Information sheet for Math 504 Winter 2025

Class meets: MTRF noon in BH 201

Credits: four credits

Teacher: Branko Ćurgus, Proofessor of Mathematics

Office: BH 184A

Office Hour: MTRF 11 am or by appointment (see the class **Canvas page** for a Zoom link)

Email: `curgus@wwu.edu`

Course website: `https://faculty.curgus.wwu.edu/Courses/504_202510/504.html`

Text: **Linear Algebra Done Right** by Sheldon Axler

Material covered: We plan to cover most of the first eight chapters of the textbook. The emphasis will be on finite dimensional *complex* vector spaces.

Student learning outcomes: By the end of this course, a successful student will demonstrate: (1) Knowledge of the following definitions: an abstract vector space over an arbitrary field, a subspace of a vector space, a sum and a direct sum of subspaces, a span of a set of vectors, a finite dimensional vector space, linear independence of a set of vectors, a basis and the dimension of a finite dimensional vector space. (2) Knowledge of the basic theorems and their rigorous proofs involving the concepts in (1); for example, the Steinitz exchange lemma, the Basis theorem. (3) Knowledge of the definitions of a linear operator and the definitions of the following related concepts: the null space, range, invertibility and the matrix of a linear operator relative to given bases. (4) Knowledge of the basic theorems and their rigorous proofs involving the concepts in (3); for example the Rank-nullity theorem. (5) Ability to prove the existence of an eigenvalue and a fan basis for a linear operator on a complex vector space. (6) Knowledge of the definitions of a positive definite inner product and a norm and the basic theorems involving these concepts: the abstract Pythagorean theorem, parallelogram low, Cauchy-Bunyakovsky-Schwarz inequality, polarization identities. (7) Understanding of the importance of orthonormal bases and Gram-Schmidt orthogonalization process in inner product spaces, the Bessel inequality, the proof of orthogonal complement theorem. (8) Knowledge of the definitions of a linear functional, an adjoint operator, a self-adjoint operator, a normal operator, a unitary operator and an isometry. (9) Knowledge (with a rigorous proof) of the spectral theorem for normal operators on finite dimensional inner product complex vector spaces and of the applications of the spectral theorem to positive operators, isometries, polar and singular-value decomposition. (10) Knowledge of the following definitions: a characteristic polynomials of an operator, a generalized eigenvector, a Jordan chain, a Jordan basis of a vector space relative to an operator. (11) Knowledge of the basic theorems and their rigorous proofs involving the concepts in (10); for example, Cayley-Hamilton theorem, the Primary Decomposition theorem, the existence of Jordan basis. (12) Ability to use concepts and theorems covered in the course to solve related problems and prove new related propositions.

Homework: Your daily homework should consist of studying the material covered in class. Proofs that I will present in class will often differ from the proofs in the textbook. Study both: your class notes and the book. Analyze the similarities and the differences. This will help you to internalize the concepts and the methods that are being studied. Exercises in the book are there to enhance and challenge the learning process. Use them. You should try to do all the exercises in the textbook. Report those exercises that you find difficult in **Discussions on Canvas**.

Exam: There will be only one exam on Friday, March 7, 2025. The questions will be four important proofs and several definitions that were presented during the class. To make this learning assignment more manageable, I will post a list of the proofs and definitions from which I will select four for the exam.

I would like to start the exam at 11 am and give you 2 hours to work on the questions. Please let me know if you cannot free your schedule from 11 am to noon on Friday, March 7, 2025.

Assignments: There will be three assignments for which you will have one week to finish. The work that you submit in your assignments must be your own. You can ask clarifying questions about the assignment problems in **Discussions on Canvas**. If your question involves a part of your solution, you can ask me during office hours. You can discuss problems on the assignments with other students in general terms only. You should not share your solutions with others.

The due dates for the first two assignments will be given on the class Canvas page. The final assignment will be posted on Canvas on the last Tuesday of classes and it will be due on Friday, March 21 at 11:59 pm.

