Section 3.4 version October 13, 2011 at 09:16

Assigned problems: 1-10.

1. $10\left(1-e^{-t/2}\right)$

2. $\frac{25}{2}e^{-t/10}\left(1-e^{-2t/5}\right)$

3. $\frac{10}{17}\left(4e^{-t/2}-4\cos(2t)+\sin(2t)\right)$

4. $\frac{10}{37}\left(-e^{-t/2}+\cos(3t)+6\sin(3t)\right)$

5. $\frac{51}{5}\left(1-e^{-t/2}\right)-\frac{t}{10}$

6. $\frac{5}{2}\left(4+e^{-t/2}-5e^{-t/10}\right)$

7. $10\left(1-e^{-t/10}\right)$

8. $t\,e^{-t/10}$

9. $\frac{50}{1+400\pi^2}\left(20\pi\, e^{-t/10}-20\pi\,\cos(2\pi t)+\sin(2\pi t)\right)$

10. $\frac{40}{901}\left(-e^{-t/10}+\cos(3t)+30\sin(3t)\right)$

11. $300\left(1-e^{-t/10}\right)-20\,t$

12. $100\left(1-e^{-t/20}\right)^2$

13. $CE\left(1-e^{-t/(CR)}\right)$

14. $\frac{E}{R}+e^{-Rt/L}\left(I_0-\frac{E}{R}\right)$

16. The current in the circuit is $10e^{-t/20}\left(1-e^{-t/20}\right)$. The maximum is $5/2$ and occurs at time $20\ln 2\approx 13.863$ seconds.

18. The general solution of the differential equation modeling this circuit is

$$c_1 e^{-t/2} - \frac{2}{1+16\pi^2}\big(4\pi\ \cos(2\pi t) - \sin(2\pi t)\big).$$

Here c_1 is an arbitrary constant. Since the function with c_1 becomes negligible for large t the steady state response, that is the significant part of all solutions for large t, is

$$-\frac{2}{1+16\pi^2}\big(4\pi\ \cos(2\pi t) - \sin(2\pi t)\big).$$

The period of this function is 1. Hence its frequency is also 1.

20. By the capacitance law the voltage drop $V(t)$ across a capacitor is related to the charge $Q(t)$ on the capacitor by the following formula: $Q(t) = CV(t)$, where C is the capacitance, which is constant. Therefore $Q'(t) = CV'(t)$. Now we substitute the last two equations in

$$RQ'(t) + \frac{1}{C}Q(t) = E\cos(\omega t)$$

and get

$$RCV'(t) + \frac{1}{C}CV(t) = E\cos(\omega t)$$

which simplifies to

$$RCV'(t) + V(t) = E\cos(\omega t).$$

The general solution of the last differential equation is

$$V(t) = c_1 e^{-t/(RC)} + \frac{E}{1+(RC\omega)^2}\big(RC\omega\sin(\omega t) + \cos(\omega t)\big).$$

Here c_1 is an arbitrary constant. As before, the function with c_1 becomes negligible for large t. Therefore the steady state response, that is the significant part of all solutions for large t, is

$$\frac{E}{1+(RC\omega)^2}\big(RC\omega\sin(\omega t) + \cos(\omega t)\big).$$