

$$2.3.3 \quad \bigcap \{ (a-u, a+u) : u > 0 \} = \{ a \}$$

Let A be the intersection, let $x \in A$. Then $x \in (a-u, a+u)$

So by definition of intersection, $x < a+u$ and $x > a-u$.

$$x-u < a \quad \text{and} \quad x+u > a.$$

$$a \in (x-u, x+u), u > 0$$

Suppose $x > a$. Then $x = a + y$ for some $y > 0$.

then $a \in (a+y-u, a+y+u)$.

choose u where $u \leq y$. then $a+y-u > a$ and

$a \notin (x-u, x+u)$, contradiction

Suppose $x < a$. Then $x = a - z$ for some $z > 0$.

then $a \in (a-z-u, a-z+u)$.

choose u where $u \leq z$, then $a-z+u < a$ and

$a \notin (x-u, x+u)$, contradiction.

Because $x \not< a$ and $x \not> a$, $x = a$. Since the only

x for which $x \in A$ is $x = a$, $\bigcap \{ a-u, a+u : u > 0 \} = \{ a \}$