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37. [M] The vector space $H = \text{Span}\{1, \cos^2 t, \cos^4 t, \cos^6 t\}$ contains at least two interesting functions that will be used in a later exercise:

$$\mathbf{f}(t) = 1 - 8\cos^2 t + 8\cos^4 t$$
$$\mathbf{g}(t) = -1 + 18\cos^2 t - 48\cos^4 t + 32\cos^6 t$$

Study the graph of $\mathbf{f}$ for $0 \le t \le 2\pi$, and guess a simple formula for $\mathbf{f}(t)$. Verify your conjecture by graphing the difference between $1 + \mathbf{f}(t)$ and your formula for $\mathbf{f}(t)$. (Hopefully, you will see the constant function 1.) Repeat for $\mathbf{g}$.

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38. [M] Show that $\{1, \cos t, \cos^2 t, \dots, \cos^6 t\}$ is a linearly independent set of functions defined on $\mathbb{R}$. Use the method of Exercise 37. (This result will be needed in Exercise 34 in Section 4.5.)

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34. [M] Let $\mathcal{B} = \{1, \cos t, \cos^2 t, \dots, \cos^6 t\}$ and $\mathcal{C} = \{1, \cos t, \cos 2t, \dots, \cos 6t\}$. Assume the following trigonometric identities (see Exercise 37 in Section 4.1).

$$\begin{aligned}
\cos 2t &= -1 + 2\cos^2 t \\
\cos 3t &= -3\cos t + 4\cos^3 t \\
\cos 4t &= 1 - 8\cos^2 t + 8\cos^4 t \\
\cos 5t &= 5\cos t - 20\cos^3 t + 16\cos^5 t \\
\cos 6t &= -1 + 18\cos^2 t - 48\cos^4 t + 32\cos^6 t
\end{aligned}$$

Let H be the subspace of functions spanned by the functions in $\mathcal{B}$. Then $\mathcal{B}$ is a basis for H, by Exercise 38 in Section 4.3.

a. Write the $\mathcal{B}$-coordinate vectors of the vectors in $\mathcal{C}$, and use them to show that $\mathcal{C}$ is a linearly independent set in H.

b. Explain why $\mathcal{C}$ is a basis for H.

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17. [M] Let $\mathcal{B} = \{\mathbf{x}_0, \dots, \mathbf{x}_6\}$ and $\mathcal{C} = \{\mathbf{y}_0, \dots, \mathbf{y}_6\}$, where $\mathbf{x}_k$ is the function $\cos^k t$ and $\mathbf{y}_k$ is the function $\cos kt$. Exercise 34 in Section 4.5 showed that both $\mathcal{B}$ and $\mathcal{C}$ are bases for the vector space $H = \text{Span}\{\mathbf{x}_0, \dots, \mathbf{x}_6\}$.

a. Set $P = \begin{bmatrix} [\mathbf{y}_0]_{\mathcal{B}} & \cdots & [\mathbf{y}_6]_{\mathcal{B}} \end{bmatrix}$, and calculate P^{-1}.

b. Explain why the columns of P^{-1} are the $\mathcal{C}$-coordinate vectors of $\mathbf{x}_0, \dots, \mathbf{x}_6$. Then use these coordinate vectors to write trigonometric identities that express powers of $\cos t$ in terms of the functions in $\mathcal{C}$.