2.3.4: Let $a, b \in \mathbb{R}$ and $a < b$.
Prove that $\bigcap \{(a, b+u) : u > 0\} \boxed{=} (a, b]$

Proof. ① Let $a, b \in \mathbb{R}$ and $a < b$.
② Let $M = \bigcap \{(a, b+u) : u > 0\}$

Part 1:

③ Assume that $x \in M$.
④ By Def'n of Intersection, $x \in (a, b+u)\ \forall u > 0$.
⑤ By Def'n of Open interval, $a < x$ AND $x < b+u\ \forall u > 0$.
⑥ By ⑤, $x - b < u\ \forall u > 0$. $x - b < u$
⑦ By Axiom 13, $x - b < 0$.
$(a<b,\ b<c \Rightarrow a<c)$
⑧ $x \leq b$ by Axiom 14
⑨ By ⑤, ⑧, and Def'n of Half-open interval, $x \in (a, b]$.
⑩ By ③, ⑨, ~~$(a, b] \subseteq M$~~. $M \subseteq (a, b]$.
~~⑪~~

Part 2:

⑪ Assume that $x \in (a, b]$.
⑫ By Def'n of Half-open interval, $a < x$ and $x \leq b$
⑬ By ⑫, $x - b \leq 0$.
⑭ ~~By~~ Let $u > 0$ be arbitrary.
⑮ By Axiom 13, $x - b < u\ \ \forall u > 0$
⑯ By ⑮, $x < b + u$. $\forall u > 0$
⑰ By Def'n of Open interval, ⑫, ⑯, $x \in (a, b+u)\ \forall u > 0$
⑱ By ②, ⑰, Def'n of intersection, $x \in M$.
⑲ $\nsubseteq$ By ⑪, ⑱, $(a, b] \subseteq M$.
⑳ By ⑩, ⑲, $M = (a, b]$. ☒