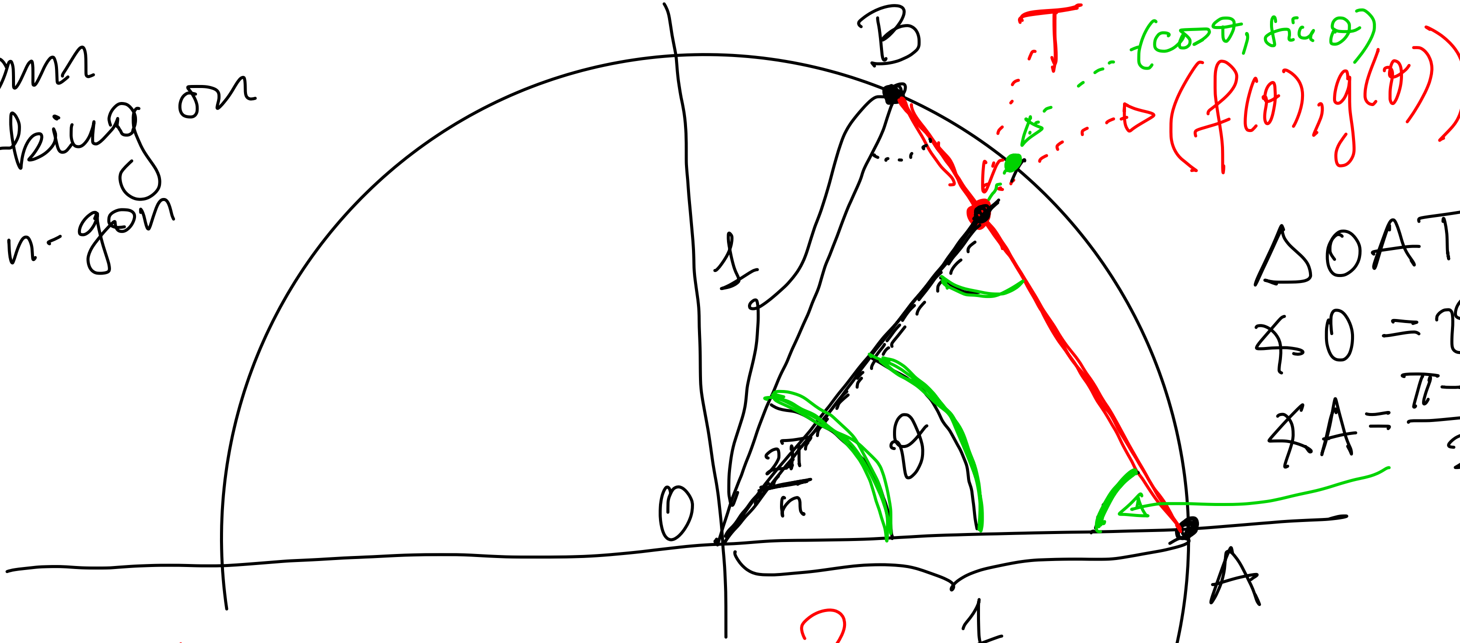
regular n-gons as Funny Circles

I am working on an n-gon



ΔOAT

$\measuredangle O = \theta$

$\measuredangle A = \dfrac{\pi - \frac{2\pi}{n}}{2}$

How long is $\overline{OT}$?

ΔOAB is "isocasil"

$OA = OB = 1$

We use law of sines in

$\Delta OAT \quad \dfrac{\sin \measuredangle T}{1} = \dfrac{\sin \measuredangle A}{OT}$

