Section 2.4 version October 12, 2011 at 11:20

Assigned problems: 1-10, 13-19, 21.

1. $2 + Ce^{-x}$
2. $-\dfrac{5}{3} + C\, e^{3\,x}$
3. $\dfrac{C}{x^2} + \dfrac{\sin(x)}{x^2}$
4. $\dfrac{5}{2} + Ce^{-t^2}$
5. $(1+t)^2\,(t+C)$
6. $t^4\ (C + \ln(t))$
7. $\dfrac{C + \sin(x)}{1+x}$
8. $\dfrac{1}{3}\left(1+x^3\right)\,\left(C + \ln(1+x^3)\right)$

   Solution. The equation in the normal form is

$$(1+t^3)y' = 3x^2y + t^2(1+t^3).$$

   To use the integrating factor we write it as

$$\begin{gathered}(1+t^3)y' - 3x^2y = t^2(1+t^3)\\ y' + \frac{-3x^2}{1+t^3}y = t^2\\ u\,y' + \frac{-3t^2}{1+t^3}\,u\,y = t^2\,u\end{gathered}$$

   We need u such that

$$u' = \frac{-3t^2}{1+t^3}\,u.$$

Thus

$$\begin{aligned}
\frac{u'}{u} &= \frac{-3t^2}{1+t^3} \\
\frac{d}{dt}(\ln u) &= \frac{-3t^2}{1+t^3} \\
\ln u &= \int \frac{-3t^2}{1+t^3} dt \\
\ln u &= -\ln(1+t^3) \\
\ln u &= \ln(1+t^3)^{-1} \\
u &= \frac{1}{1+t^3}.
\end{aligned}$$

Now the equation is

$$\begin{aligned}
\frac{1}{1+t^3}\,y' + \frac{-3t^2}{(1+t^3)^2}\,y &= \frac{t^2}{1+t^3} \\
\frac{d}{dt}\left(\frac{1}{1+t^3}\,y\right) &= \frac{t^2}{1+t^3} \\
\frac{1}{1+t^3}\,y &= \int \frac{t^2}{1+t^3} dt \\
\frac{1}{1+t^3}\,y &= \frac{1}{3}\ln(1+t^3) + C \\
y(t) &= (1+t^3)\left(\frac{1}{3}\ln(1+t^3) + C\right)
\end{aligned}$$

9. $\dfrac{E}{R} + Ce^{-\frac{R}{L}t}$

10. $e^{m\,x}\,(c_1\,x + C)$

13. $1 + C\,e^{-\sin(x)}$

14. $e^x\,(5 + 2\,e^x\,(x-1))$

15. $2 - \dfrac{3}{(1+x^2)^{\frac{3}{2}}}$

16. $\dfrac{\arctan(t) - \pi/4}{(1+t^2)^2}$

17. $-1 + \sin(t) + 2e^{-\sin(t)}$

18. $\dfrac{\sin(x) - x\ \cos(x) - 1}{x^2}$ The interval of existence is $(0, +\infty)$.

19. $\dfrac{1}{2}\sqrt{3 + 2\,x}\ \ln(3 + 2\,x)$ The interval of existence is $\left(-\frac{3}{2}, +\infty\right)$.

21. $\dfrac{1 + \sin(t)}{1 + t}$ The interval of existence is $(-\infty, -1)$.