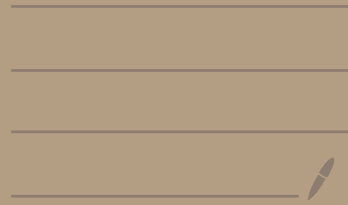


Sturm-Liouville

Eigenvalue

Problems



$$-A''(x) = \lambda A(x)$$

$$A(0) = 0$$

$$A(L) = 0$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(0, t) = 0$$

$$u(L, t) = 0$$

BCs

I.Cs

→ different BCs
are possible

Always homogeneous

→ What is more of a problem, more
complicated differential equation
occurs.

Heat Equation

$$\rho(x) \frac{\partial u}{\partial t}(x,t) = \frac{\partial}{\partial x} \left(K_0(x) \frac{\partial u}{\partial x}(x,t) \right) + \overset{\text{Special}}{Q(x,t)} u(x,t)$$

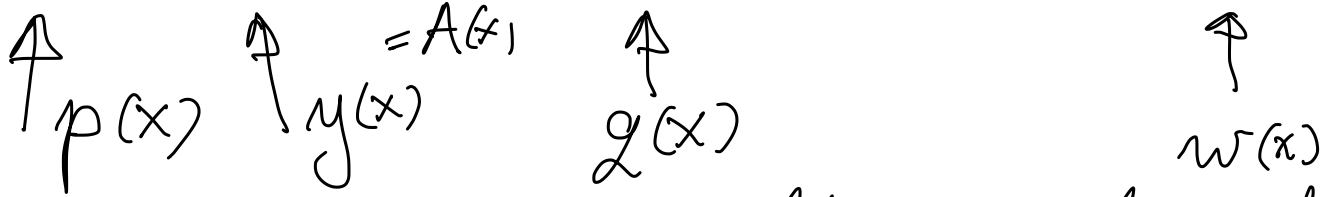
Separate variables: $A(x) B(t)$

$$\rho(x) A(x) B'(t) = \left(K_0(x) A'(x) \right)' B(t) + \alpha(x) A(x) B(t)$$

$$\frac{B'(t)}{B(t)} = \frac{\left(K_0(x) A'(x) \right)' + \alpha(x) A(x)}{\rho(x) A(x)} = -\lambda$$

ind variables are separated ☺

$$\underbrace{\left(K_0(x) A'(x) \right)'}_{\text{coeff. unknown for}} + \underbrace{\alpha(x)}_{\text{known c}} \underbrace{A(x)}_{\text{unknown}} = -\lambda \underbrace{\rho(x) A(x)}_{\text{known coeff}}$$



The general Sturm-Liouville eigenvalue problem

$$-\left(p(x)y'(x)\right)' + q(x)y(x) = \lambda w(x)y(x)$$

+ BC \rightsquigarrow $y(0), y(L)$
 $y'(0), y'(L)$

polar coordinates
 Laplacian in polar

circular symmetric

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$$

$$\frac{\partial u}{\partial t} = \partial \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right)$$

$$u(r, t) = A(r) B(t)$$

$$A(r) B'(t) = \partial \frac{1}{r} \left(r A'(r) \right)' B(t)$$

$$\frac{B'(t)}{\partial B(t)} = \frac{\frac{1}{r} \left(r A'(r) \right)'}{A(r)} = -\lambda$$

Eigenvalue D. E.

$$\frac{1}{r} \left(r A'(r) \right)' = -\lambda A(r)$$

$$\rightarrow (x y'(x))' = \lambda x y(x)$$

$p(x)$ $q(x) = 0$ $w(x)$ weight