In one of the problems in the book we are asked to prove that the set of seven functions

```
{1, Cos[t], (Cos[t])^2, (Cos[t])^3, (Cos[t])^4, (Cos[t])^5,
  (Cos[t])^6}
```

is linearly independent. In Mathematica such a set can be formed as follows:

In[1]:=
```
Table[(Cos[t])^k, {k, 1, 6}]
```

Out[1]=
```
{Cos[t], Cos[t]^2, Cos[t]^3, Cos[t]^4, Cos[t]^5, Cos[t]^6}
```

In[ ]:=
```
Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1]
```

Out[ ]=
```
{1, Cos[t], Cos[t]^2, Cos[t]^3, Cos[t]^4, Cos[t]^5, Cos[t]^6}
```

Above, I am a little paranoid not to use

```
(Cos[t])^0
```

instead of 1, since zero to the power zero is not defined. So, formally the expression in the preceding line is not well-defined. Therefore, I use Prepend

Now we want to assign seven values to t. We do it as follows:

In[3]:=
```
Table[t, {t, 0, 6 Pi/7, Pi/7}]
```

Out[3]=
```
{0, π/7, 2 π/7, 3 π/7, 4 π/7, 5 π/7, 6 π/7}
```

In[4]:=
```
Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1],
 {t, 0, 6 Pi/7, Pi/7}]
```

Out[4]=
$$\left\{\{1, 1, 1, 1, 1, 1, 1\}, \left\{1, \cos\left[\frac{\pi}{7}\right], \cos\left[\frac{\pi}{7}\right]^2, \cos\left[\frac{\pi}{7}\right]^3, \cos\left[\frac{\pi}{7}\right]^4, \cos\left[\frac{\pi}{7}\right]^5, \cos\left[\frac{\pi}{7}\right]^6\right\}, \left\{1, \sin\left[\frac{3\pi}{14}\right], \sin\left[\frac{3\pi}{14}\right]^2, \sin\left[\frac{3\pi}{14}\right]^3, \sin\left[\frac{3\pi}{14}\right]^4, \sin\left[\frac{3\pi}{14}\right]^5, \sin\left[\frac{3\pi}{14}\right]^6\right\}, \left\{1, \sin\left[\frac{\pi}{14}\right], \sin\left[\frac{\pi}{14}\right]^2, \sin\left[\frac{\pi}{14}\right]^3, \sin\left[\frac{\pi}{14}\right]^4, \sin\left[\frac{\pi}{14}\right]^5, \sin\left[\frac{\pi}{14}\right]^6\right\}, \left\{1, -\sin\left[\frac{\pi}{14}\right], \sin\left[\frac{\pi}{14}\right]^2, -\sin\left[\frac{\pi}{14}\right]^3, \sin\left[\frac{\pi}{14}\right]^4, -\sin\left[\frac{\pi}{14}\right]^5, \sin\left[\frac{\pi}{14}\right]^6\right\}, \left\{1, -\sin\left[\frac{3\pi}{14}\right], \sin\left[\frac{3\pi}{14}\right]^2, -\sin\left[\frac{3\pi}{14}\right]^3, \sin\left[\frac{3\pi}{14}\right]^4, -\sin\left[\frac{3\pi}{14}\right]^5, \sin\left[\frac{3\pi}{14}\right]^6\right\}, \left\{1, -\cos\left[\frac{\pi}{7}\right], \cos\left[\frac{\pi}{7}\right]^2, -\cos\left[\frac{\pi}{7}\right]^3, \cos\left[\frac{\pi}{7}\right]^4, -\cos\left[\frac{\pi}{7}\right]^5, \cos\left[\frac{\pi}{7}\right]^6\right\}\right\}$$

The above expression is exactly the matrix that we need to study:

