

This notebook is saved with all output deleted. To recreate all the calculations and the pictures go to the menu item

Evaluation-> Evaluate notebook (the shortcut is Alt v + o)

To evaluate individual cells use Shift+Enter

When you are done, before saving the notebook delete all output by menu item Cell->Delete all output (shortcut Alt c + l)

$$y u_x - x u_y = u, u(x, 0) = x^2$$

The solution that we on the website

$$\text{In[1]:= } uu[x_, y_] := (x^2 + y^2) \text{Exp}\left[-\text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right]$$

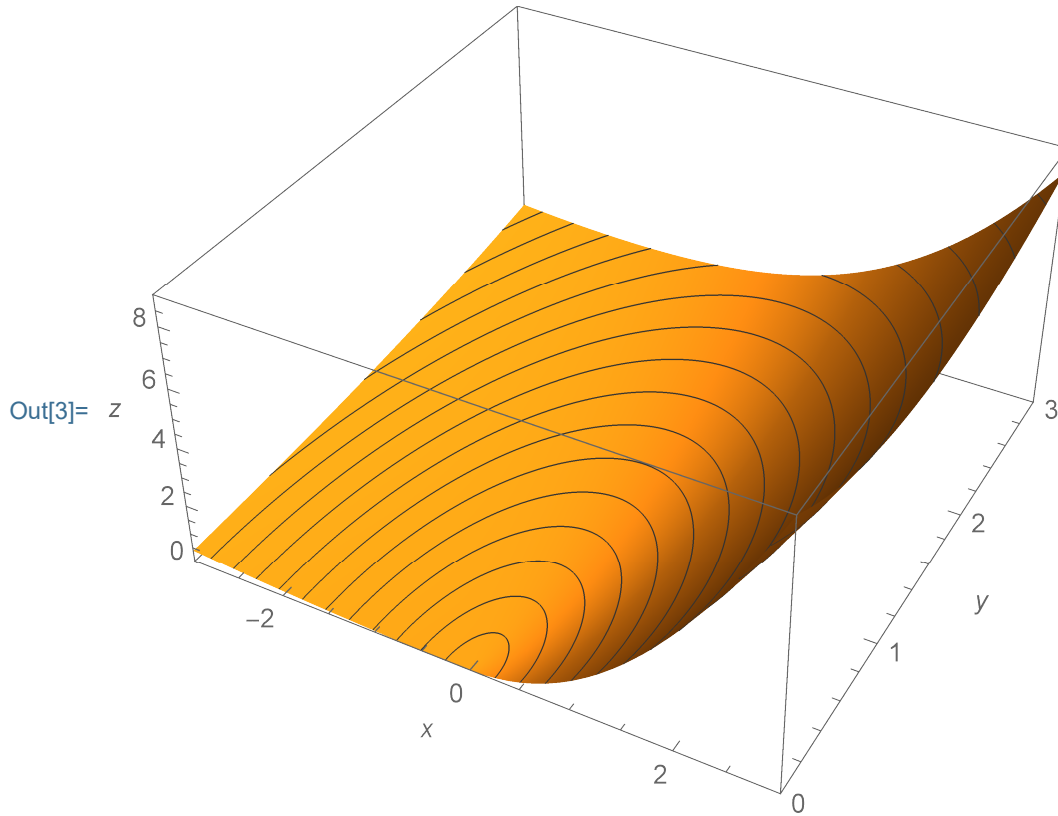
Verify by Mathematica:

$$\text{In[2]:= } \text{FullSimplify}[y D[uu[x, y], x] - x D[uu[x, y], y] - uu[x, y], \text{And}[y > 0, x \in \text{Reals}]]$$

$$\text{Out[2]= } 0$$

A graph of the surface formed by the characteristics; you can see characteristics (helix-like curves) as mesh lines on the surface