Grading: The exam and each assignment will be graded by an integer between 0 and 100. Your final grade will be determined using the following formula

$$\mathtt{FG} = \lceil (\mathtt{A1} + \mathtt{A2} + \mathtt{A3} + \mathtt{E})/4 \rceil,$$

where $\mathtt{A1}$, $\mathtt{A2}$, $\mathtt{A3}$, are the grades for the assignments and $\mathtt{E}$ is the grade for the exam. In the above formula the symbol $\lceil x \rceil$ denotes the ceiling of a real number x. Your letter grade will be assigned according to the following table:

F : 0 - 49	D : 50 - 54	C−: 55 - 59	C : 60 - 64	C+: 65 - 69
B−: 70 - 74	B : 75 - 79	B+: 80 - 84	A−: 85 - 89	A : 90 - 100

On Your Written Work: Students must submit their work electronically through Canvas Assignments. The only allowable file type is pdf. I cannot grade work submitted by email. Please make sure that you produce a high-quality, readable pdf file of your work. LaTeX is a free software designed for typesetting high-quality mathematical documents. I encourage you to learn LaTeX and use it for your writing. If you submit your handwritten work, write neatly on paper with a light-colored background using a dark pencil or ink. Please use a good scanning app to produce a high-quality, readable pdf file.

Since you will have enough time to work on the homework and assignments, your papers should be well-written. Presenting calculations alone without the context in which they occur and explanations of your reasoning is not sufficient for the full credit. I believe that writing mathematics in complete sentences organized in meaningful paragraphs enhances the learning process. As a guide for writing, you can use examples in the textbook or my writing on the class website.

This course is a fast-paced course. It builds on the concepts that you learned in undergraduate linear algebra courses and some ideas from calculus. It is essential that you keep up with the material presented every day. Do the exercises at the end of each chapter. Look for help if you encounter difficulties.

How to succeed: Doing well in mathematics builds on understanding, not memorizing. To understand, practice critical thinking in all mathematical activities and question everything until clarity has been achieved. Put your questions in writing, share with the class in **Discussions on Canvas**.

Diversity, Equity, Inclusion: Welcome to my class. I promise to keep my mind open for all the diverse mathematical experiences you bring to this class. I want to help each of you use those personal experiences creatively to build your own understanding of the content studied in this class. I will bring diverse approaches to most concepts. For example, to make this class more diverse, I looked into the history of linear algebra. Amazingly, the first known system of linear equations appears on the old Babylonian tablet VAT 8389, which is between 3600 and 4000 years old (2000-1600 BC). The second oldest one is from ancient Egypt in the Rhind papyrus, dating back to around 1550 BC. This system involves five unknowns, but the solution in the papyrus is cryptic. The oldest treatment of systems of linear equations from antiquity which uses a method that resembles matrices is in Chapter 8 of the Chinese textbook *Nine Chapters of the Mathematical Art* which is at least 1800 years old.

My celebration of the history of mathematics in one long sentence: Throughout history, human civilizations continue to develop and share mathematical knowledge. Succeeding civilizations recognize and admire the contributions of the preceding ones. Prior mathematical creations serve as a foundation and inspiration for further advancements, all in unity, giving mathematics the true spirit of a collective endeavor of the entirety of humanity through history, the present, and the future.

I understand that each of you comes to this class with a diverse mathematical background. I believe that mathematics is so universally diverse that it offers a path to understanding to everybody. The only prerequisite is to be open to the human worth of rigorous thinking, which is practiced in Mathematics. I want to help you build your own understanding of Linear Algebra. The goal is to create an environment where you can succeed in Mathematics and be proud of your achievement.

Academic Honesty Policy: Academic dishonesty is not tolerated at Western Washington University. Representing the work of another as one's own is an act of academic dishonesty. For a full description of the academic honesty policy and procedures at Western, see Appendix D in the University Catalog.

Flexibility Statement: This syllabus is subject to change. Changes, if any, will be announced in class or online. Students will be held responsible for all changes.

Syllabi@WWU Please also check `https://syllabi.wwu.edu/`