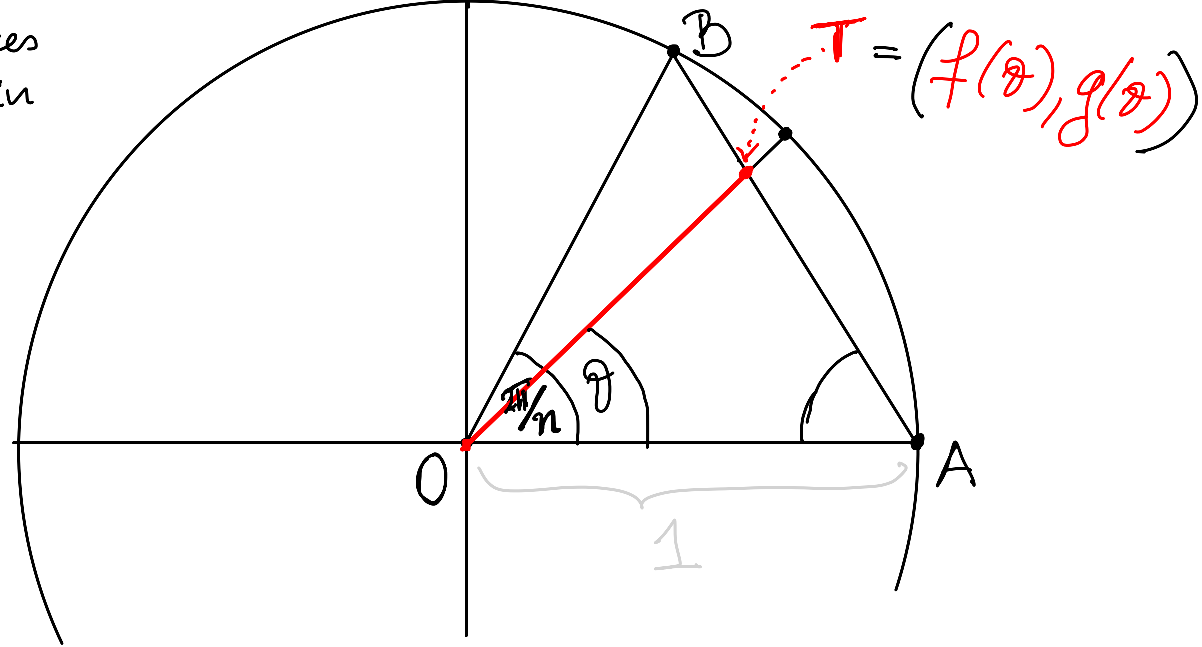
$$\overline{OT} = \frac{\sin \sphericalangle A}{\sin \sphericalangle T} = \frac{\sin\left(\frac{\pi - 2\pi/n}{2}\right)}{\sin\left(\pi - \theta - \frac{\pi - \frac{2\pi}{n}}{2}\right)}$$

$$f(\theta) = \overline{OT} \cdot \cos(\theta)$$

$$g(\theta) = \overline{OT} \cdot \sin(\theta)$$

See below, for hopefully clearer presentation:

A, B are vertices of the n-gon in the unit circle. So, the angle $\measuredangle BOA = \frac{2\pi}{n}$



We want to find the length $\overline{OT}$ as it depends on the angle θ

$\measuredangle TOA = \theta$

Use the Law of Sines in ΔOAT:

$\measuredangle OAT = \frac{1}{2}\left(\pi - \frac{2\pi}{n}\right)$ since ΔAOB is isosceles $OA = OB = 1$

$\measuredangle OTA = \pi - \theta - \frac{1}{2}\left(\pi - \frac{2\pi}{n}\right)$

The Law of Sines: $\frac{\sin \measuredangle OTA}{1} = \frac{\sin \measuredangle OAT}{\overline{OT}}$

Thus

$\overline{OT} = \frac{\sin \measuredangle OAT}{\sin \measuredangle OTA}$

$$\sin \sphericalangle OAT = \sin\left(\frac{\pi}{2} - \frac{\pi}{n}\right) = \cos\left(\frac{\pi}{n}\right)$$

$$\sin \sphericalangle OTA = \sin\left(\pi - \theta - \frac{\pi}{2} + \frac{\pi}{n}\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{n} - \theta\right) = \cos\left(\frac{\pi}{n} - \theta\right)$$

Therefore the coordinates of the point T are

$$\frac{\cos(\pi/n)}{\cos(\pi/n - \theta)}\left(\cos(\theta), \sin(\theta)\right).$$

Therefore

$$f(\theta) = \frac{\cos(\pi/n)}{\cos(\pi/n - \theta)} \cos(\theta)$$

This is a funny cosine.

$$g(\theta) = \frac{\cos(\pi/n)}{\cos(\pi/n - \theta)} \sin(\theta)$$

This is a funny sine.

See the implementation in 20210216_A1P1.nb