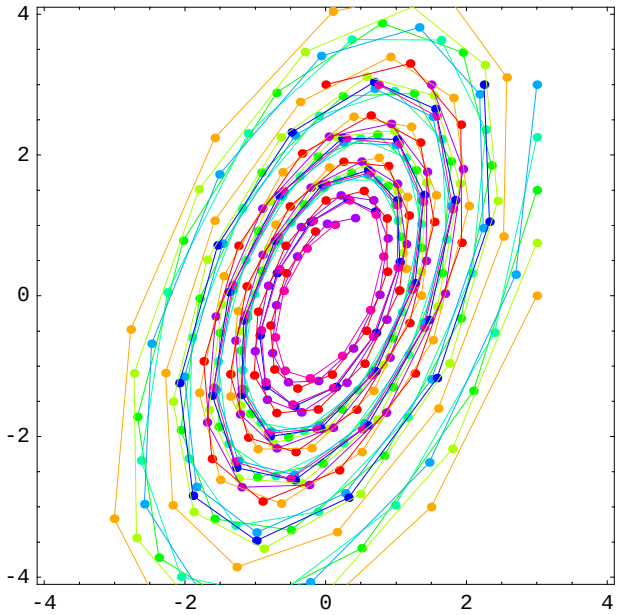
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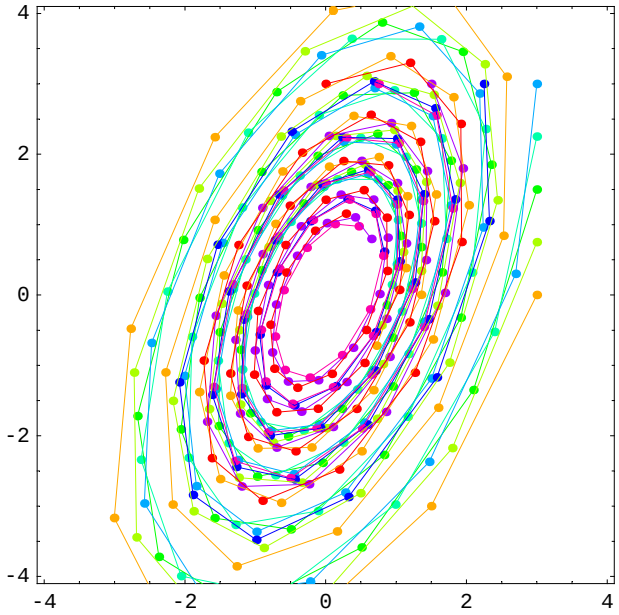
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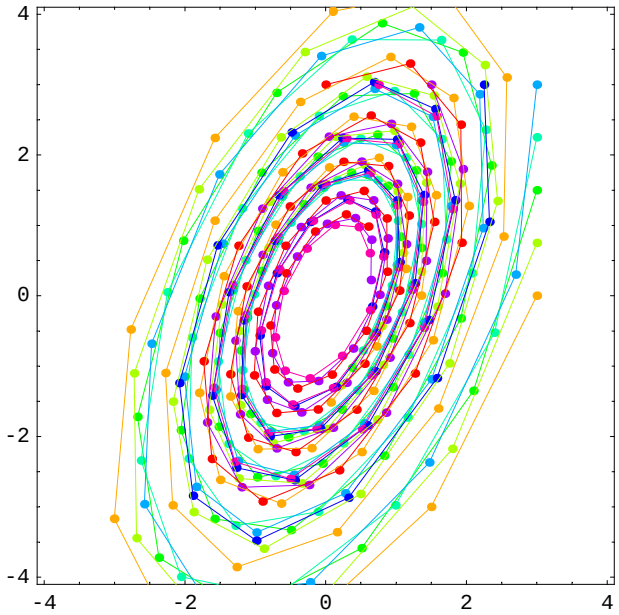
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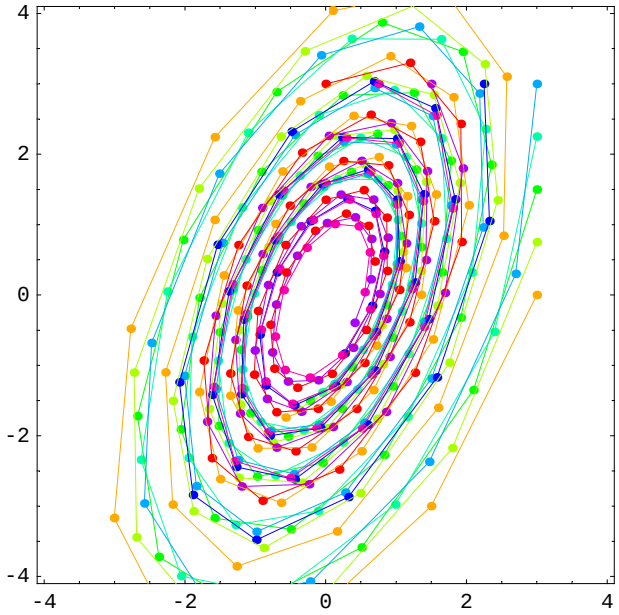
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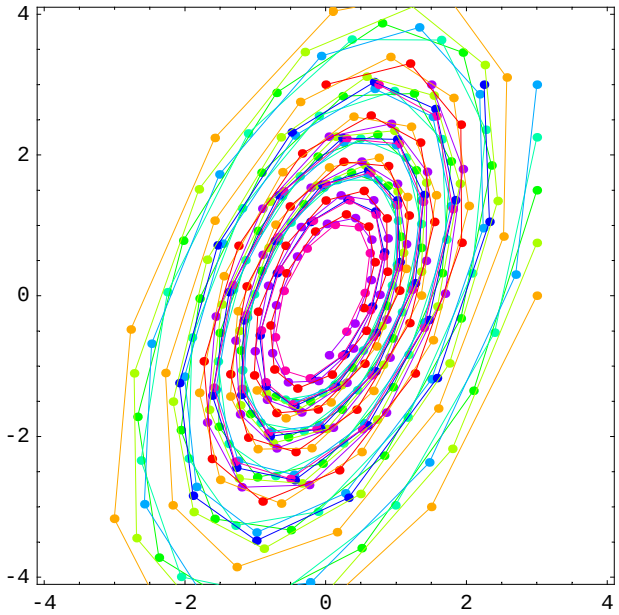
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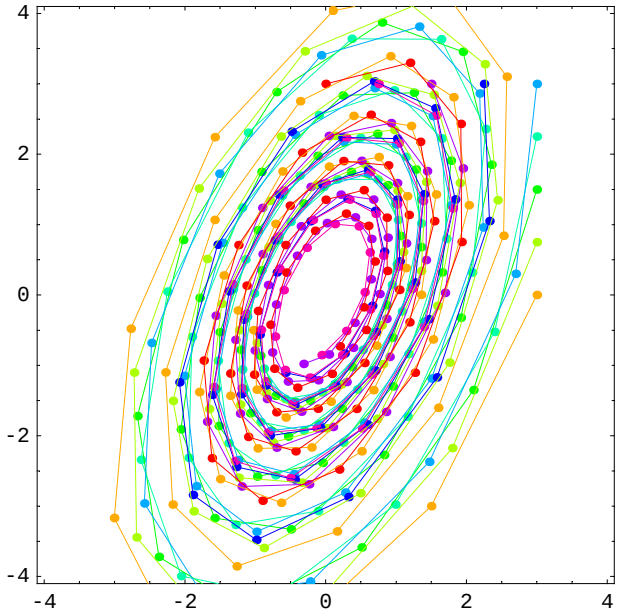
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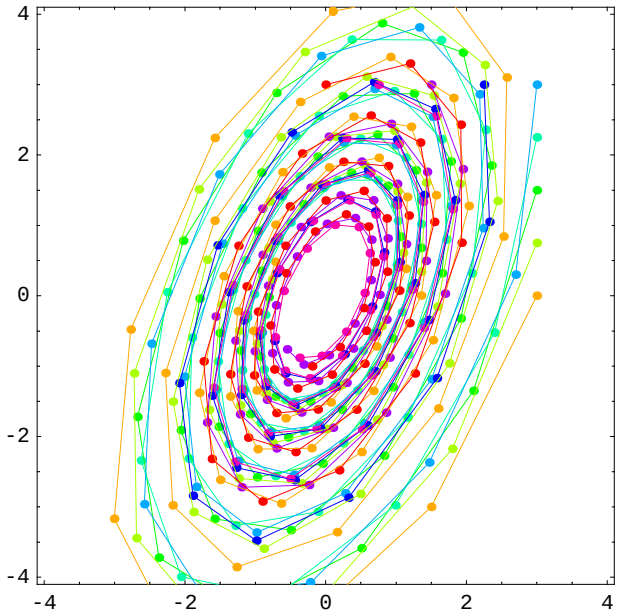
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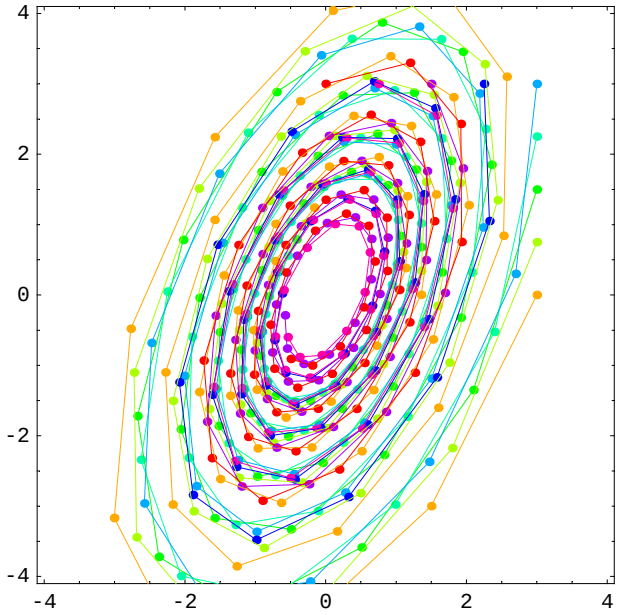
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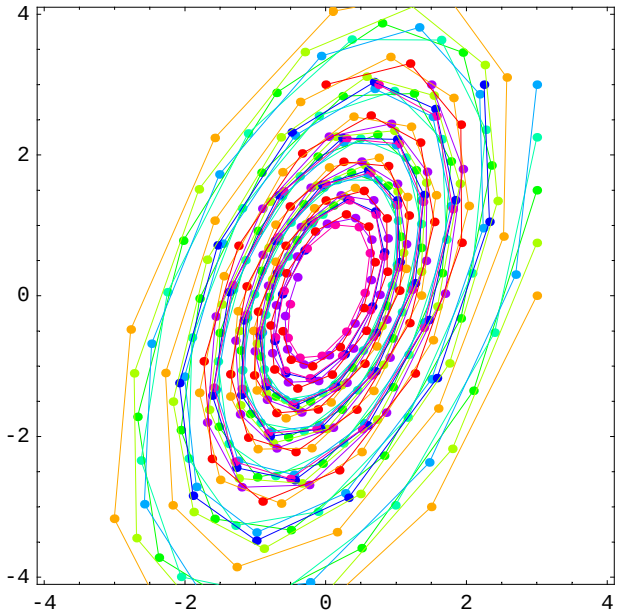
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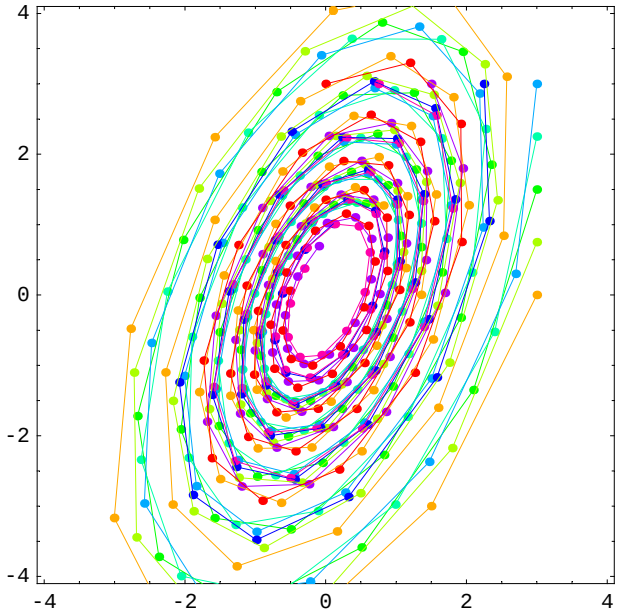


