MATH 321 Examination 1 hw October 15, 2009

Name ______________________________

Provide all **significant** steps of your reasoning.

Problem 1. Consider the following initial value problem

$$y' = \frac{1}{t(1-y)}, \qquad y(1) = -1.$$

(a) Before solving the equation state explicitly what is the slope of the solution at the point $(1, -1)$. Draw a direction field in a neighborhood of this point. Do you expect the solution to be an increasing function or a decreasing function? Why?

(b) Solve the given initial value problem. Determine the exact interval of existence of the solution.

(c) Plot the solution in the direction field. Was it possible to guess the interval of existence of solution from the initial value problem? Explain!

Problem 2. Consider the following initial value problem

$$ty' = y + \frac{2\,t^2}{t-2}, \qquad y(1) = 1.$$

(a) Before solving the equation state explicitly what is the slope of the solution at the point $(1, 1)$.

(b) Write the given equation in the normal form. Can you guess the interval of existence of the solution? Explain!

(c) Solve the given initial value problem.

(d) Determine the exact interval of existence of the solution.

Problem 3. A ball with mass $\frac{1}{4}$ kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 meters high. The air offers resistance proportional to velocity with the constant of proportionality $\frac{1}{32}$. For simplicity assume that the gravitational acceleration is 10 m/sec^2.

(a) Set up the initial value problem for the velocity of the ball as a function of time.

(b) Solve the initial value problem in (a).

(c) Calculate the formula for the position of the ball as a function of time.

(d) Find the maximum height above the ground that the ball reaches.

(e) How long does it take for the ball to return to the ground? (This equation cannot be solved exactly. Give 5 significant digits of the solution.)

(f) Solve the same problem ignoring resistance. Compare the results. Comment on the results?

Problem 4. A 400-gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the than at the rate of 5 gal/min, and well-mixed brine in the tank flows out of the tank at the rate of 3 gal/min.

(a) Set up an initial value problem for the amount $S(t)$ of salt in the tank.

(b) Solve the initial value problem.

(c) How much salt will the tank contain when it is full of brine?

(d) Provide a ballpark estimate for the number in (c). In fact provide two ballpark estimates, one that will be guaranteed to be larger than the number in (c) and one that will be guaranteed to be smaller than the number in (c). The most important part here is your brief explanation how you found these estimates.