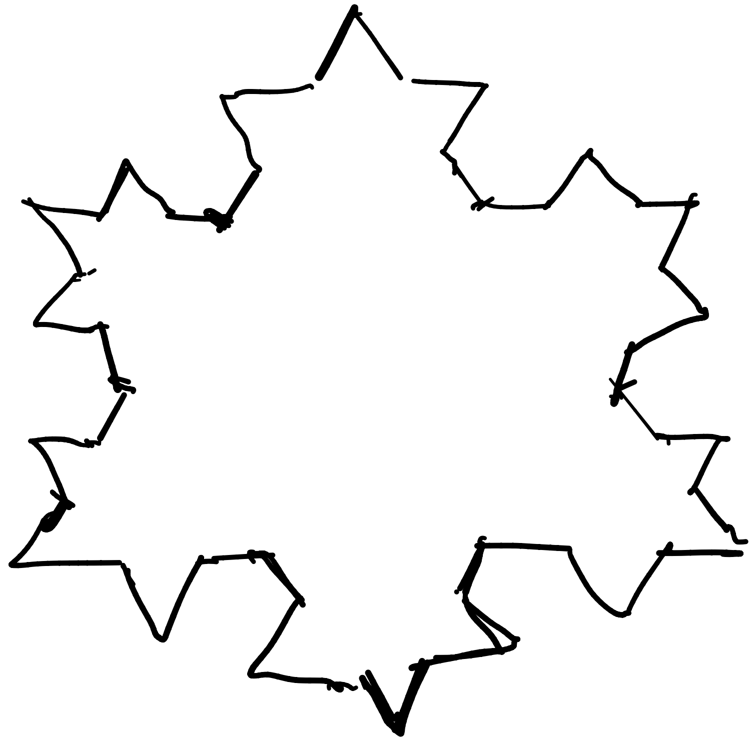
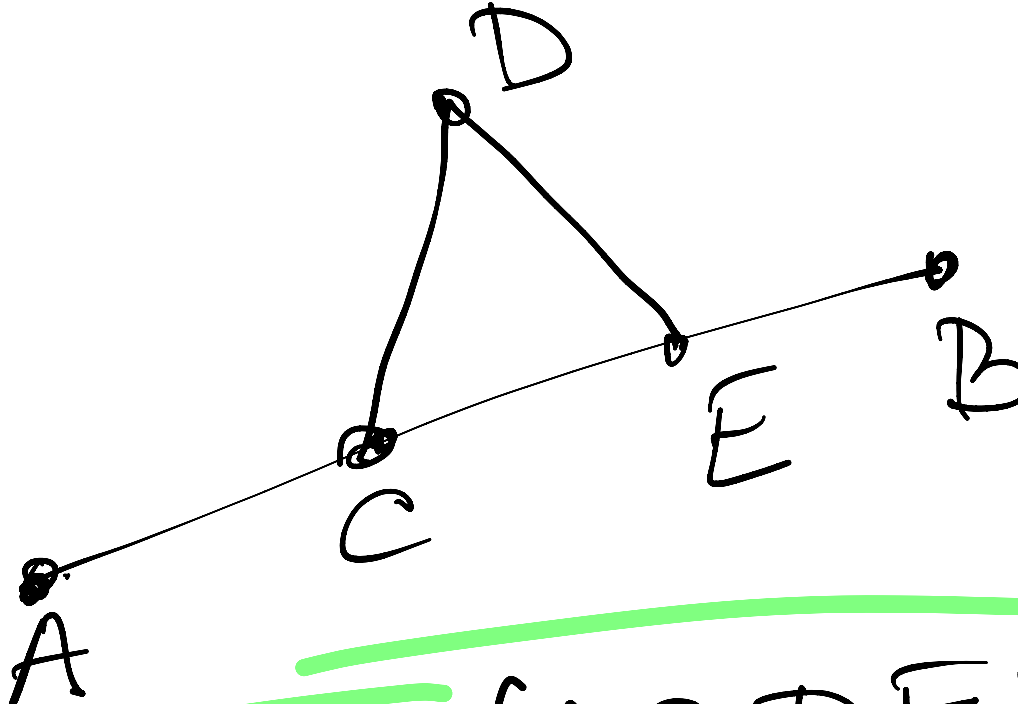
The first step towards a Mathematica construction of the von Koch curve