Rotation

In[64]:=
```
mA = 0.95 {{Cos[Pi/3], -Sin[Pi/3]}, {Sin[Pi/3], Cos[Pi/3.]}}
```

Out[64]=
```
{{0.475, -0.822724}, {0.822724, 0.475}}
```

In[65]:=
```
Eigensystem[mA]
```

Out[65]=
```
{{0.475 + 0.822724 i, 0.475 - 0.822724 i},
 {{0.707107 + 0. i, 1.6263 × 10^-17 - 0.707107 i}, {0.707107 + 0. i, 1.6263 × 10^-17 + 0.707107 i}}}
```

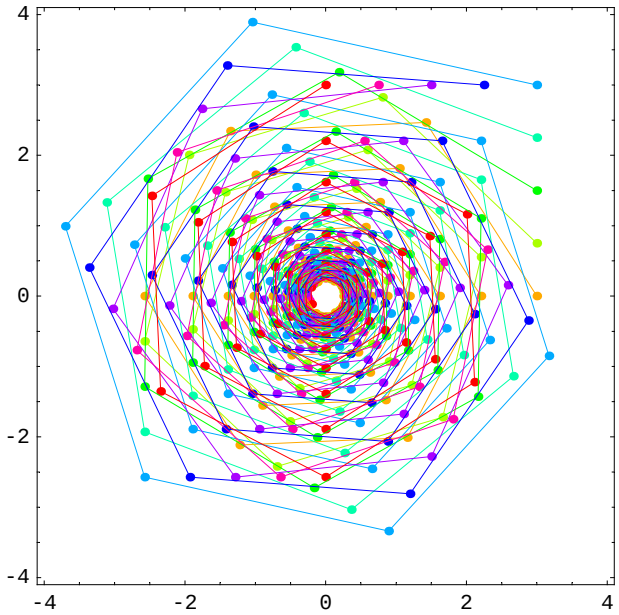
In[66]:=
```
Abs[Eigensystem[mA][[1, 1]]]
```

Out[66]=
```
0.95
```

In[67]:=
```
steps = 50;
ips =
  {{3, 0}, {3, .75}, {3, 1.5}, {3, 2.25}, {3, 3}, {2.25, 3}, {1.5, 3}, {.75, 3}, {0, 3}};

PR = {{-4.1, 4.1}, {-4.1, 4.1}};

Show[
  Graphics[{
    {PointSize[0.015], Hue[#[[2]]/Length[ips]], Table[Point[MatrixPower[mA, k].#[[1]]],
        {k, 0, steps}]} & /@ Transpose[{ips, Range[Length[ips]]}],
