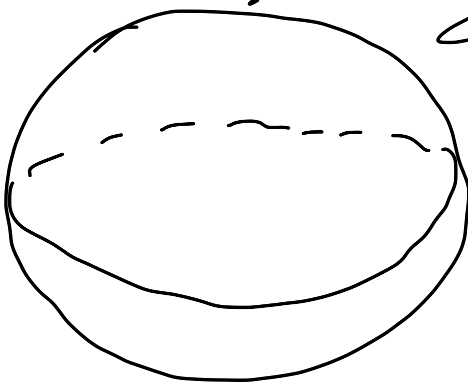
Equilibrium solutions of the Heat Equation

MoC gives at first step gives you

$$\underbrace{\langle X(s,\xi),\ Y(s,\xi),\ Z(s,\xi)\rangle}$$

$s \in ?$ $\xi \in ?$ vect. eq of a surface



ϑ, φ

$z \geq 0$

$z = \sqrt{1-x^2-y^2}$

1-d heat equation

$c(x)$, $\rho(x)$, $K_0(x)$ $Q(x,t)$

$$\boxed{c(x)\rho(x)\frac{\partial u}{\partial t}(x,t) = \frac{\partial}{\partial x}\left(K_0(x)\frac{\partial u}{\partial x}(x,t)\right) + Q(x,t)}$$

$u(x,t)$ is the temp @ x pos. t time

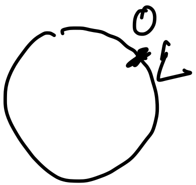
I.C. $\boxed{u(x,0) = f(x)}$ $\quad 0 \le x \le L$

B.C.s some info about $u(0,t)$, $u(L,t)$, $\frac{\partial u}{\partial x}(0,t)$, $\frac{\partial u}{\partial x}(L,t)$ $\to$ 2

for all $t \geq 0$

Direchlet $\boxed{u(0,t)=0\ ,\ u(L,t)=0}$ $\forall t \geq 0$

Neuman $\boxed{\frac{\partial u}{\partial x}(0,t)=0\ ,\ \frac{\partial u}{\partial x}(L,t)=0}$ $\forall t \geq 0$



Periodic BCs

$$u(0,t) = u(L,t) \quad \forall t \geq 0$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) \quad \forall t \geq 0$$

Equilibrium Solution

$u(x,t)$ which does not depend on time.

$$\frac{\partial u}{\partial t}(x,t) = 0 \iff$$ $u(x,t)$ is independent of t just a function of x $u(x)$ and satisfies H.E.

$$\frac{\partial}{\partial x}\left(K_0(x)\frac{\partial u}{\partial x}(x)\right) + Q(x) = 0$$

3.3.1 → since we started from 1-d heat Eq.
2-d problem & 3-d we assume that coefficients
$c(\ )$, $\rho(\ )$, $K_0(\ )$ are constants in space variables

▷ H. Eq is $\dfrac{\partial u}{\partial t} = \underbrace{\dfrac{K_0}{c\rho}}_{k}\underbrace{\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right)}_{\text{Laplacian}}$

Laplacian Δu

$\nabla^2 u$

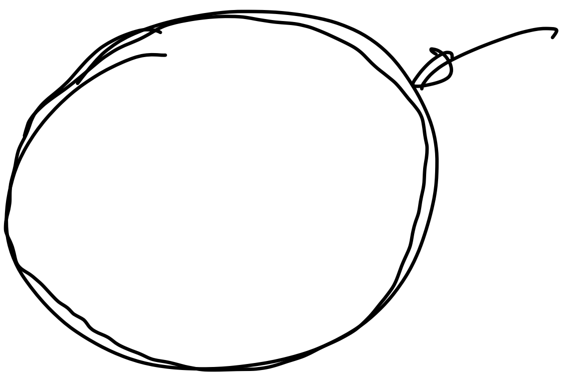
$$\boxed{\frac{\partial u}{\partial t} = k\,\Delta u}$$

Equilibrium sol. in 2-d. or 3-d.

$$\Delta u = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

PDE



Laplacian in polar coordinates