May 5, 2020



Helge von Koch

fractal is the limit



$$\{A,B\} \longmapsto \{A,C,D,E,B\}$$

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AC} = \frac{1}{3}\vec{AB}$$

$$\vec{OC} = \vec{OA} + \frac{1}{3}\vec{AB} = \vec{OA} + \frac{1}{3}\left(\vec{OB} - \vec{OA}\right)$$

$$\vec{AE} = \frac{2}{3}\vec{AB}$$

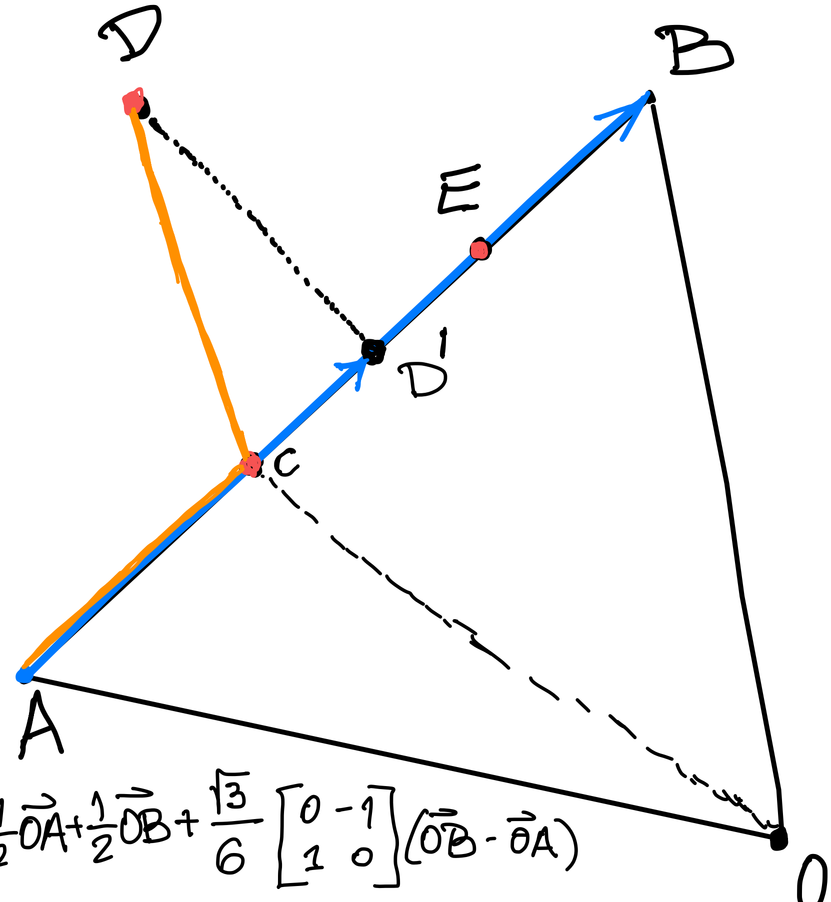
$$\vec{OE} = \vec{OA} + \vec{AE} = \vec{OA} + \frac{2}{3}\left(\vec{OB} - \vec{OA}\right)$$

$$\vec{AD'} = \frac{1}{2}\vec{AB}$$

$$\vec{OD'} = \vec{OA} + \frac{1}{2}\left(\vec{OB} - \vec{OA}\right) = \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OB}$$

$$\vec{D'D} = \frac{\sqrt{3}}{2} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \left(\frac{1}{3}\left(\vec{OB} - \vec{OA}\right)\right)}_{\text{orthogonal to } \vec{AB} \text{, the same length as } \vec{CE}}$$

length of the height of the equilateral triangle



$$\vec{OD} = \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OB} + \frac{\sqrt{3}}{6}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\left(\vec{OB} - \vec{OA}\right)$$

So, the formulas for pC, pD, pE based on pA and pB are

$pC = \frac{2}{3} pA + \frac{1}{3} pB$, $pD = \frac{1}{2} pA + \frac{1}{2} pB + \frac{\sqrt{3}}{6} \{\{0,-1\},\{1,0\}\}.(pB - pA)$

$pE = \frac{1}{3} pA + \frac{2}{3} pB$