    {Thickness[0.002], Hue[#[[2]]/Length[ips]], Line[Table[MatrixPower[mA, k].#[[1]],
        {k, 0, steps}]]} & /@ Transpose[{ips, Range[Length[ips]]}]
   }],
  PlotRange → PR, AspectRatio → Automatic, Frame → True
 ];
```



In[71]:=
```
Table[Show[
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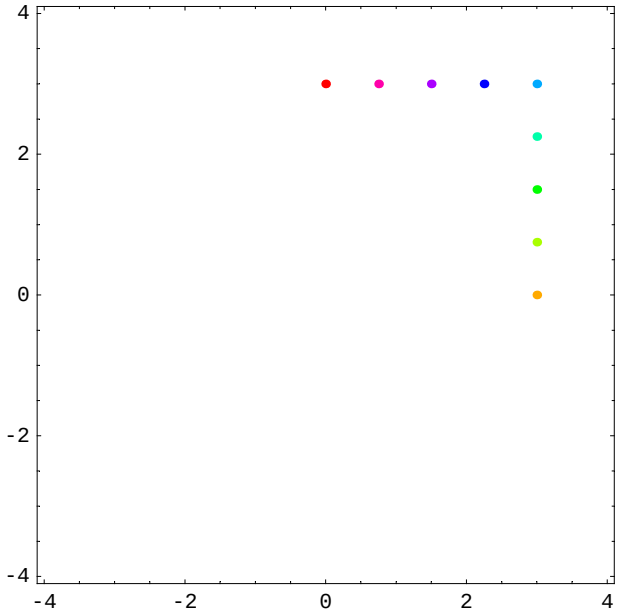
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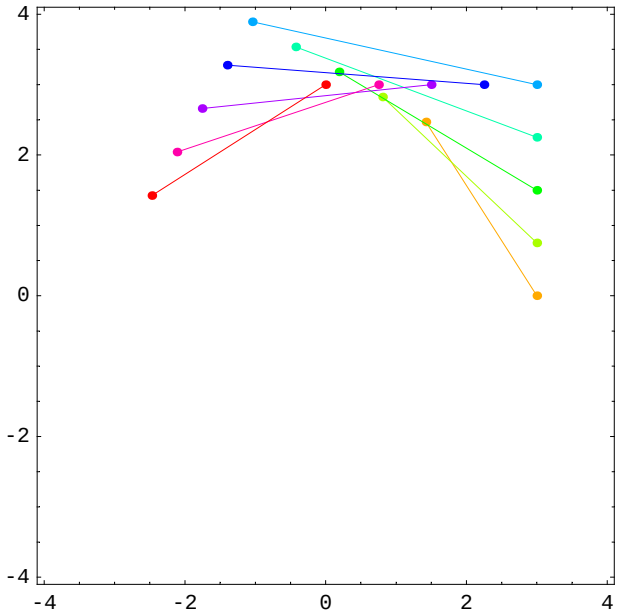
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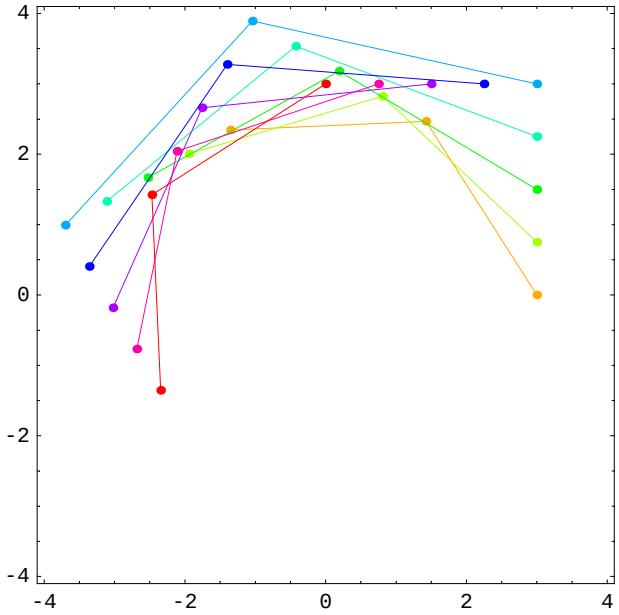
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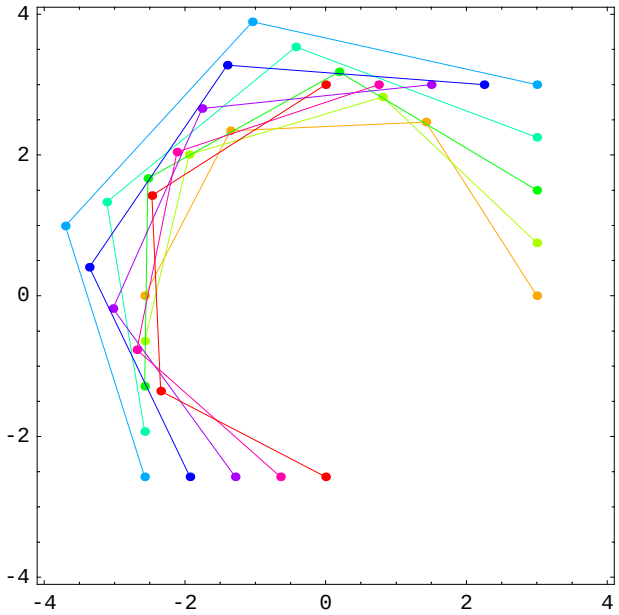
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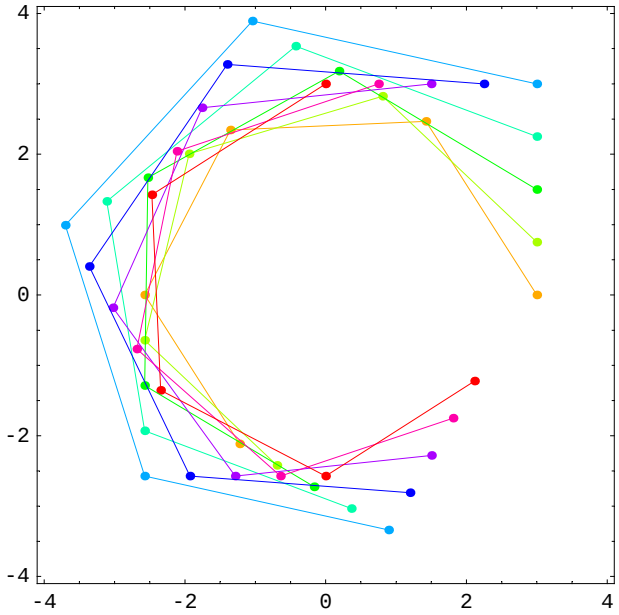
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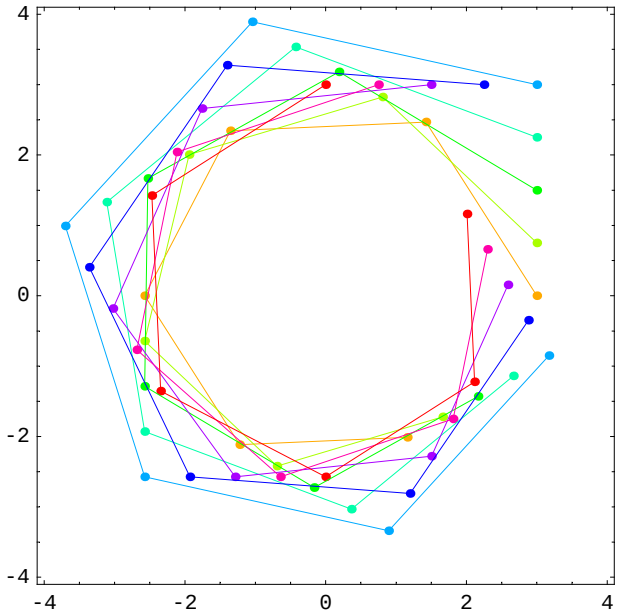
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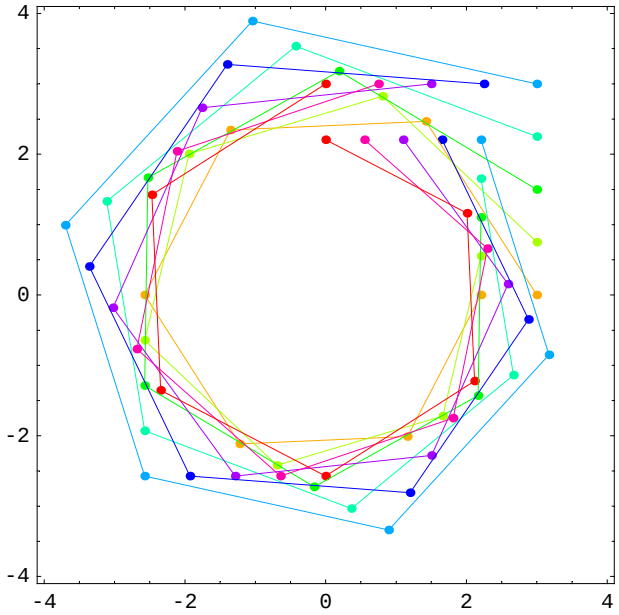
