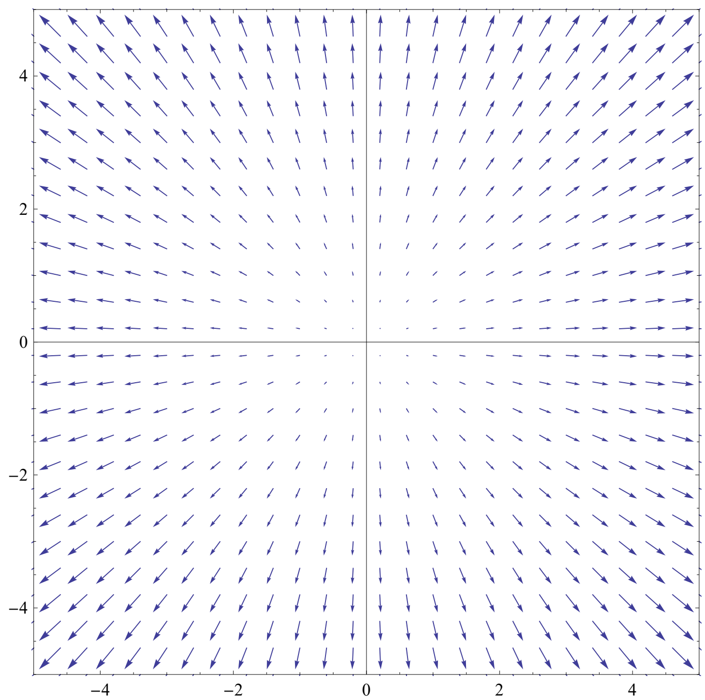
In[1]:=
```
NotebookDirectory[]
```
Out[1]=
```
C:\Dropbox\Work\myweb\Courses\Math_pages\Math_225\
```

Must know vector fields

- **“Exploding” vector field**

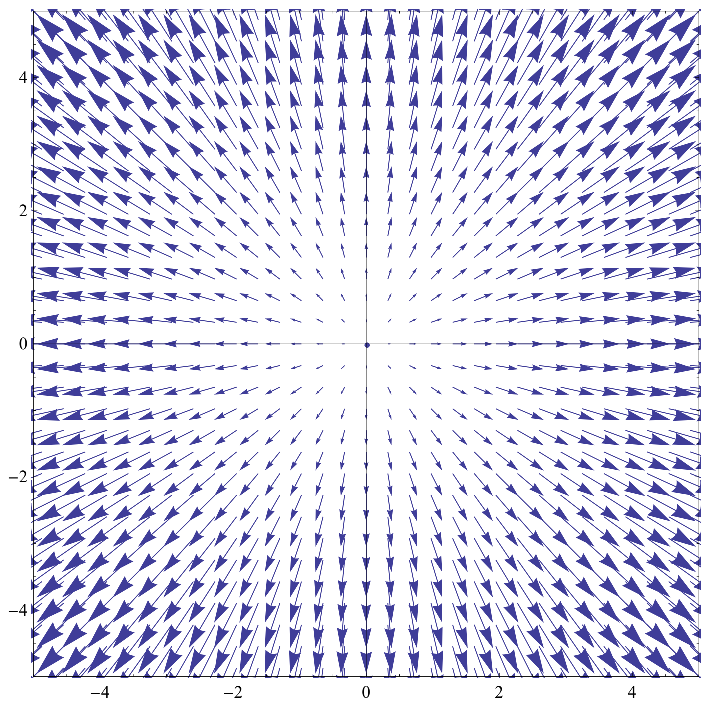
In[2]:=
```
VectorPlot[{x, y},
      {x, -6,  6}, {y, -6, 6},
 VectorPoints → 30,
    Axes → True, Frame → True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```
Out[2]=



Below is a modified version in which we specify a predefined scaling of vectors in a vector field.

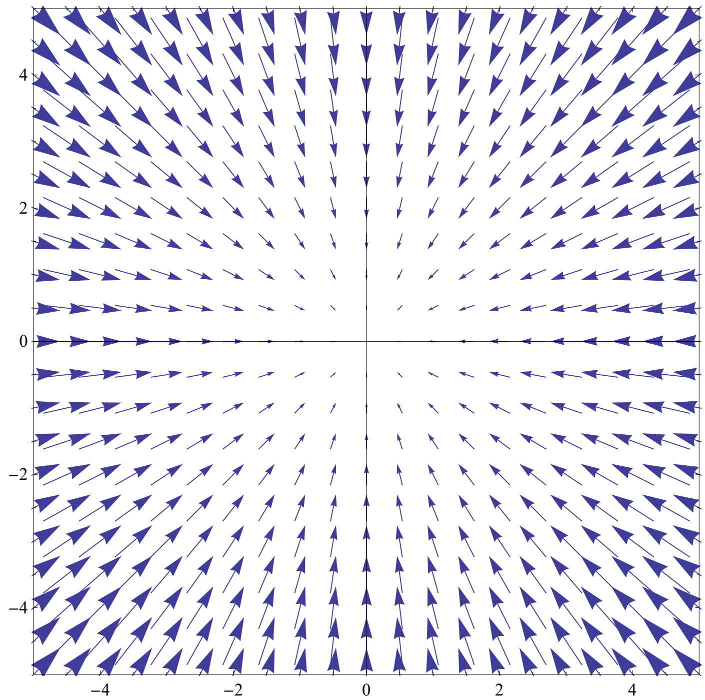
In[3]:=
```
VectorPlot[{x, y},
      {x, -6, 6}, {y, -6, 6},
    VectorPoints → 35,
 VectorScale → Medium, Axes -> True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[3]=



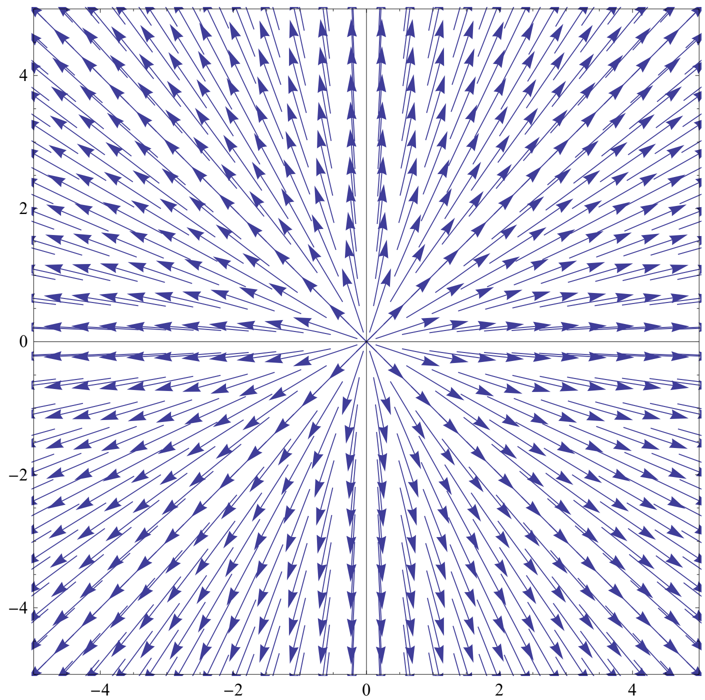
In[4]:=
```
VectorPlot[{-x, -y},
    {x, -6, 6}, {y, -6, 6},
  VectorPoints → 25,
VectorScale → Medium, Axes -> True, Frame → True,
     PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[4]=



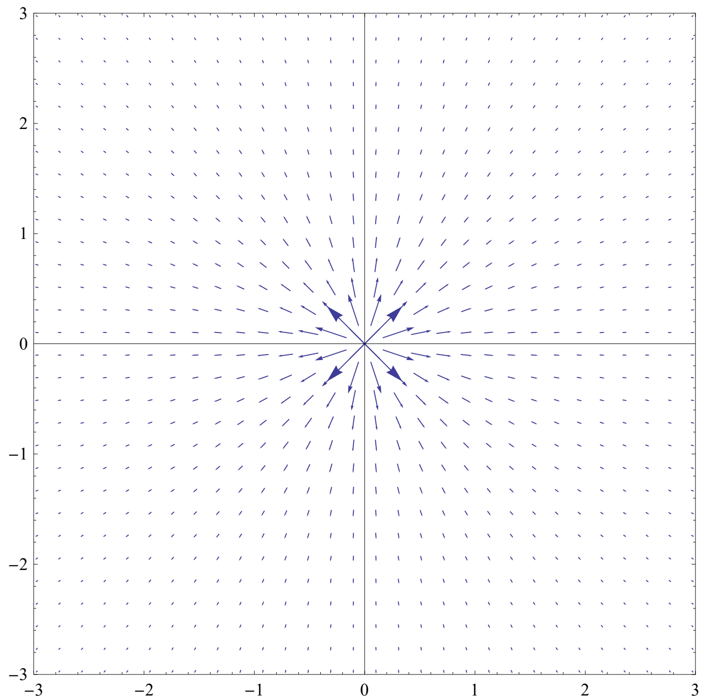
In[5]:=
```
VectorPlot[{x/Sqrt[x^2 + y^2], y/Sqrt[x^2 + y^2]},
    {x, -6, 6}, {y, -6, 6},
  VectorPoints → 30,
VectorScale → Small, Axes -> True, Frame → True,
  PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[5]=



