2.3.6 Let $a, b \in \mathbb{R}$ & $a < b$. Prove $\bigcup\{[a+u, b) : 0 < u < b-a\} = (a,b)$

1 Let $a, b \in \mathbb{R}$ s.t. $a < b$.

2 Let $A = \bigcup\{[a+u, b] : 0 < u < b-a\}$

Part 1

3 Let $x \in A$.

4 By 2, 3, the def'n of an open interval, & the def'n of a closed interval $\exists u \in (0, b-a)$ s.t. $x \geq a+u > a+0 = a$ and $x < b$.

5 So by 4 $a < x < b$ & so $x \in (a,b)$.

6 By 5 $A \subseteq (a,b)$.

Part 2

7 Assume that $x \in (a,b)$

8 By 7 & def'n of an open interval, $x > a$ & $a < b$.

9 By 8 $x - a > 0$.

10 Assume $\exists u \in (0, b-a)$ s.t. $x \in [a+u, b)$.

11 By 10 $x \geq a+u$

12 So by 11, 8 $a+u < x < b$ & so $x \in [a+u, b)$.

13 By 12 $(a,b) \subseteq A$

14 By 13, 6 $A = (a,b)$

we cannot assume this. Either it is true or not. But setting $u = \frac{\cancel{x}-a}{2}$ we have $u > 0$ and $u < \cancel{x}-a$. Thus it is TRUE that $\exists\, u \in (0, b-a)$ such that $x \in [a+u, b)$