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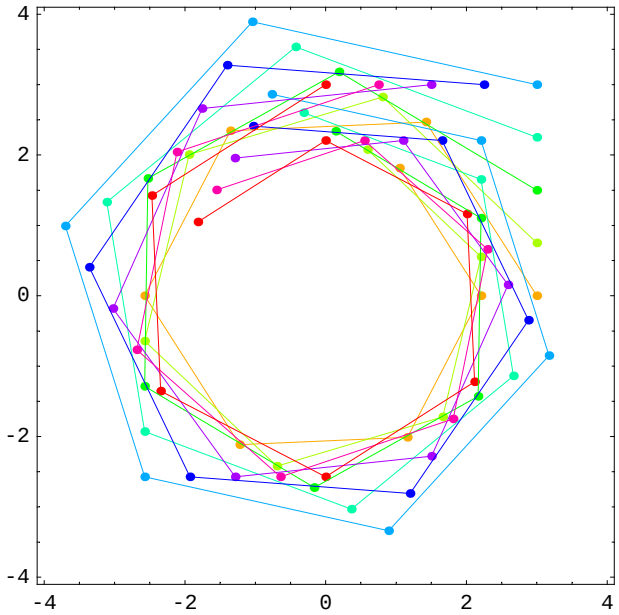
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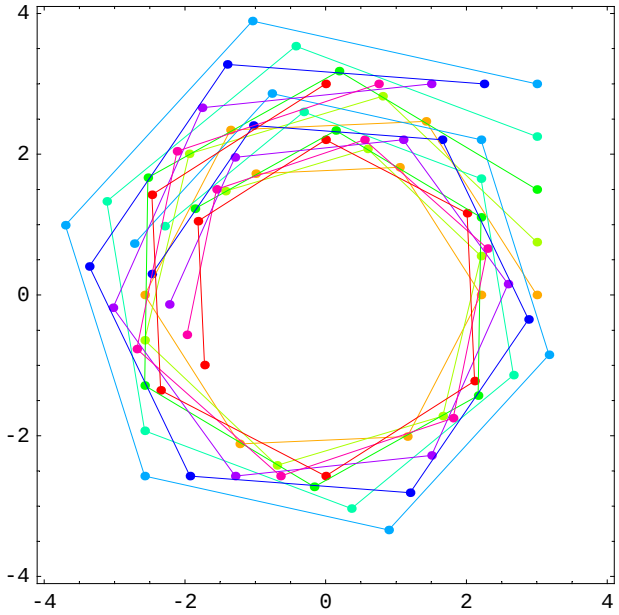
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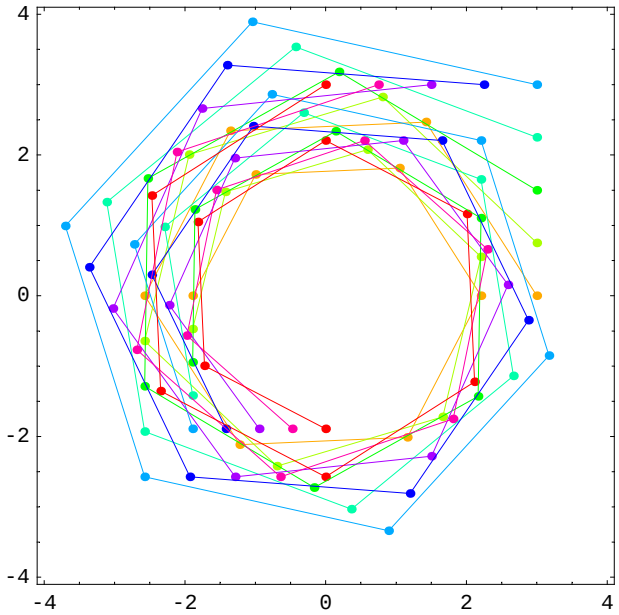
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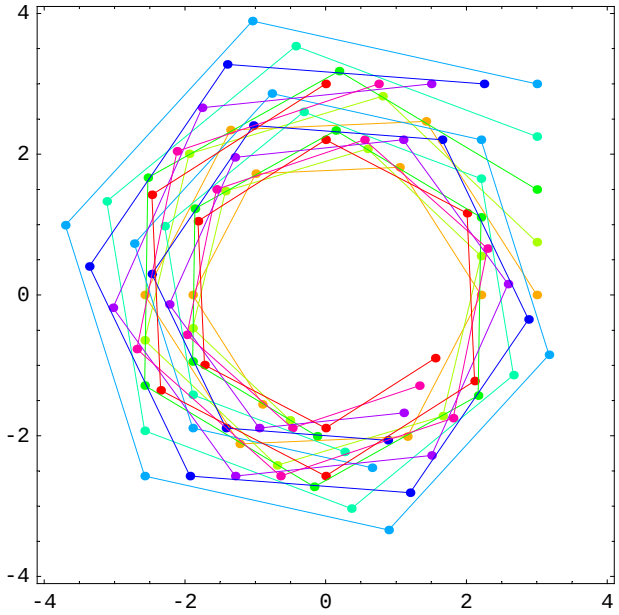
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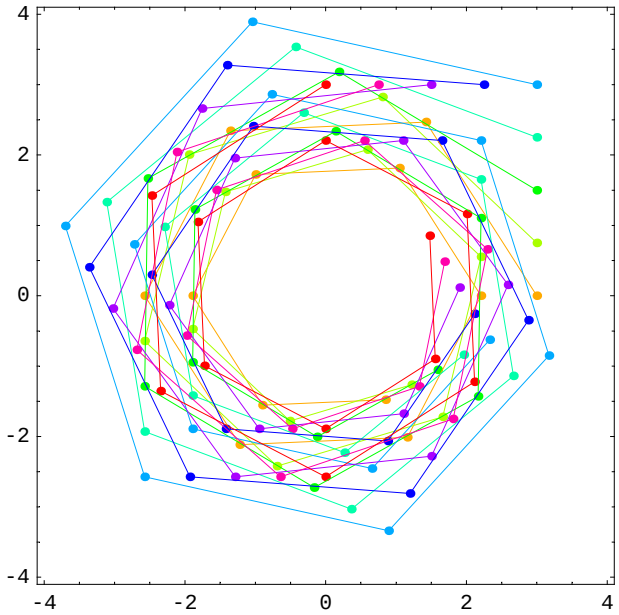
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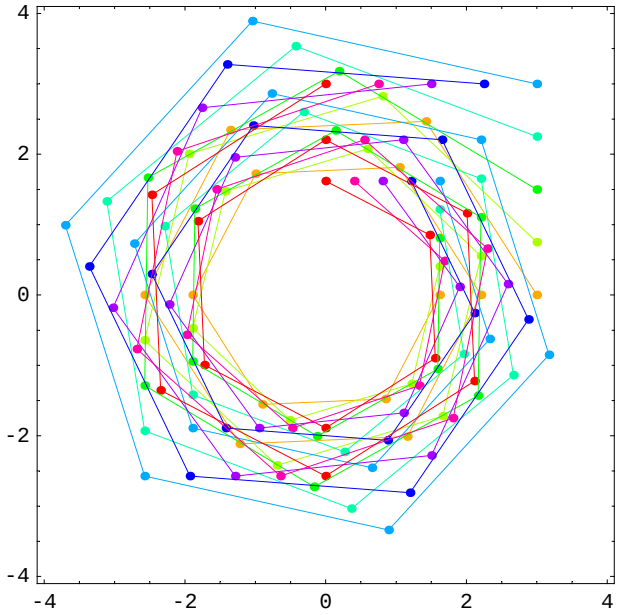
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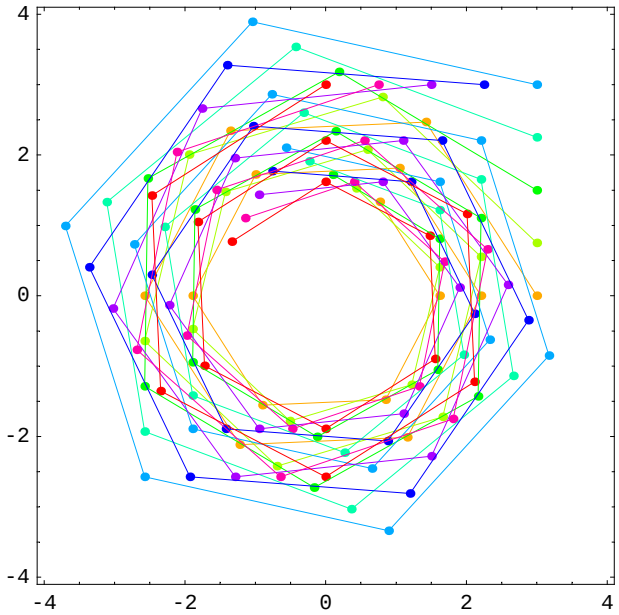
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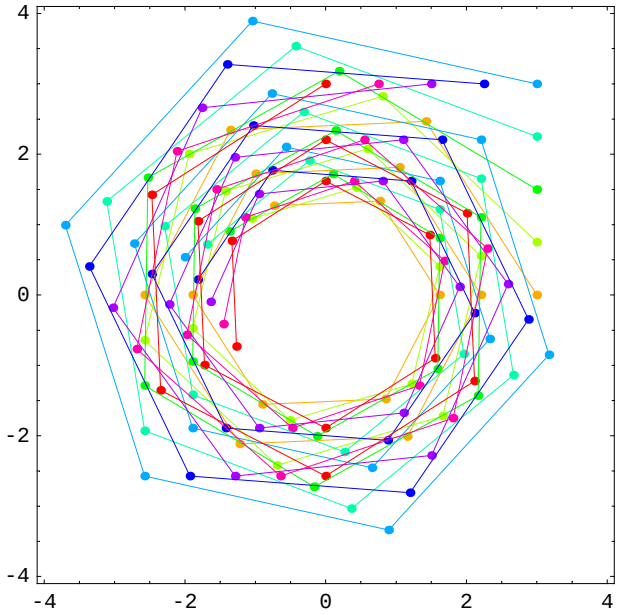
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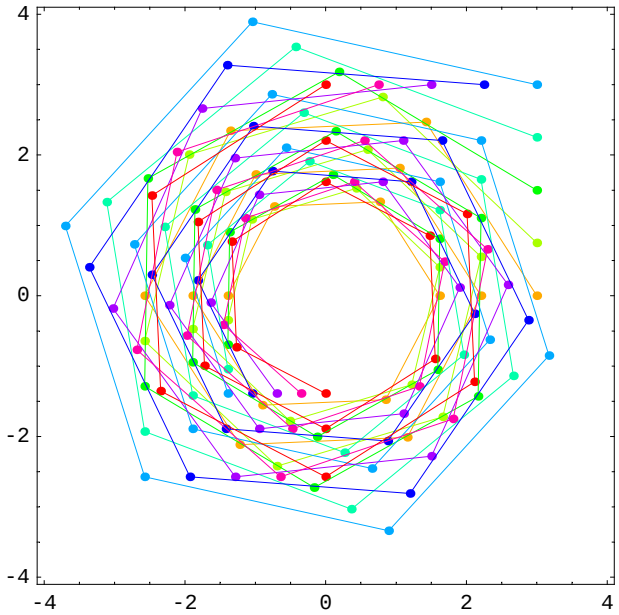