In[6]:=
```
VectorPlot[{x/(x^2 + y^2), y/(x^2 + y^2)},
   {x, -4, 4}, {y, -4, 4},
  VectorPoints → 40,
 VectorScale → Small, Axes -> True, Frame → True,
  PlotRange -> {{-3, 3}, {-3, 3}}]
```

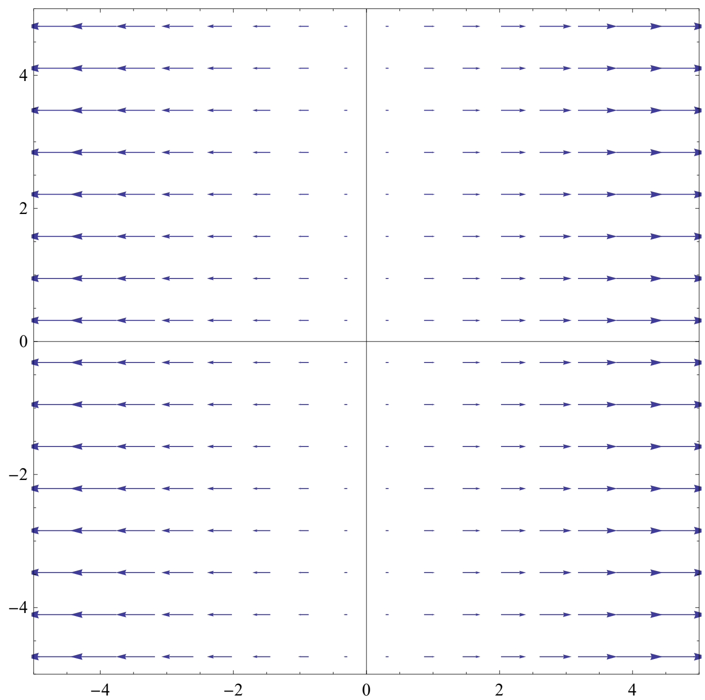
Out[6]=



■ One component constant

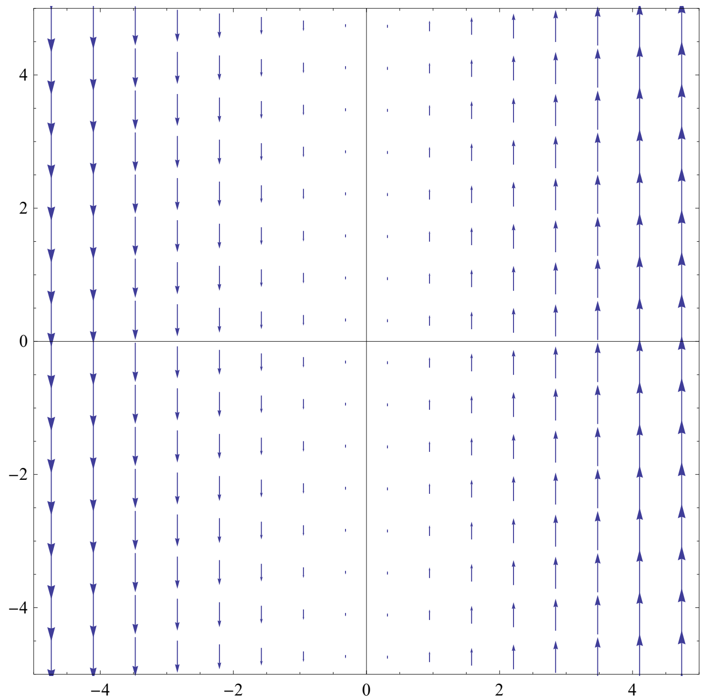
In[7]:=
```
VectorPlot[{x, 0},
     {x, -6,  6}, {y, -6, 6},
    VectorPoints → 20,
 VectorScale → Small, Axes → True, Frame → True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[7]=



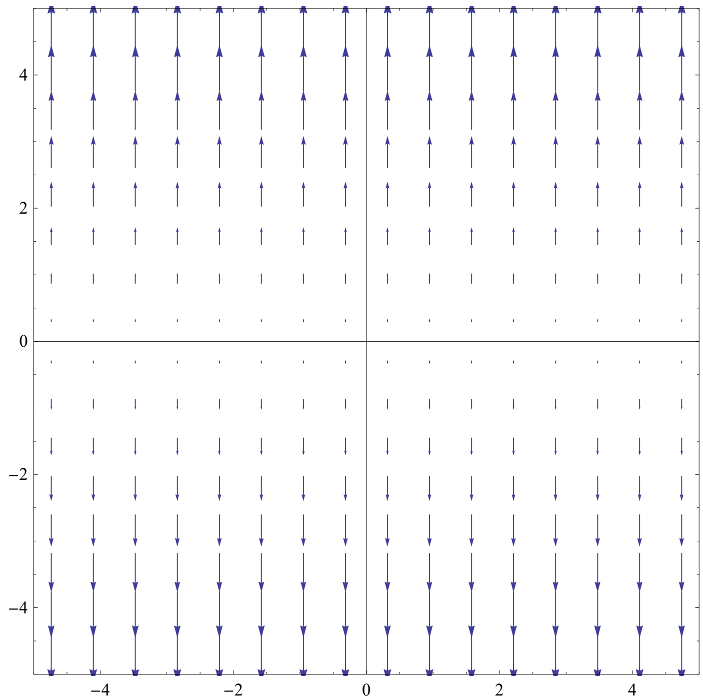
In[8]:=
```
VectorPlot[{0, x},
    {x, -6, 6}, {y, -6, 6},
   VectorPoints → 20,
 VectorScale → Small, Axes → True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[8]=



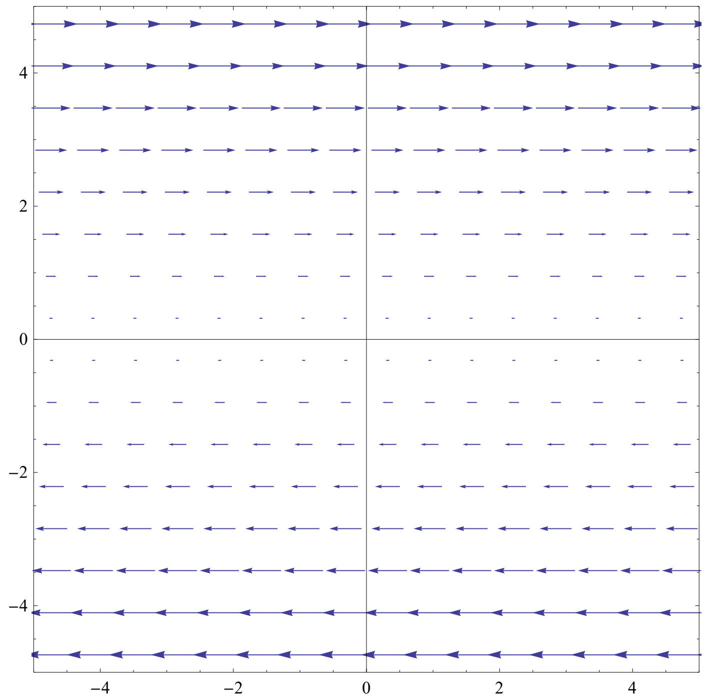
In[9]:=
```
VectorPlot[{0, y},
    {x, -6, 6}, {y, -6, 6},
   VectorPoints → 20,
VectorScale → Small, Axes → True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[9]=



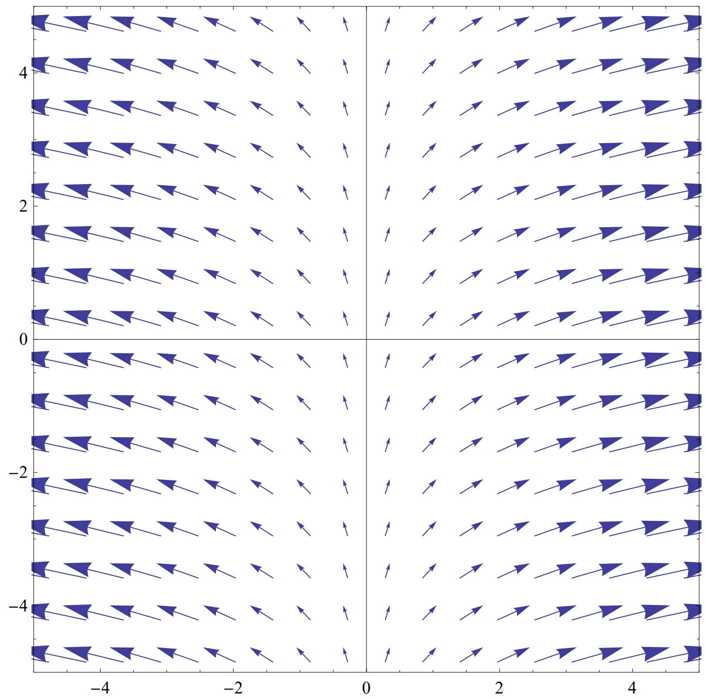
In[10]:=
```
VectorPlot[{y, 0},
    {x, -6, 6}, {y, -6, 6},
  VectorPoints → 20,
VectorScale → Small, Axes → True, Frame → True,
  PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[10]=