In[8]:=
```
MatrixForm[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1],
  {t, 0, 6 Pi/7, Pi/7}]]
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \cos\left[\frac{\pi}{7}\right] & \cos\left[\frac{\pi}{7}\right]^2 & \cos\left[\frac{\pi}{7}\right]^3 & \cos\left[\frac{\pi}{7}\right]^4 & \cos\left[\frac{\pi}{7}\right]^5 & \cos\left[\\ 1 & \sin\left[\frac{3\pi}{14}\right] & \sin\left[\frac{3\pi}{14}\right]^2 & \sin\left[\frac{3\pi}{14}\right]^3 & \sin\left[\frac{3\pi}{14}\right]^4 & \sin\left[\frac{3\pi}{14}\right]^5 & \sin\left[\\ 1 & \sin\left[\frac{\pi}{14}\right] & \sin\left[\frac{\pi}{14}\right]^2 & \sin\left[\frac{\pi}{14}\right]^3 & \sin\left[\frac{\pi}{14}\right]^4 & \sin\left[\frac{\pi}{14}\right]^5 & \sin\left[\\ 1 & -\sin\left[\frac{\pi}{14}\right] & \sin\left[\frac{\pi}{14}\right]^2 & -\sin\left[\frac{\pi}{14}\right]^3 & \sin\left[\frac{\pi}{14}\right]^4 & -\sin\left[\frac{\pi}{14}\right]^5 & \sin\left[\\ 1 & -\sin\left[\frac{3\pi}{14}\right] & \sin\left[\frac{3\pi}{14}\right]^2 & -\sin\left[\frac{3\pi}{14}\right]^3 & \sin\left[\frac{3\pi}{14}\right]^4 & -\sin\left[\frac{3\pi}{14}\right]^5 & \sin\left[\\ 1 & -\cos\left[\frac{\pi}{7}\right] & \cos\left[\frac{\pi}{7}\right]^2 & -\cos\left[\frac{\pi}{7}\right]^3 & \cos\left[\frac{\pi}{7}\right]^4 & -\cos\left[\frac{\pi}{7}\right]^5 & \cos\left[\end{pmatrix}$$

Calculate the determinant:

In[5]:=
```
FullSimplify[
  Det[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1],
    {t, 0, 6 Pi/7, Pi/7}]]]
```

Out[5]=
$$-\frac{343}{262\,144}$$

Row reduce:

In[•]:=
```
RowReduce[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1], {t, 0, 6 Pi/7, Pi/7}]]
```

Out[•]=
```
{{1, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1}}
```

We could have used some other values of t. For example

In[14]:=
```
TeXForm[Table[t, {t, 0, Pi, Pi/6}]]
```

Out[14]//TeXForm=
```
\left\{0,\frac{\pi }{6},\frac{\pi }{3},\frac{\pi }{2},\frac{2
   \pi }{3},\frac{5 \pi }{6},\pi \right\}
```

In[9]:=
```
MatrixForm[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1], {t, 0, Pi, Pi/6}]]
```

Out[9]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{\sqrt{3}}{2} & \frac{3}{4} & \frac{3\sqrt{3}}{8} & \frac{9}{16} & \frac{9\sqrt{3}}{32} & \frac{27}{64} \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} & \frac{1}{32} & \frac{1}{64} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{8} & \frac{1}{16} & -\frac{1}{32} & \frac{1}{64} \\ 1 & -\frac{\sqrt{3}}{2} & \frac{3}{4} & -\frac{3\sqrt{3}}{8} & \frac{9}{16} & -\frac{9\sqrt{3}}{32} & \frac{27}{64} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix}$$

In[15]:=
```
TeXForm[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1],
  {t, 0, Pi, Pi/6}]]
```

Out[15]//TeXForm=
```
\left(
\begin{array}{ccccccc}
 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 1 & \frac{\sqrt{3}}{2} & \frac{3}{4} & \frac{3 \sqrt{3}}{8}
   & \frac{9}{16} & \frac{9 \sqrt{3}}{32} & \frac{27}{64} \\
 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} &
   \frac{1}{32} & \frac{1}{64} \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{8} & \frac{1}{16}
   & -\frac{1}{32} & \frac{1}{64} \\
 1 & -\frac{\sqrt{3}}{2} & \frac{3}{4} & -\frac{3
   \sqrt{3}}{8} & \frac{9}{16} & -\frac{9 \sqrt{3}}{32} &
   \frac{27}{64} \\
 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
\end{array}
\right)
```

In[17]:=
```
Det[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1],
  {t, 0, Pi, Pi/6}]]
```

Out[17]=
$$-\frac{27\sqrt{3}}{8192}$$

In[16]:=
```
TeXForm[
 Det[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1],
   {t, 0, Pi, Pi/6}]]]
```

Out[16]//TeXForm=
```
-\frac{27 \sqrt{3}}{8192}
```

In[12]:=
```
RowReduce[Table[Prepend[Table[(Cos[t])^k, {k, 1, 6}], 1],
  {t, 0, Pi, Pi/6}]]
```

Out[12]=
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$