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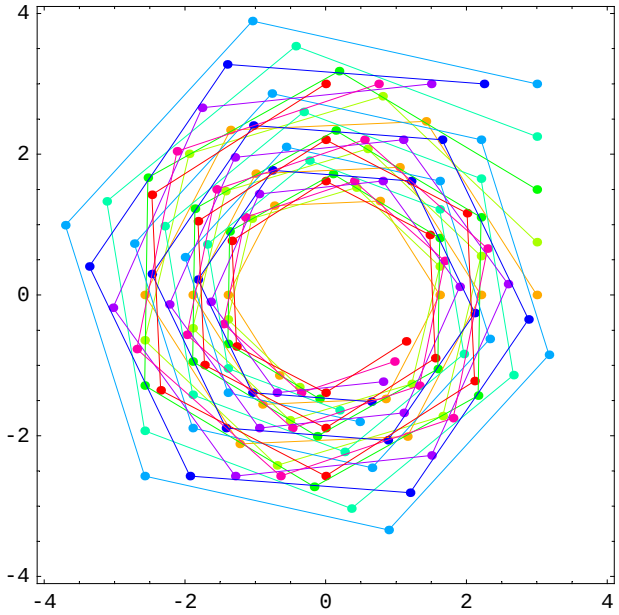
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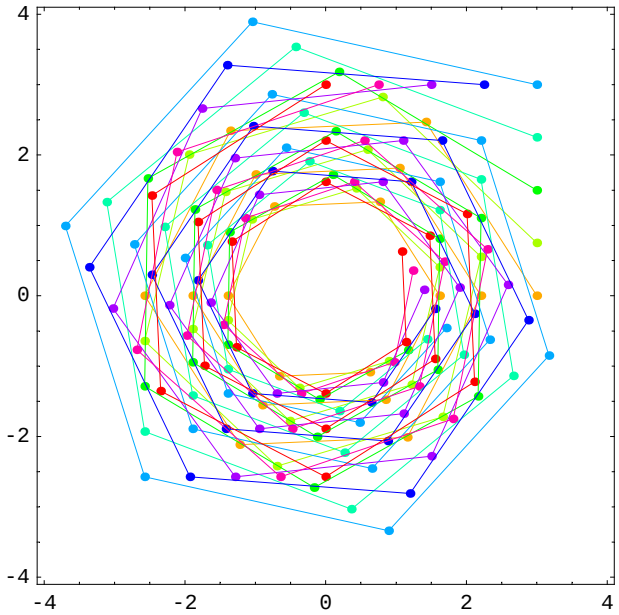
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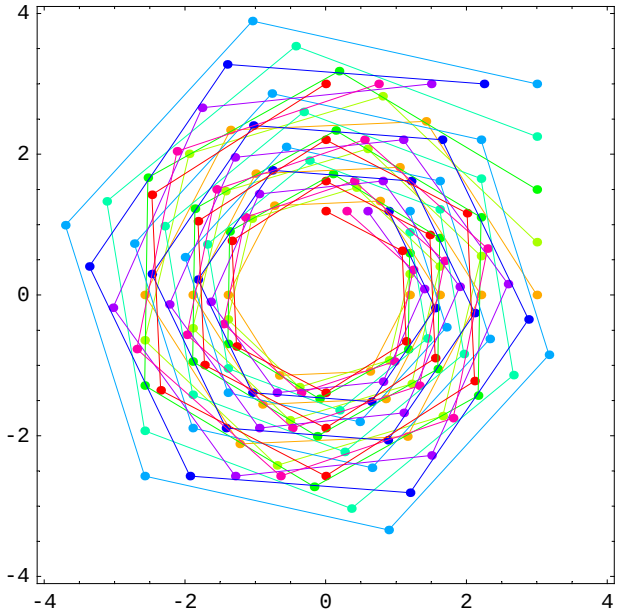
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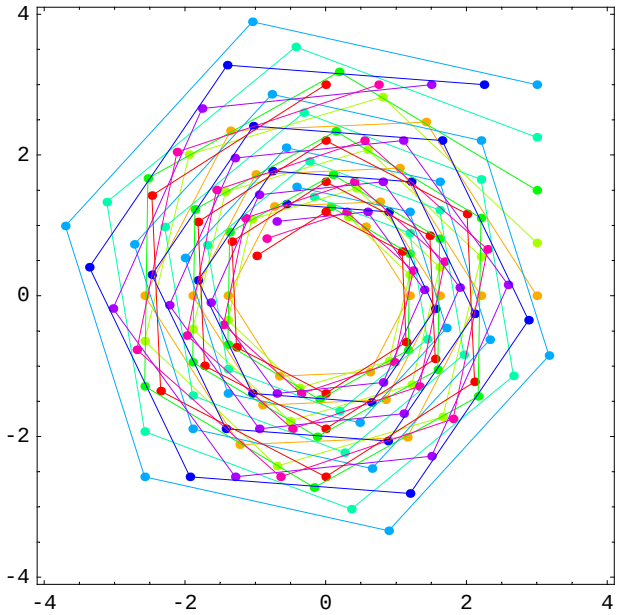
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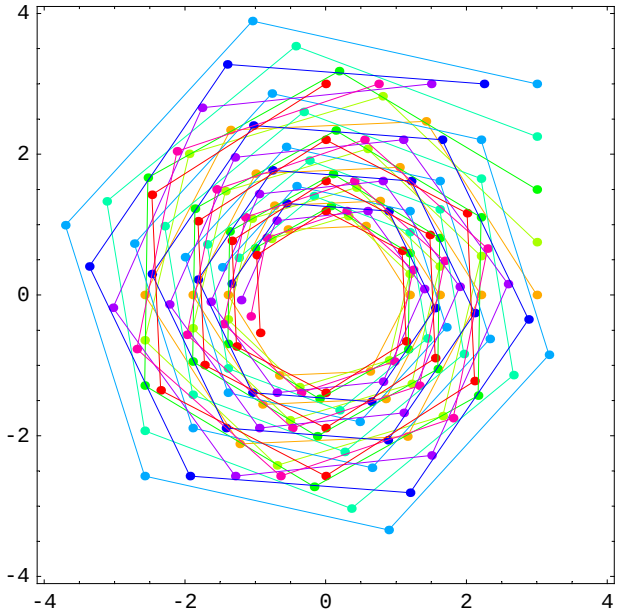
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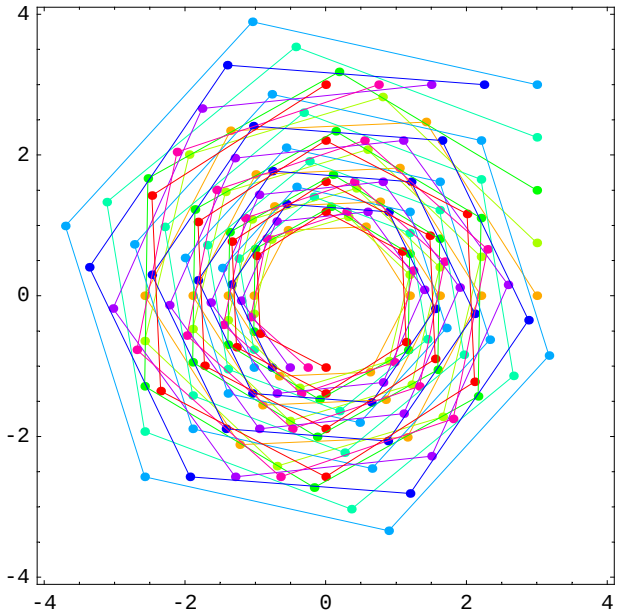
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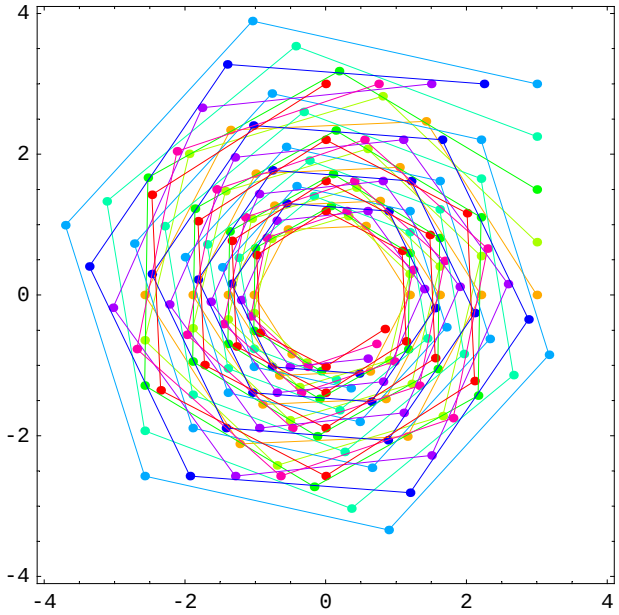
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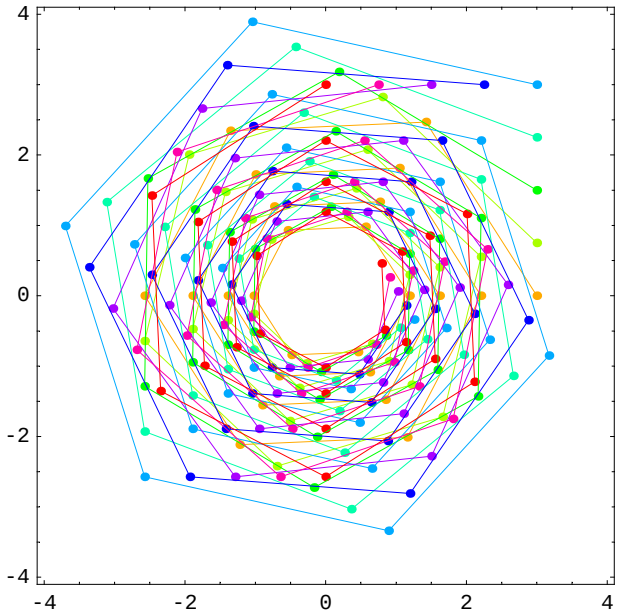
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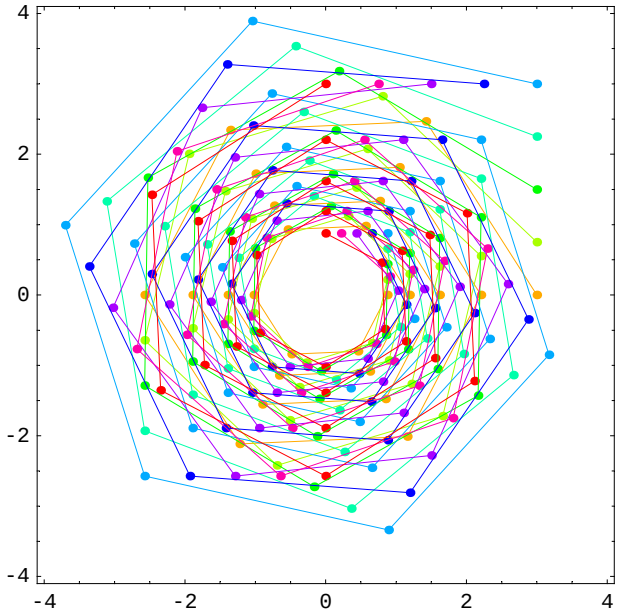