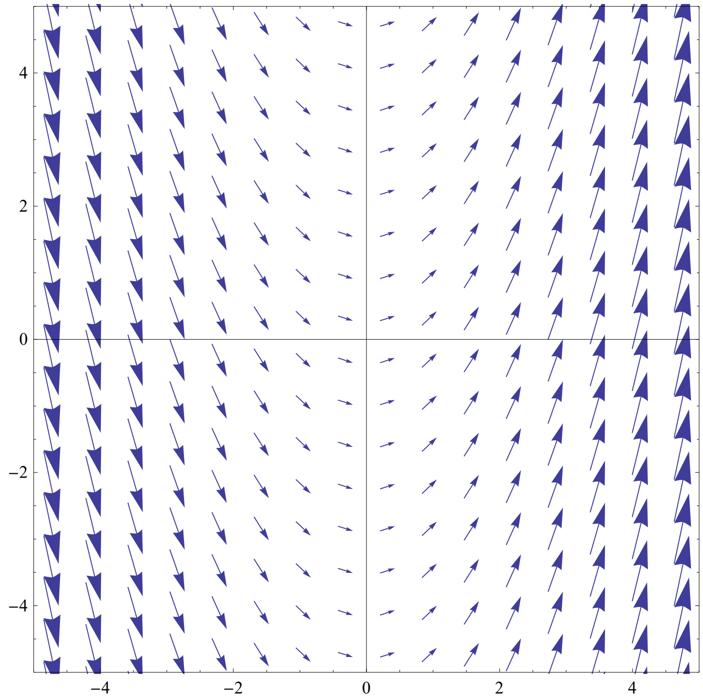
In[11]:=
```
VectorPlot[{x, 1},
    {x, -6, 6}, {y, -6, 6},
  VectorPoints → 20,
VectorScale → Medium, Axes → True, Frame → True,
  PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[11]=



In[12]:=
```
VectorPlot[{1, x},
    {x, -6, 6}, {y, -6, 6},
  VectorPoints → 20,
VectorScale → Medium, Axes → True, Frame → True,
  PlotRange -> {{-5, 5}, {-5, 5}}]
```

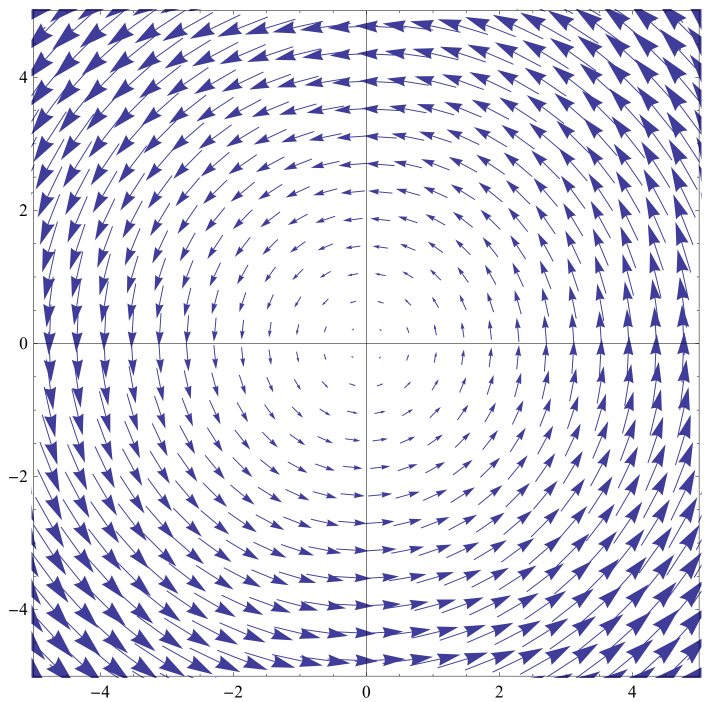
Out[12]=



■ Rotational vector field

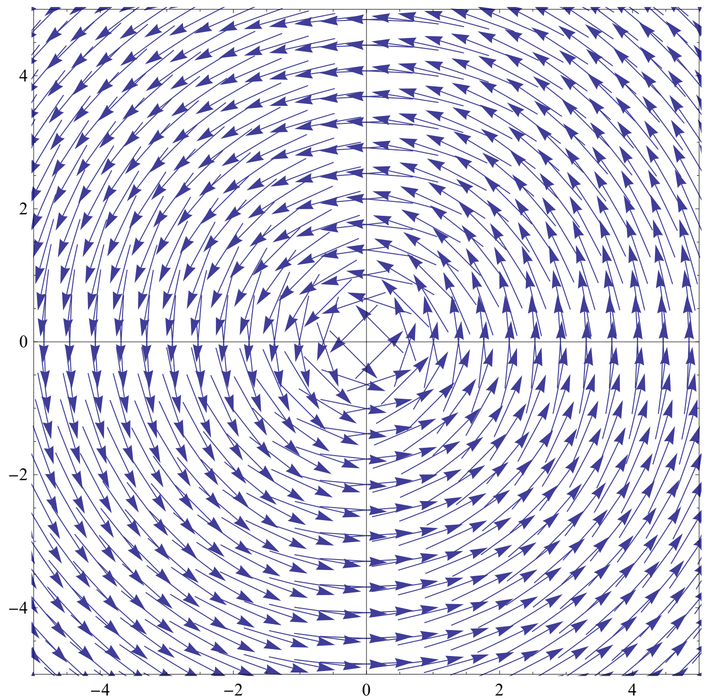
In[13]:=
```
VectorPlot[{-y, x},
     {x, -6, 6}, {y, -6, 6},
   VectorPoints → 30,
 VectorScale → Medium, Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[13]=



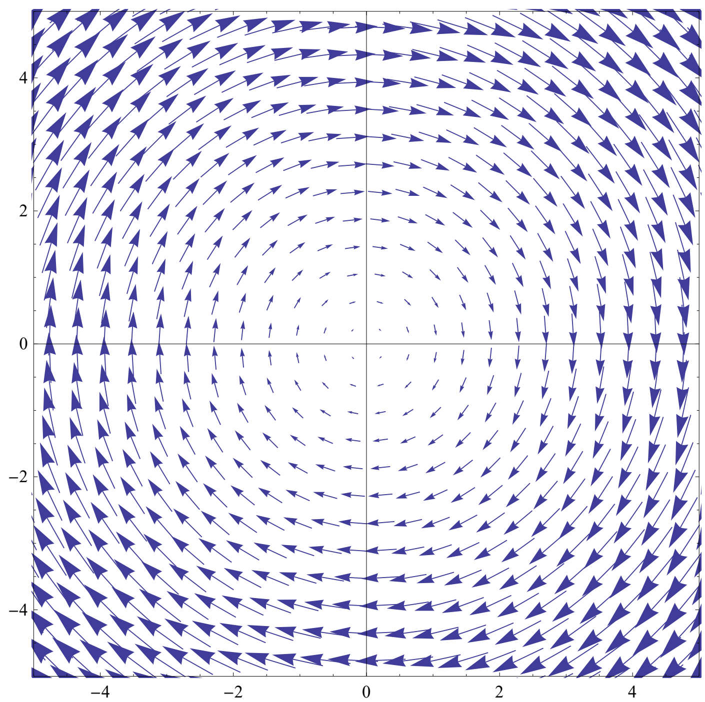
In[14]:=
```
VectorPlot[{-y/Sqrt[x^2 + y^2], x/Sqrt[x^2 + y^2]},
  {x, -6, 6}, {y, -6, 6},
  VectorPoints → 32,
 VectorScale → Small, Axes -> True, Frame → True,
  PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[14]=



In[15]:=
```
VectorPlot[{y, -x},
    {x, -6, 6}, {y, -6, 6},
   VectorPoints → 30,
 VectorScale → Medium, Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