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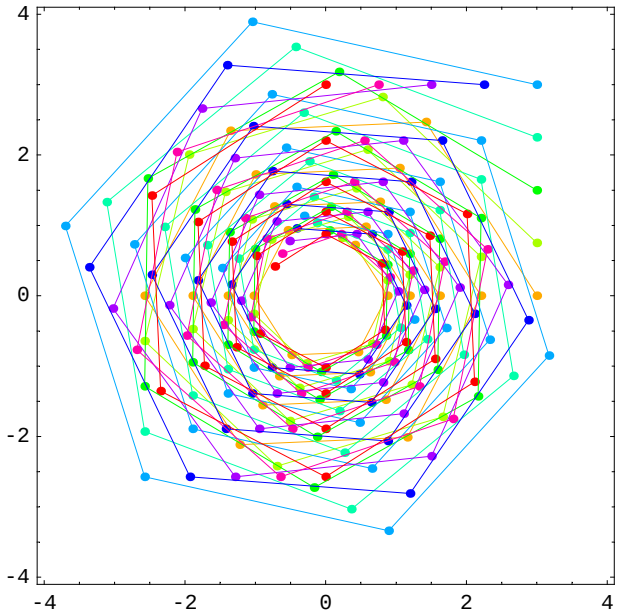
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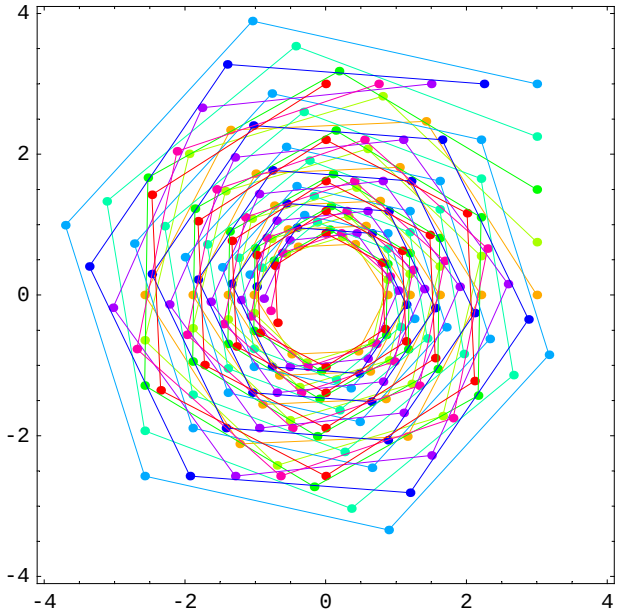
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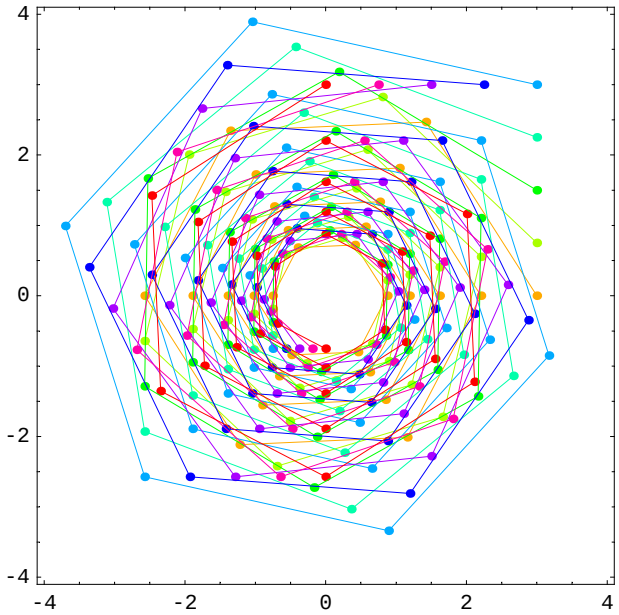
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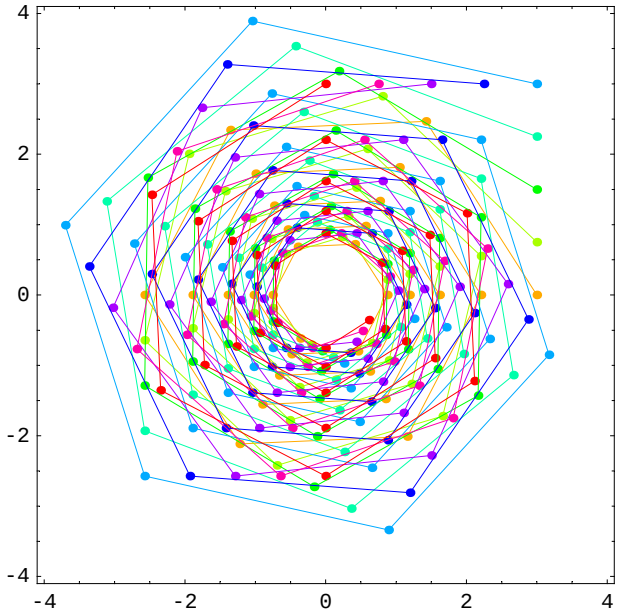
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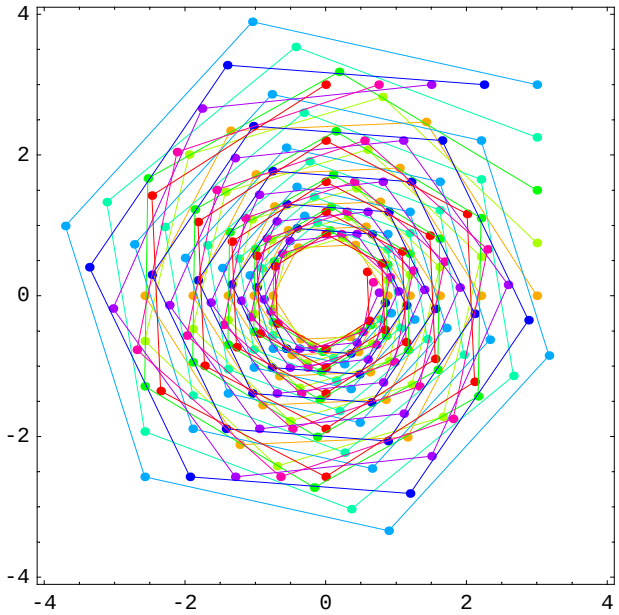
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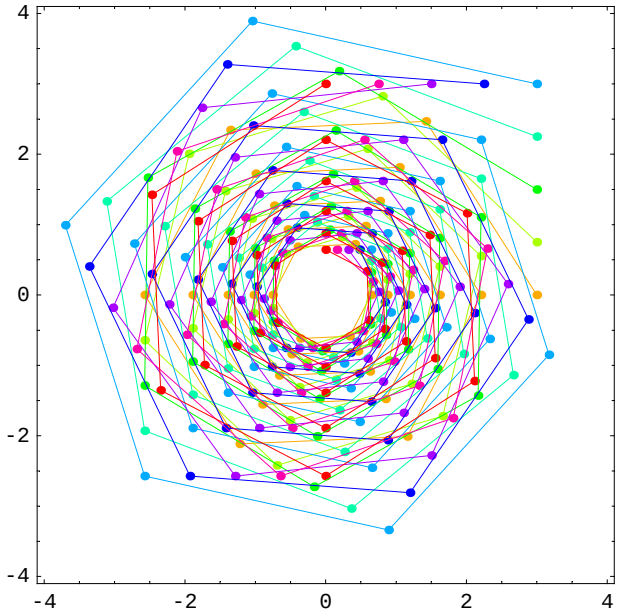
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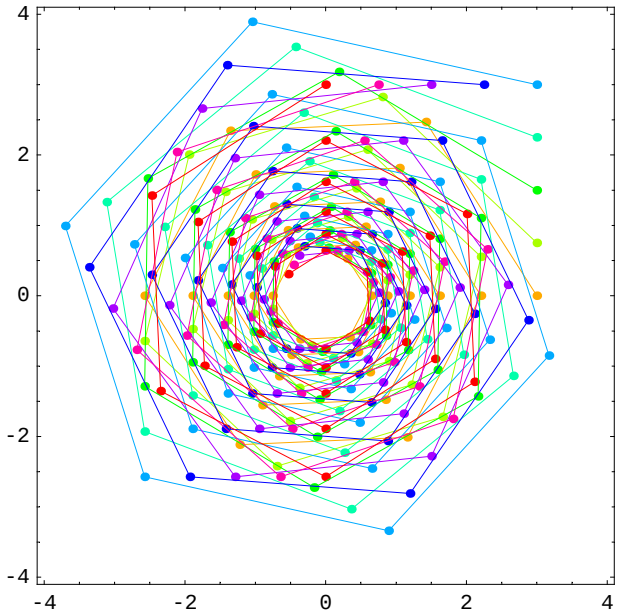
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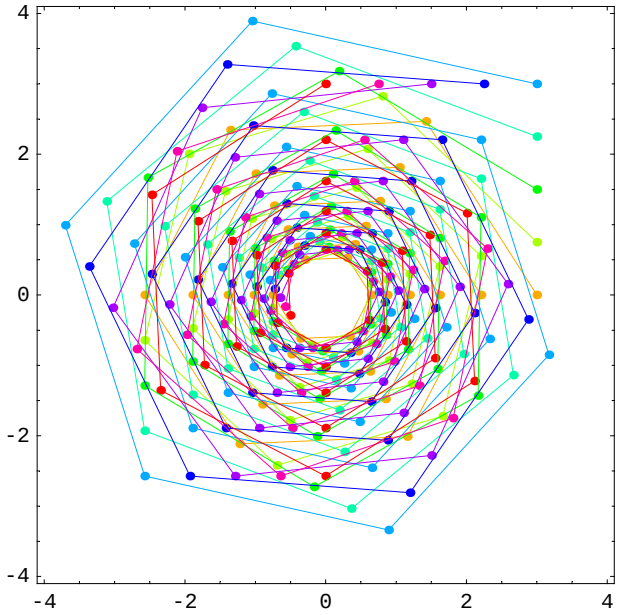
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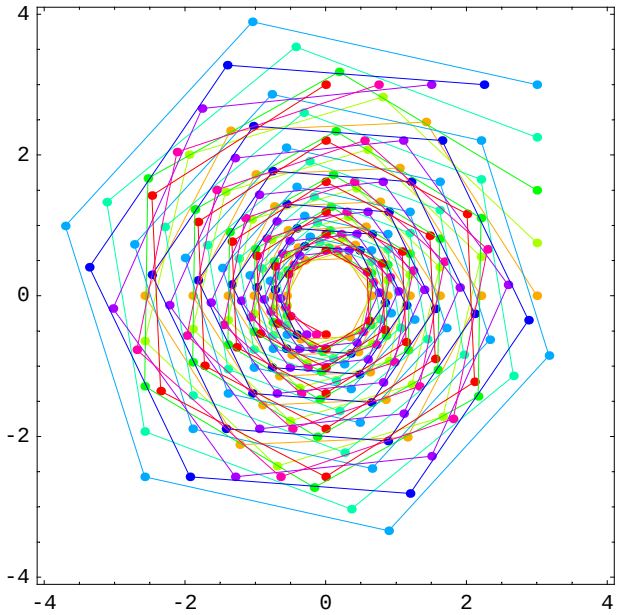