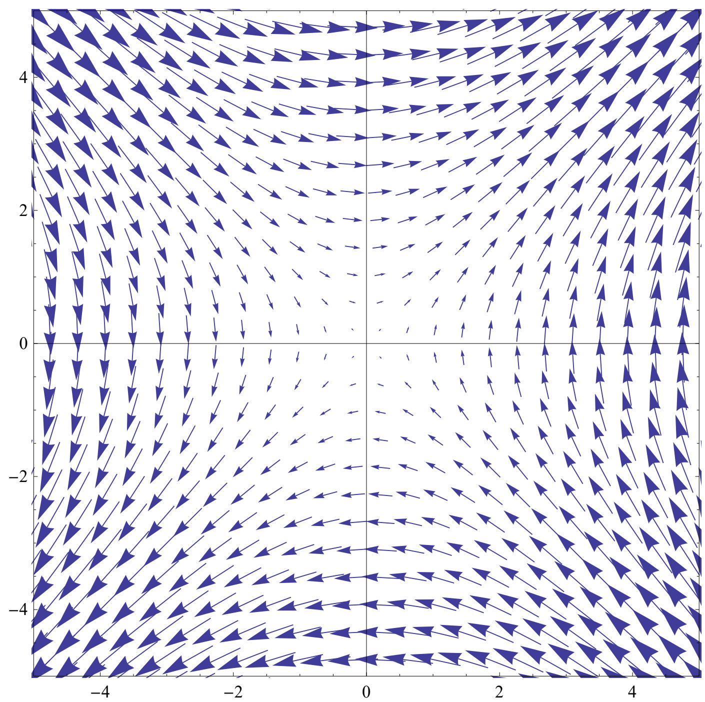
Out[15]=



▪ Hyperbolic vector field

In[16]:=
```
VectorPlot[{y, x},
     {x, -6, 6}, {y, -6, 6},
   VectorPoints → 30,
 VectorScale → Medium, Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

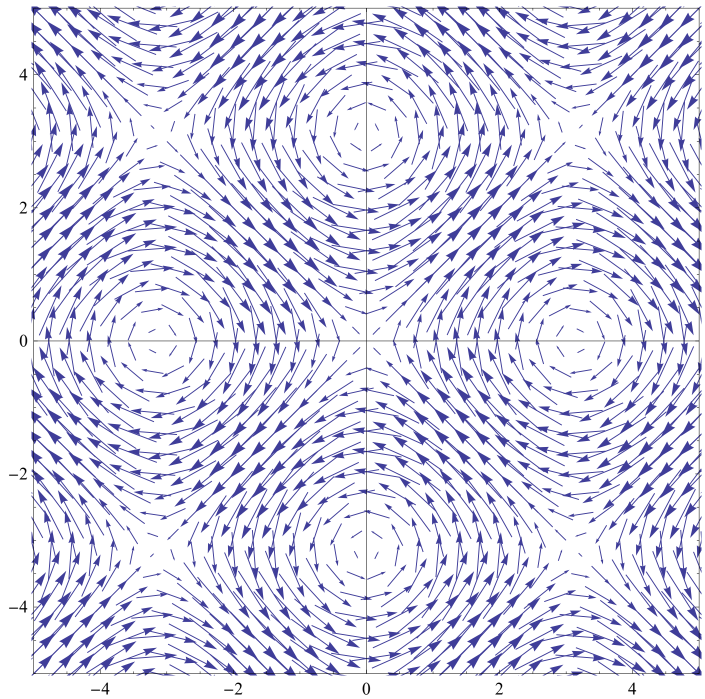
Out[16]=



More vector fields

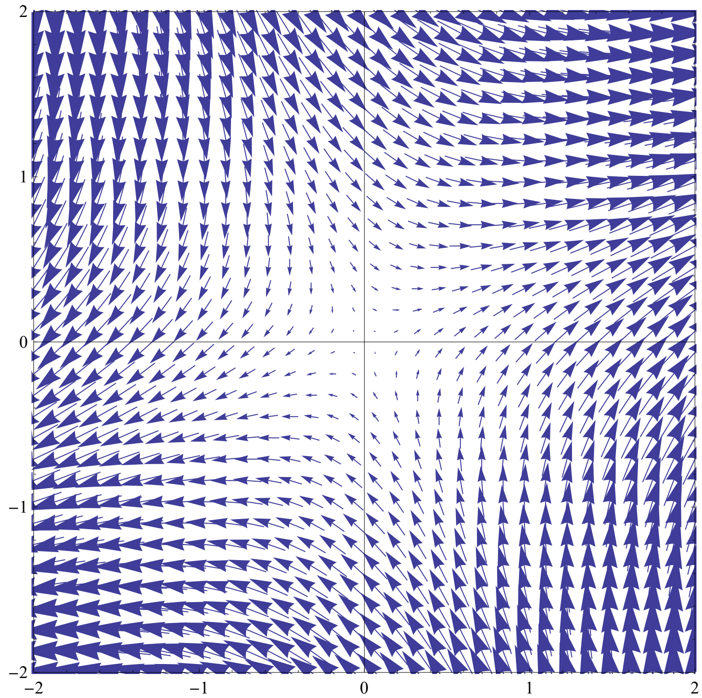
In[17]:=
```
VectorPlot[{Sin[y], Sin[x]},
    {x, -2 π, 2 π}, {y, -2 π, 2 π},
   VectorPoints → 42,
 VectorScale → Small, Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[17]=



In[18]:=
```
VectorPlot[{x + y, x - y},
     {x, -2, 2 }, {y, -2, 2},
   VectorPoints → 32,
 VectorScale → Medium, Axes → True, Frame → True,
   PlotRange → {{-2, 2}, {-2, 2}}]
```

Out[18]=



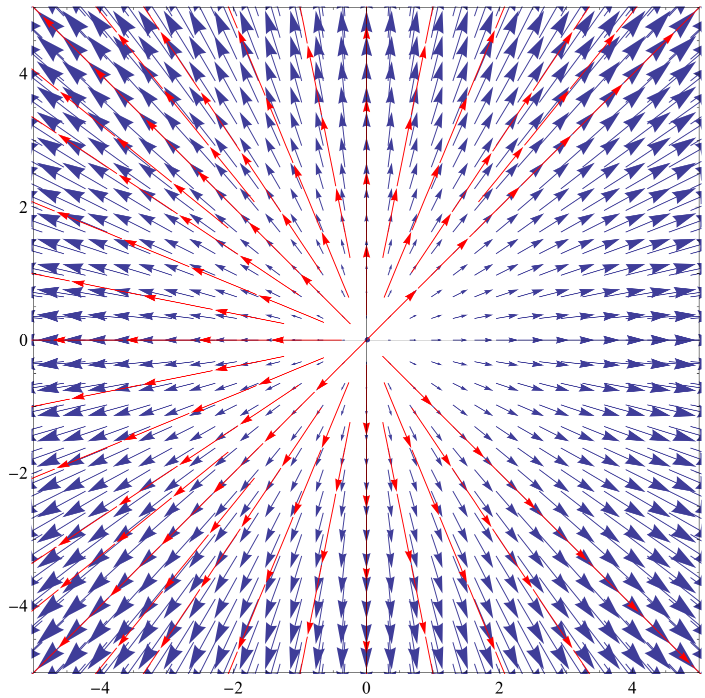
In[19]:=

Must know vector fields with flow lines

■ Exploding vector field

In[20]:=
```
VectorPlot[{x, y},
      {x, -6, 6}, {y, -6, 6},
    VectorPoints → 35, VectorScale → Medium,
 StreamPoints → 30, StreamStyle → Red,
 Axes -> True, PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[20]=



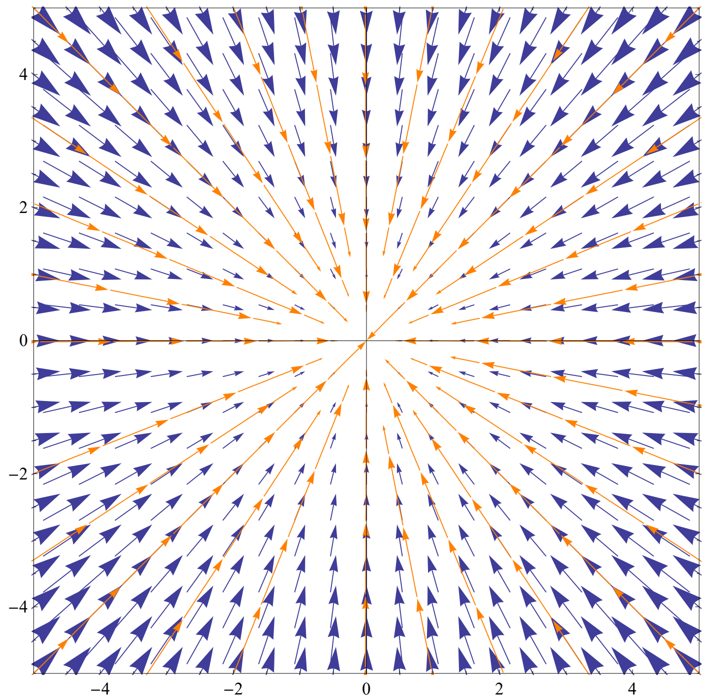
To find the exact formulas for flow lines we need to solve two differential equations

In[21]:=
```
DSolve[{x'[t] == x[t], y'[t] == y[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[21]= $\{x[t] \to e^{t}\, x0,\ y[t] \to e^{t}\, y0\}$

In[22]:=
```
VectorPlot[{-x, -y},
      {x, -6, 6}, {y, -6, 6},
    VectorPoints → 25,
 VectorScale → Medium,
 StreamPoints → 30, StreamStyle → Orange,
 Axes -> True, Frame → True,
        PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[22]=



To find the exact formulas for flow lines we need to solve two differential equations

In[23]:=
```
DSolve[{x'[t] == -x[t], y'[t] == -y[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

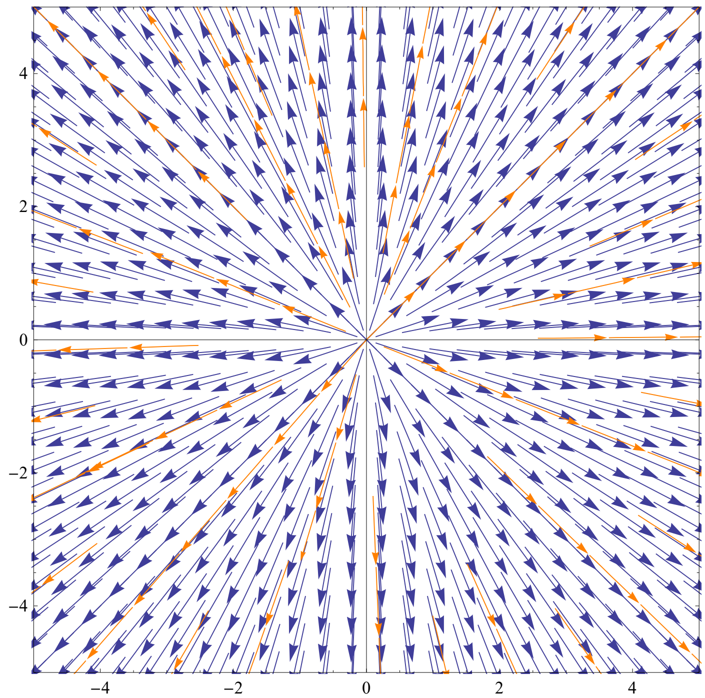
Out[23]= $\{x[t] \to e^{-t}\, x0,\ y[t] \to e^{-t}\, y0\}$

In[24]:=
```
{x[t], y[t]} /.
 DSolve[{x'[t] == -x[t], y'[t] == -y[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[24]= $\{e^{-t}\, x0,\ e^{-t}\, y0\}$

In[25]:=
```
VectorPlot[{x/Sqrt[x^2 + y^2], y/Sqrt[x^2 + y^2]},
    {x, -6, 6}, {y, -6, 6},
   VectorPoints → 30,
 VectorScale → Small,
 StreamPoints → 35, StreamStyle → Orange,
 Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[25]=



To find the exact formulas for flow lines we need to solve a system of two differential equations

In[26]:=
```
FullSimplify[DSolve[{x'[t] == x[t]/Sqrt[x[t]^2 + y[t]^2],
   y'[t] == y[t]/Sqrt[x[t]^2 + y[t]^2], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]]
```

Out[26]= $\left\{y[t] \to y0 - \frac{t\, y0}{x0 \sqrt{1 + \frac{y0^2}{x0^2}}},\ x[t] \to x0 - \frac{t}{\sqrt{1 + \frac{y0^2}{x0^2}}}\right\}$

In fact this solution is wrong. We can see that this solution is wrong by substituting x0 = 1, y0 = 1

In[27]:= $\left\{y[t] \to y0 - \frac{t\, y0}{x0 \sqrt{1 + \frac{y0^2}{x0^2}}},\ x[t] \to x0 - \frac{t}{\sqrt{1 + \frac{y0^2}{x0^2}}}\right\}$ /. {x0 → 1, y0 → 1}

Out[27]= $\left\{y[t] \to 1 - \frac{t}{\sqrt{2}},\ x[t] \to 1 - \frac{t}{\sqrt{2}}\right\}$

For $t = 0$ we get the point (1, 1), but with increasing t the point moves towards the origin not away from the origin.

Interestingly, based on *Mathematica* solution we can guess the correct solution. It is

In[28]:= $\left\{y[t] \to y0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right), x[t] \to x0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right)\right\}$

Out[28]= $\left\{y[t] \to y0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right), x[t] \to x0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right)\right\}$

and this solution is valid for all $t > -\sqrt{x0^2 + y0^2}$. To prove that this is the correct solution, I take the derivative

In[29]:= $D\left[x0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right), t\right]$

Out[29]= $\frac{x0}{\sqrt{x0^2+y0^2}}$

In[30]:= $\text{FullSimplify}\left[\frac{x0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right)}{\sqrt{\left(x0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right)\right)^2+\left(y0\left(1+\frac{t}{\sqrt{x0^2+y0^2}}\right)\right)^2}}\right]$

Out[30]= $\frac{x0\left(t+\sqrt{x0^2+y0^2}\right)}{\sqrt{x0^2+y0^2}\sqrt{t^2+x0^2+y0^2+2\,t\sqrt{x0^2+y0^2}}}$

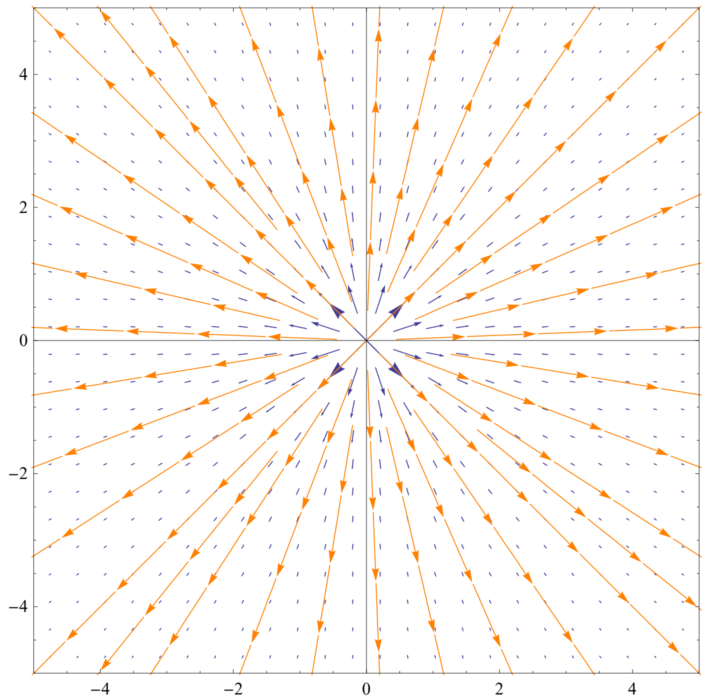
Mathematica does not simplify this fully. Notice that $t^2 + x0^2 + y0^2 + 2\,t\sqrt{x0^2+y0^2} = \left(t + \sqrt{x0^2+y0^2}\right)^2$ and thus

$\sqrt{t^2 + x0^2 + y0^2 + 2\,t\sqrt{x0^2+y0^2}} = t + \sqrt{x0^2+y0^2}$, since $t + \sqrt{x0^2+y0^2} > 0$.

This proves that x[t] satisfies the given differential equation. Similarly y[t] satisfies the given differential equation.