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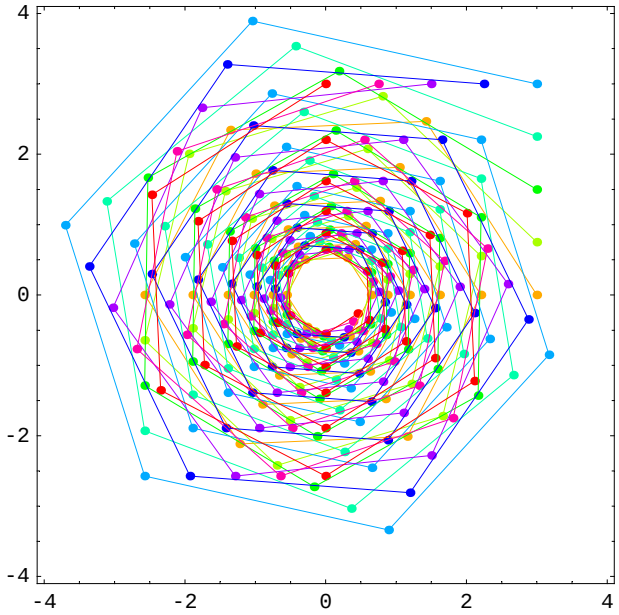
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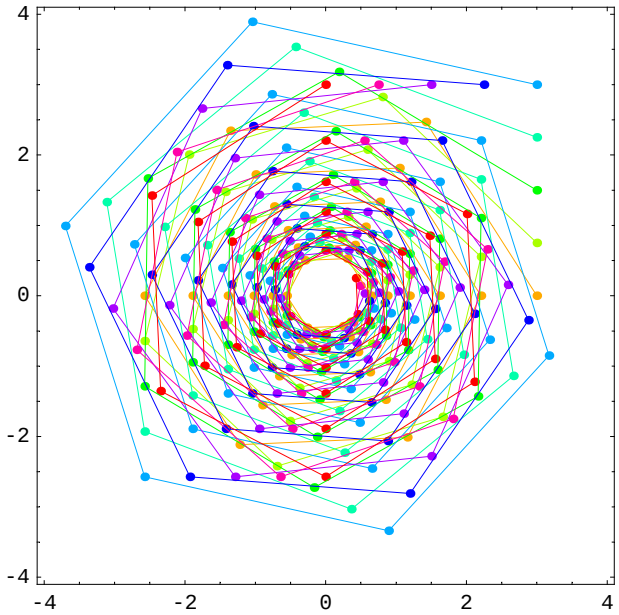
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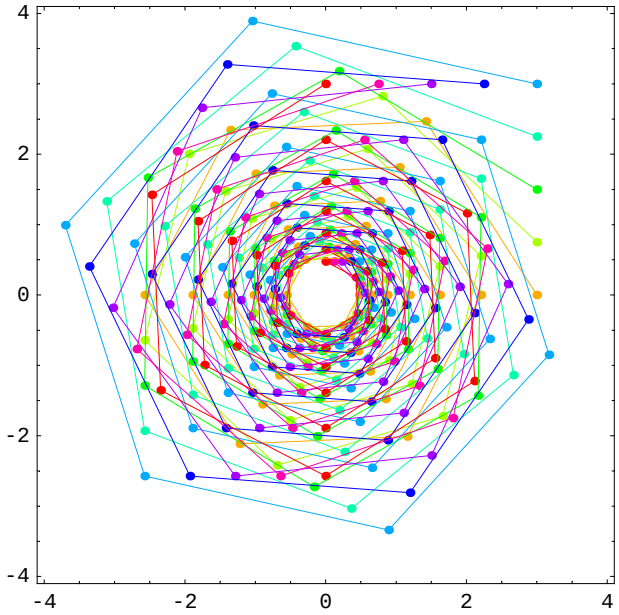
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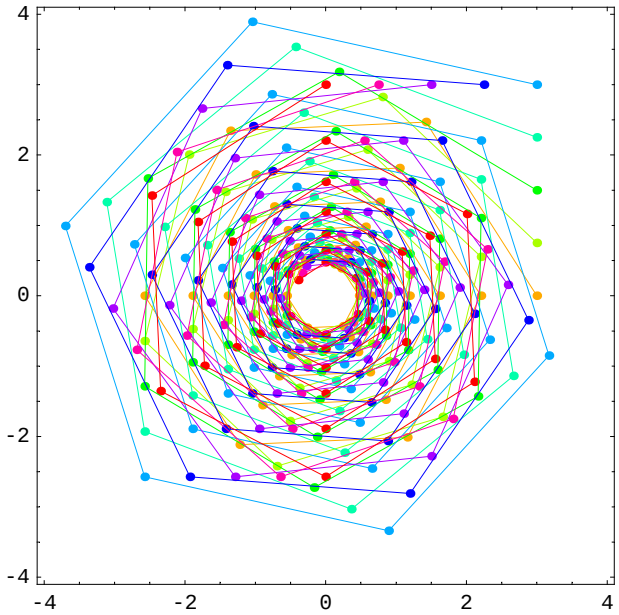
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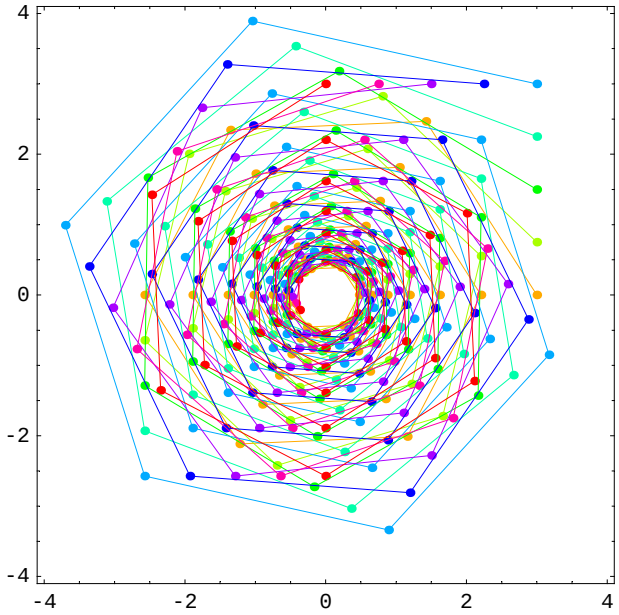
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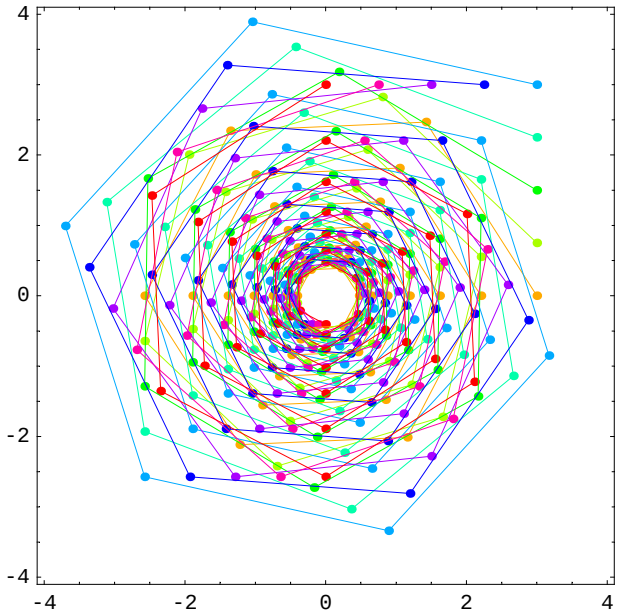
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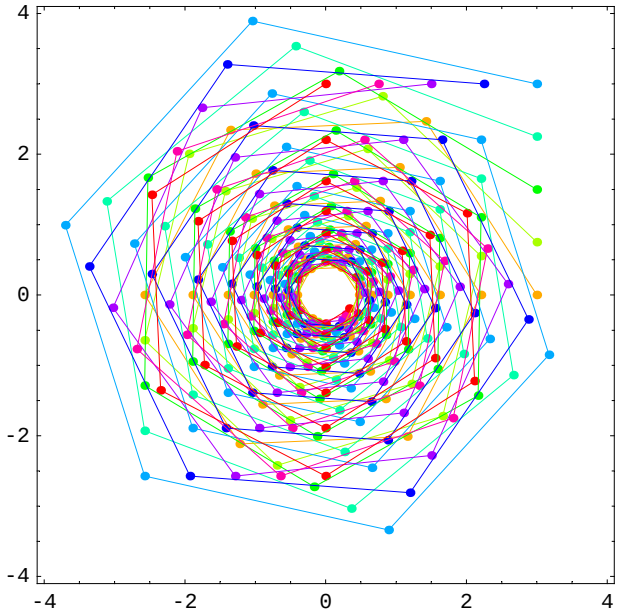
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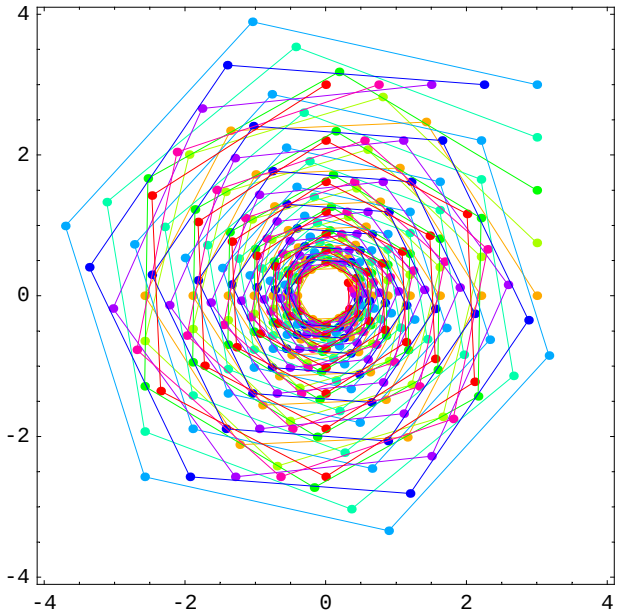
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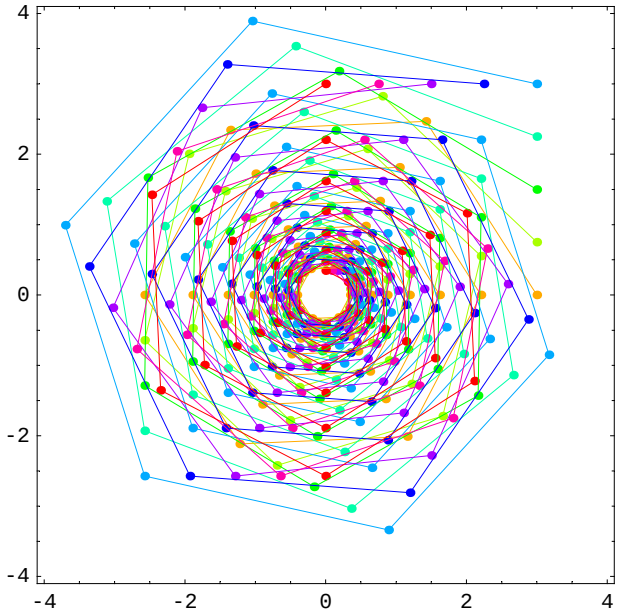