In[31]:=
```
VectorPlot[{x/(x^2 + y^2), y/(x^2 + y^2)},
    {x, -6, 6}, {y, -6, 6},
   VectorPoints → 30, VectorScale → Small,
 StreamPoints → 35, StreamStyle → Orange,
 Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[31]=



In[32]:=
```
FullSimplify[DSolve[
  {x'[t] == x[t]/(x[t]^2 + y[t]^2), y'[t] == y[t]/(x[t]^2 + y[t]^2), x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]]
```

Out[32]= $\left\{y[t] \to -\frac{y0\sqrt{2\,t + x0^2 + y0^2}}{x0\sqrt{1 + \frac{y0^2}{x0^2}}},\ x[t] \to -\frac{\sqrt{2\,t + x0^2 + y0^2}}{\sqrt{1 + \frac{y0^2}{x0^2}}}\right\}$

Again, this is wrong.

In[33]:=
```
FullSimplify[{y[t] → -(y0 Sqrt[2 t + x0^2 + y0^2])/(x0 Sqrt[1 + y0^2/x0^2]), x[t] → -Sqrt[2 t + x0^2 + y0^2]/Sqrt[1 + y0^2/x0^2]} /. {x0 → 1, y0 → 1}]
```

Out[33]= $\left\{y[t] \to -\sqrt{1 + t},\ x[t] \to -\sqrt{1 + t}\right\}$

This is wrong since with increasing t the point moves towards the origin, not away from it. As before, we can guess the correct solution

In[34]:= $\left\{y[t] \to \frac{y0 \sqrt{2\,t + x0^2 + y0^2}}{\sqrt{x0^2 + y0^2}},\ x[t] \to \frac{x0 \sqrt{2\,t + x0^2 + y0^2}}{\sqrt{x0^2 + y0^2}}\right\}$

Out[34]= $\left\{y[t] \to \frac{y0 \sqrt{2\,t + x0^2 + y0^2}}{\sqrt{x0^2 + y0^2}},\ x[t] \to \frac{x0 \sqrt{2\,t + x0^2 + y0^2}}{\sqrt{x0^2 + y0^2}}\right\}$

which is defined for $t > \left(x0^2 + y0^2\right)/2$. Now prove it:

In[35]:= $\text{FullSimplify}\left[D\left[\frac{x0 \sqrt{2\,t + x0^2 + y0^2}}{\sqrt{x0^2 + y0^2}}, t\right]\right]$

Out[35]= $\frac{x0}{\sqrt{x0^2 + y0^2}\ \sqrt{2\,t + x0^2 + y0^2}}$

In[36]:= $\text{FullSimplify}\left[\frac{x0 \frac{\sqrt{2\,t+x0^2+y0^2}}{\sqrt{x0^2+y0^2}}}{\left(x0 \frac{\sqrt{2\,t+x0^2+y0^2}}{\sqrt{x0^2+y0^2}}\right)^2 + \left(y0 \frac{\sqrt{2\,t+x0^2+y0^2}}{\sqrt{x0^2+y0^2}}\right)^2}\right]$

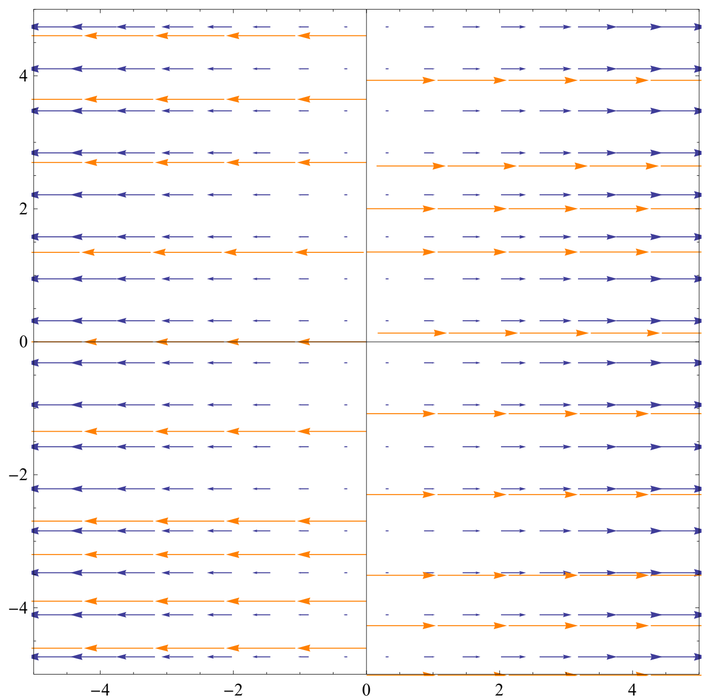
Out[36]= $\frac{x0}{\sqrt{x0^2 + y0^2}\ \sqrt{2\,t + x0^2 + y0^2}}$

Since the last two expressions are identical, we have the solution.

■ One component constant

In[37]:=
```
VectorPlot[{x, 0},
     {x, -6, 6}, {y, -6, 6},
   VectorPoints → 20,
 VectorScale → Small,
 StreamPoints → 25, StreamStyle → Orange,
 Axes → True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[37]=



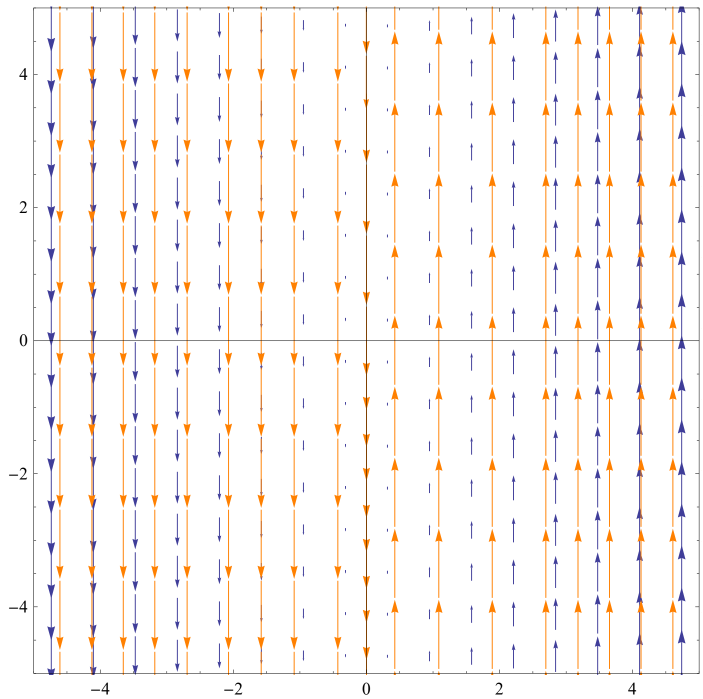
To find the exact formulas for flow lines we need to solve the differential equations (this is easily done by hand as well)

In[38]:=
```
DSolve[{x'[t] == x[t], y'[t] == 0, x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[38]= $\{x[t] \to e^{t}\, x0,\ y[t] \to y0\}$

In[39]:=
```
VectorPlot[{0, x},
      {x, -6, 6}, {y, -6, 6},
    VectorPoints → 20,
 VectorScale → Small,
  StreamPoints → 25, StreamStyle → Orange,
  Axes → True, Frame → True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[39]=

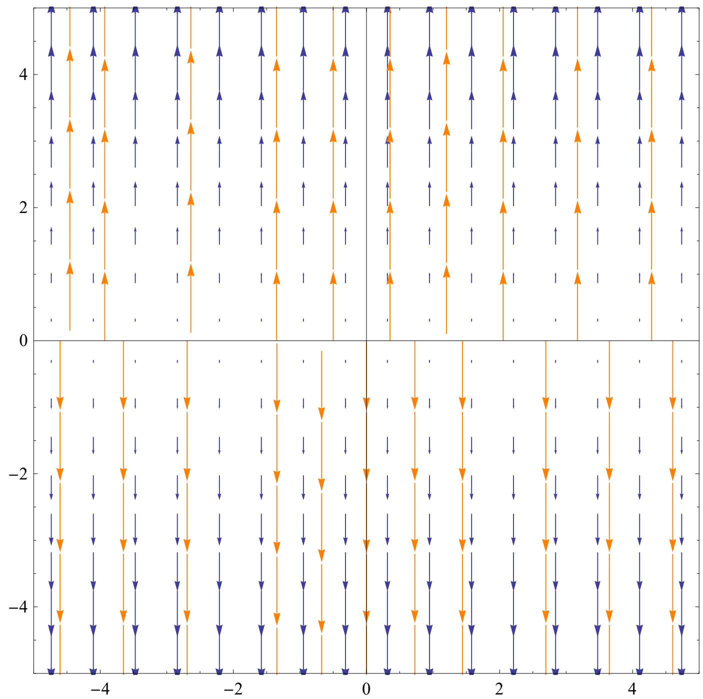


In[40]:=
```
DSolve[{x'[t] == 0, y'[t] == x[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[40]= {x[t] → x0, y[t] → t x0 + y0}

In[41]:=
```
VectorPlot[{0, y},
      {x, -6,  6}, {y, -6, 6},
    VectorPoints → 20,
 VectorScale → Small,
 StreamPoints → 25, StreamStyle → Orange,
 Axes → True, Frame → True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[41]=

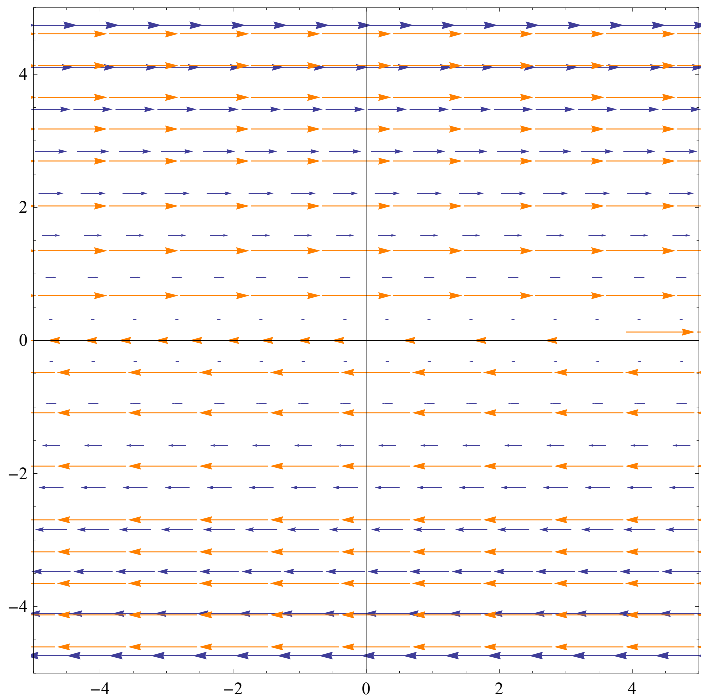


In[42]:=
```
DSolve[{x'[t] == 0, y'[t] == y[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[42]= $\{x[t] \to x0,\ y[t] \to e^{t}\, y0\}$

In[43]:=
```
VectorPlot[{y, 0},
      {x, -6, 6}, {y, -6, 6},
    VectorPoints → 20,
 VectorScale → Small,
  StreamPoints → 25, StreamStyle → Orange,
  Axes → True, Frame → True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[43]=

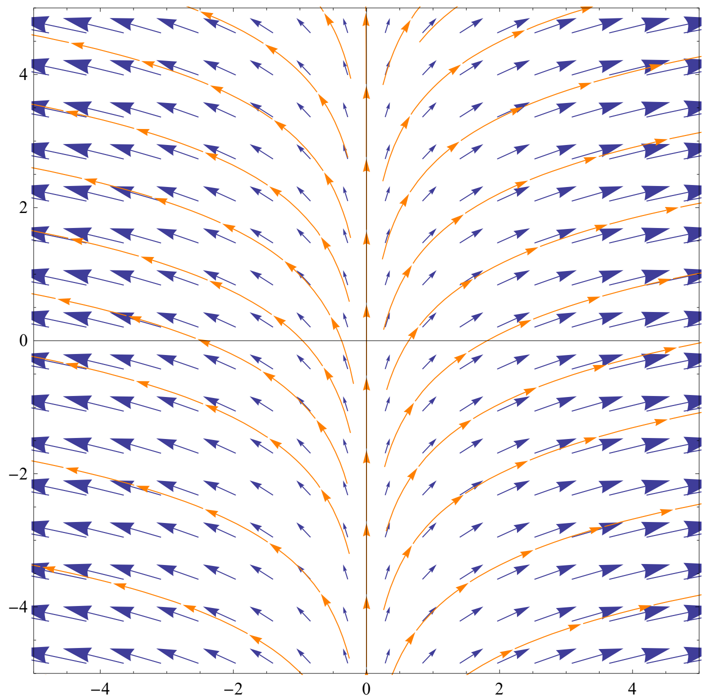


In[44]:=
```
{x[t], y[t]} /. DSolve[{x'[t] == y[t], y'[t] == 0, x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[44]= {x0 + t y0, y0}

In[45]:=
```
VectorPlot[{x, 1},
     {x, -6, 6}, {y, -6, 6},
   VectorPoints → 20,
 VectorScale → Medium,
 StreamPoints → 25, StreamStyle → Orange,
  Axes → True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[45]=

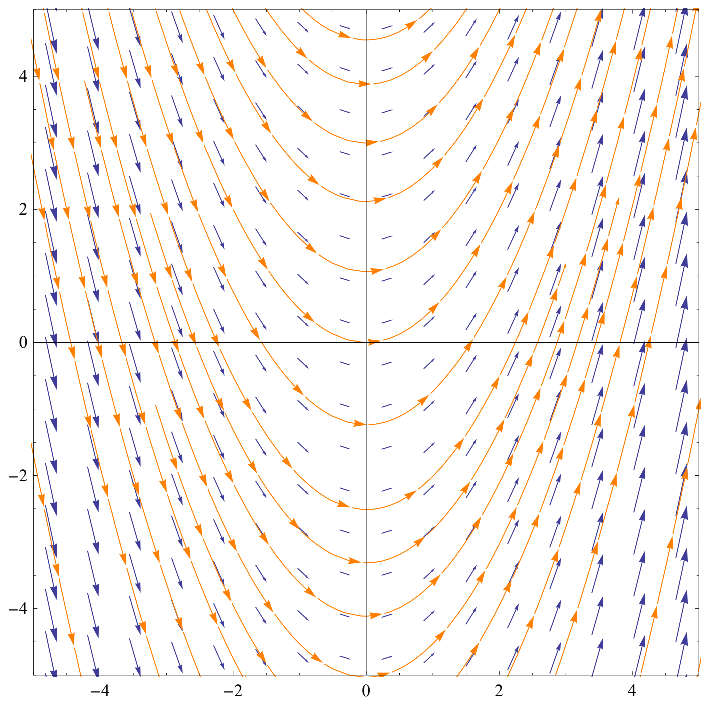


In[46]:=
```
DSolve[{x'[t] == x[t], y'[t] == 1, x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[46]= $\{x[t] \to e^{t}\, x0,\ y[t] \to t + y0\}$

In[47]:=
```
VectorPlot[{1, x},
     {x, -6, 6}, {y, -6, 6},
   VectorPoints → 20,
 VectorScale → Small,
 StreamPoints → 25, StreamStyle → Orange,
  Axes → True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[47]=



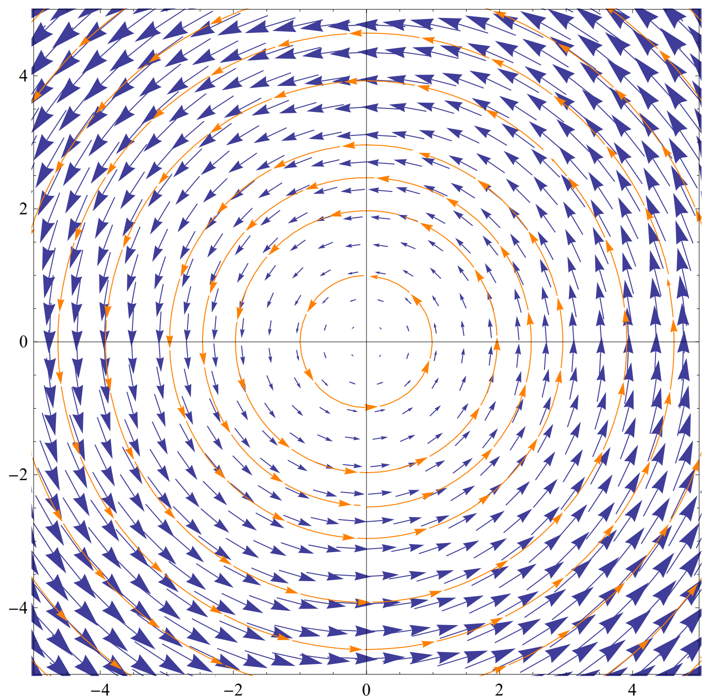
In[48]:=
```
DSolve[{x'[t] == 1, y'[t] == x[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[48]= $\left\{x[t] \to t + x0,\ y[t] \to \frac{1}{2}\left(t^2 + 2\,t\,x0 + 2\,y0\right)\right\}$

■ Rotational vector field

In[49]:=
```
VectorPlot[{-y, x},
      {x, -6,  6}, {y, -6, 6},
    VectorPoints → 30,
 VectorScale → Medium,
 StreamPoints → 25, StreamStyle → Orange,
  Axes -> True, Frame → True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[49]=



In[50]:=
```
DSolve[{x'[t] == -y[t], y'[t] == x[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]
```

Out[50]= {x[t] → x0 Cos[t] - y0 Sin[t], y[t] → y0 Cos[t] + x0 Sin[t]}

In[51]:=
```
DSolve[{x'[t] == -y[t], y'[t] == x[t], x[0] == 1, y[0] == 0}, {x[t], y[t]}, t][[1]]
```