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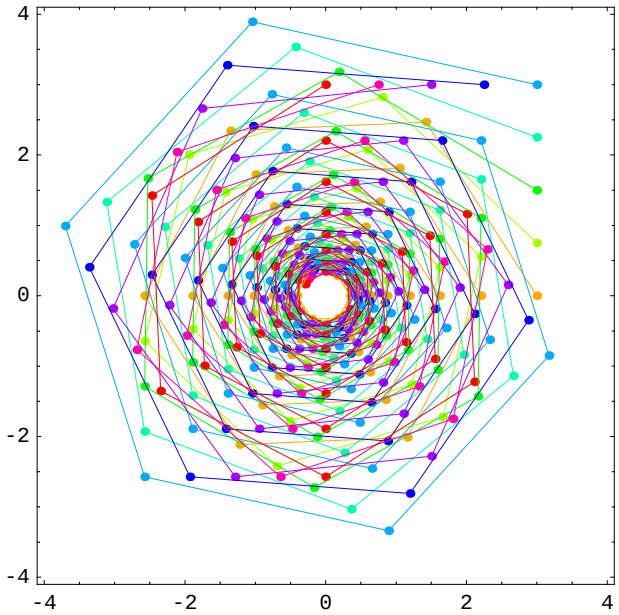
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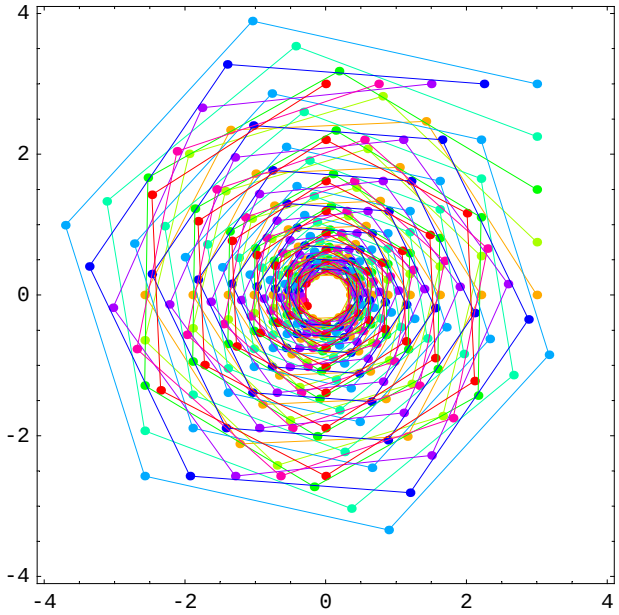
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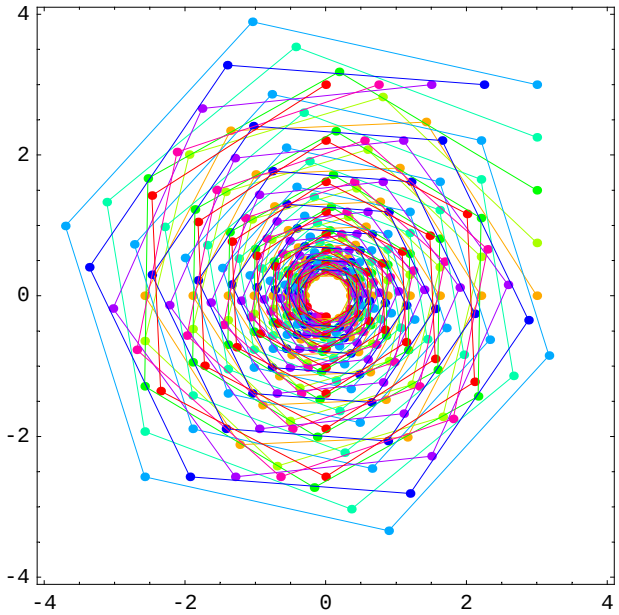
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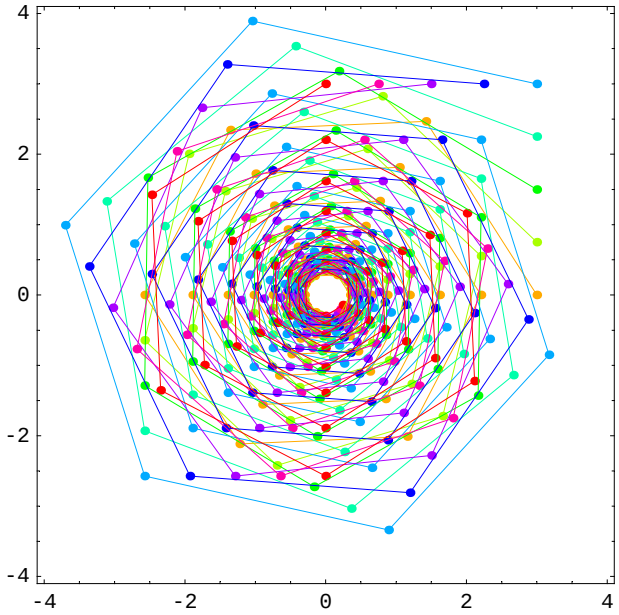
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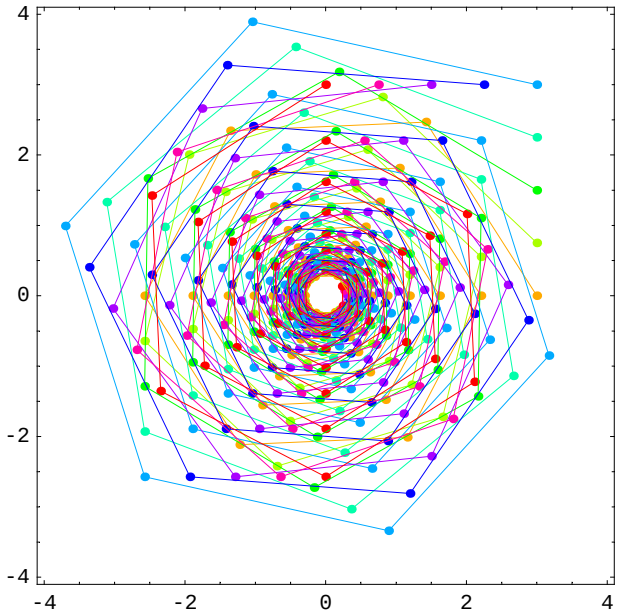
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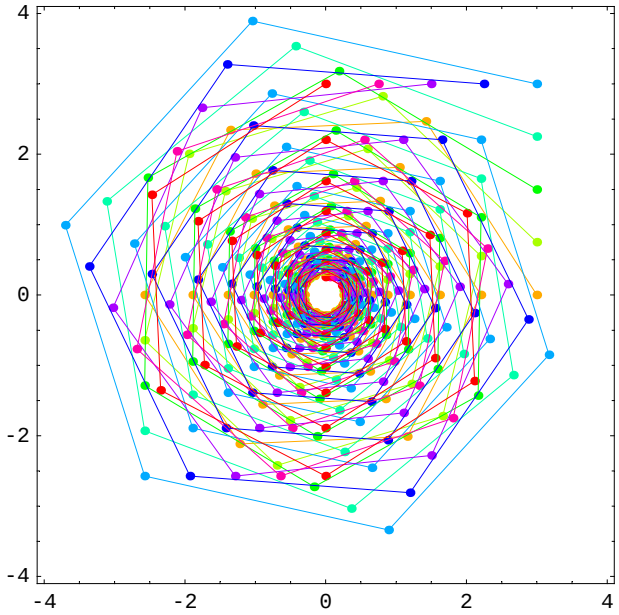
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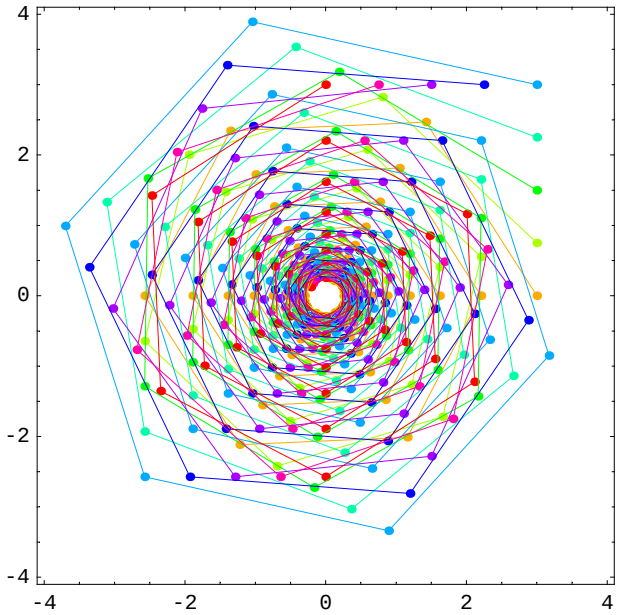
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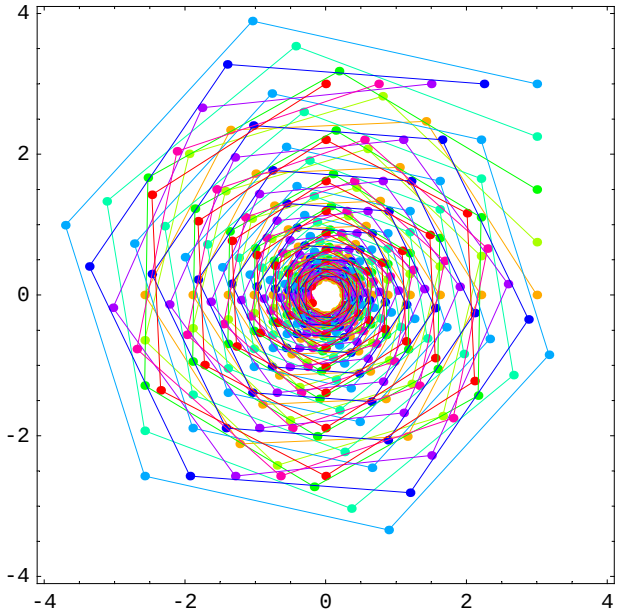
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