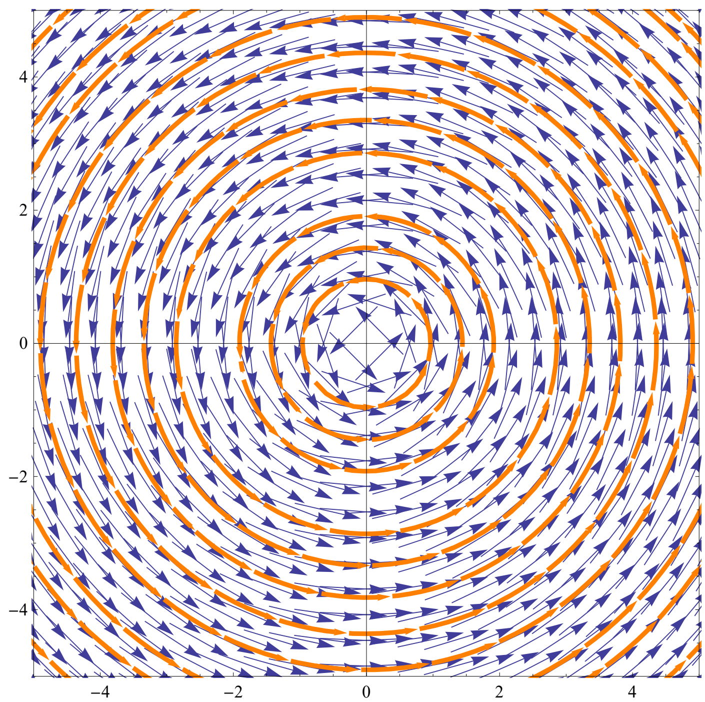
Out[51]= {x[t] → Cos[t], y[t] → Sin[t]}

In[52]:=
```
DSolve[{x'[t] == -y[t], y'[t] == x[t], x[0] == 0, y[0] == 1}, {x[t], y[t]}, t][[1]]
```

Out[52]= {x[t] → -Sin[t], y[t] → Cos[t]}

In[53]:=
```
VectorPlot[{-y/Sqrt[x^2 + y^2], x/Sqrt[x^2 + y^2]},
    {x, -6, 6}, {y, -6, 6},
   VectorPoints → 32,
 VectorScale → Small,
 StreamPoints → 25, StreamStyle → {Orange, Thickness[0.007]},
 Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[53]=



In[54]:=
```
FullSimplify[DSolve[{x'[t] == -y[t]/Sqrt[x[t]^2 + y[t]^2],
    y'[t] == x[t]/Sqrt[x[t]^2 + y[t]^2], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]]
```

Solve::ifun :
  Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

Solve::ifun :
  Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

Solve::ifun :
  Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

General::stop : Further output of Solve::ifun will be suppressed during this calculation. ≫

Out[54]=
$$\left\{y[t] \to \sqrt{\left(x0^2 + y0^2\right) \mathrm{Cos}\left[\frac{t}{\sqrt{x0^2+y0^2}} + \mathrm{ArcTan}\left[\frac{x0}{y0}\right]\right]^2},\; x[t] \to \frac{\sqrt{x0^2+y0^2}\; \mathrm{Tan}\left[\frac{t}{\sqrt{x0^2+y0^2}} + \mathrm{ArcTan}\left[\frac{x0}{y0}\right]\right]}{\sqrt{\mathrm{Sec}\left[\frac{t}{\sqrt{x0^2+y0^2}} + \mathrm{ArcTan}\left[\frac{x0}{y0}\right]\right]^2}}\right\}$$

Testing

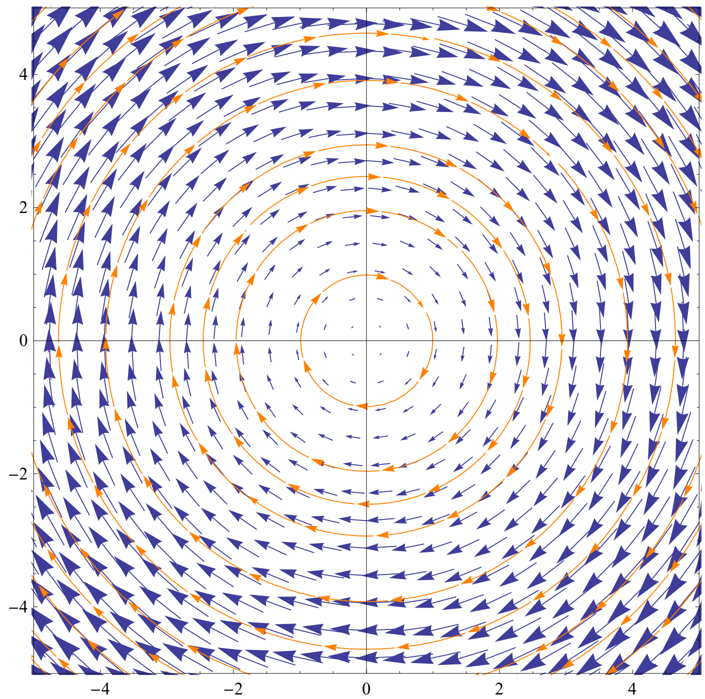
In[55]:=
```
FullSimplify[{y[t] -> Sqrt[(x0^2 + y0^2) Cos[t/Sqrt[x0^2 + y0^2] + ArcTan[x0/y0]]^2],
    x[t] -> (Sqrt[x0^2 + y0^2] Tan[t/Sqrt[x0^2 + y0^2] + ArcTan[x0/y0]])/
      Sqrt[Sec[t/Sqrt[x0^2 + y0^2] + ArcTan[x0/y0]]^2]} /. {x0 -> 0, y0 -> 1}]
```

Out[55]=
$$\left\{y[t] \to \sqrt{\mathrm{Cos}[t]^2},\; x[t] \to \frac{\mathrm{Tan}[t]}{\sqrt{\mathrm{Sec}[t]^2}}\right\}$$

Clearly these formulas have problems. However they are valid for $t \in [0, \pi/2)$. Can you guess the formulas for the correct solution?

In[56]:=
```
VectorPlot[{y, -x},
     {x, -6,  6}, {y, -6, 6},
   VectorPoints → 30,
 VectorScale → Medium,
 StreamPoints → 25, StreamStyle → Orange,
  Axes -> True, Frame → True,
   PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[56]=



In[57]:=
```
FullSimplify[DSolve[{x'[t] == y[t], y'[t] == -x[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]]
```

Out[57]= {x[t] → x0 Cos[t] + y0 Sin[t], y[t] → y0 Cos[t] − x0 Sin[t]}

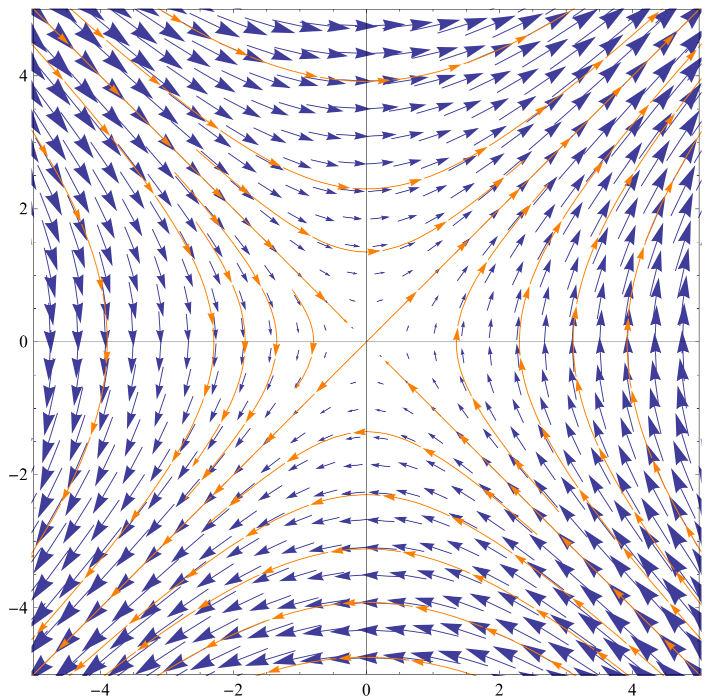
In[58]:=
```
FullSimplify[DSolve[{x'[t] == y[t], y'[t] == -x[t], x[0] == 1, y[0] == 0}, {x[t], y[t]}, t][[1]]]
```

Out[58]= {x[t] → Cos[t], y[t] → −Sin[t]}

■ Hyperbolic vector field

In[59]:=
```
VectorPlot[{y, x},
      {x, -6,  6}, {y, -6, 6},
    VectorPoints → 30,
 VectorScale → Medium,
  StreamPoints → 25, StreamStyle → Orange,
  Axes -> True, Frame → True,
    PlotRange -> {{-5, 5}, {-5, 5}}]
```

Out[59]=



In[60]:=
```
FullSimplify[DSolve[{x'[t] == y[t], y'[t] == x[t], x[0] == x0, y[0] == y0}, {x[t], y[t]}, t][[1]]]
```

Out[60]= {x[t] → x0 Cosh[t] + y0 Sinh[t], y[t] → y0 Cosh[t] + x0 Sinh[t]}

In[61]:=
```
FullSimplify[DSolve[{x'[t] == y[t], y'[t] == x[t], x[0] == 1, y[0] == 0}, {x[t], y[t]}, t][[1]]]
```

Out[61]= {x[t] → Cosh[t], y[t] → Sinh[t]}