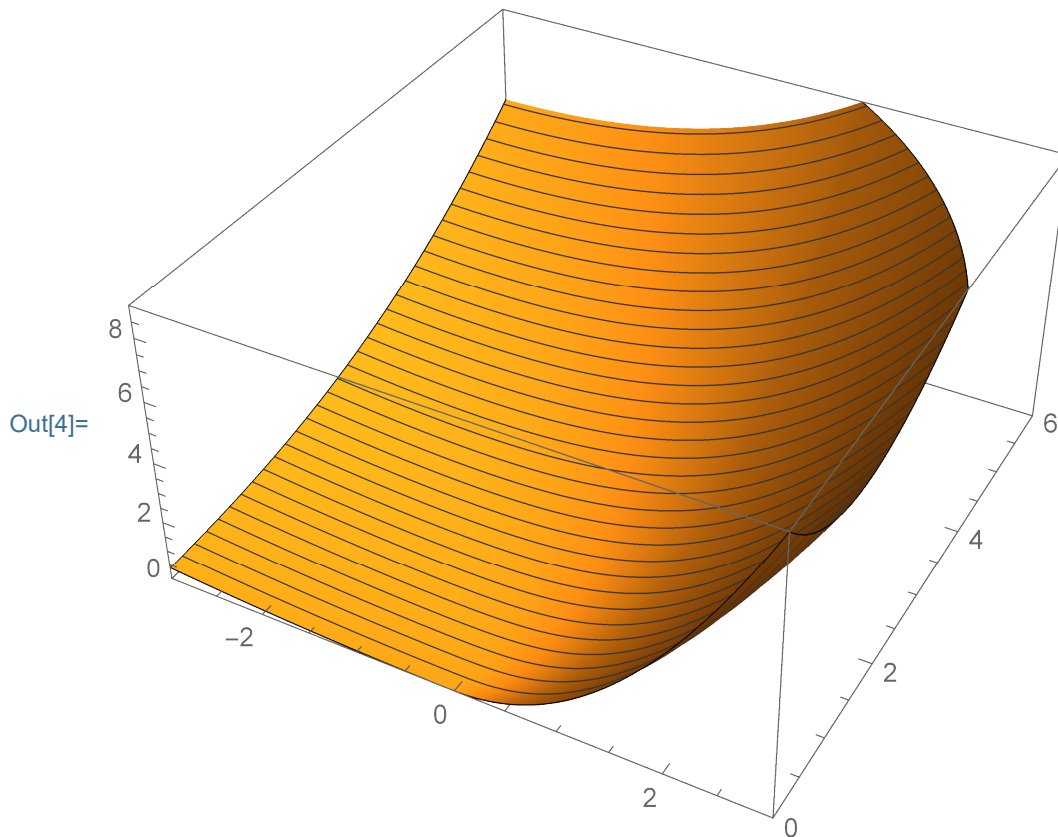
```
In[3]:= ParametricPlot3D[{ $\xi \text{Cos}[s]$ ,  $\xi \text{Sin}[s]$ ,  $\xi^2 \text{Exp}[-s]$ }, {s, 0, Pi},  
{ $\xi$ , 0, 13}, PlotPoints  $\rightarrow$  {100, 100}, Mesh  $\rightarrow$  {0, 50}, BoxRatios  $\rightarrow$  {2, 2, 1},  
AxesLabel  $\rightarrow$  {x, y, z}, PlotRange  $\rightarrow$  {{-3, 3}, {0, 3}, {0, 9}}
```



A graph of the solution; thinking of y as time, we can see how the initial condition changes in time by looking at the mesh lines on the surface

```
In[4]:= Plot3D[(x^2 + y^2) Exp[-ArcCos[x/sqrt(x^2 + y^2)]]], {x, -3, 3}, {y, 0, 15},
```

```
PlotPoints -> {100, 100}, Mesh -> {0, 40}, BoxRatios -> {2, 2, 1},
PlotRange -> {{-3, 3}, {0, 6}, {0, 9}}, ClippingStyle -> None]
```

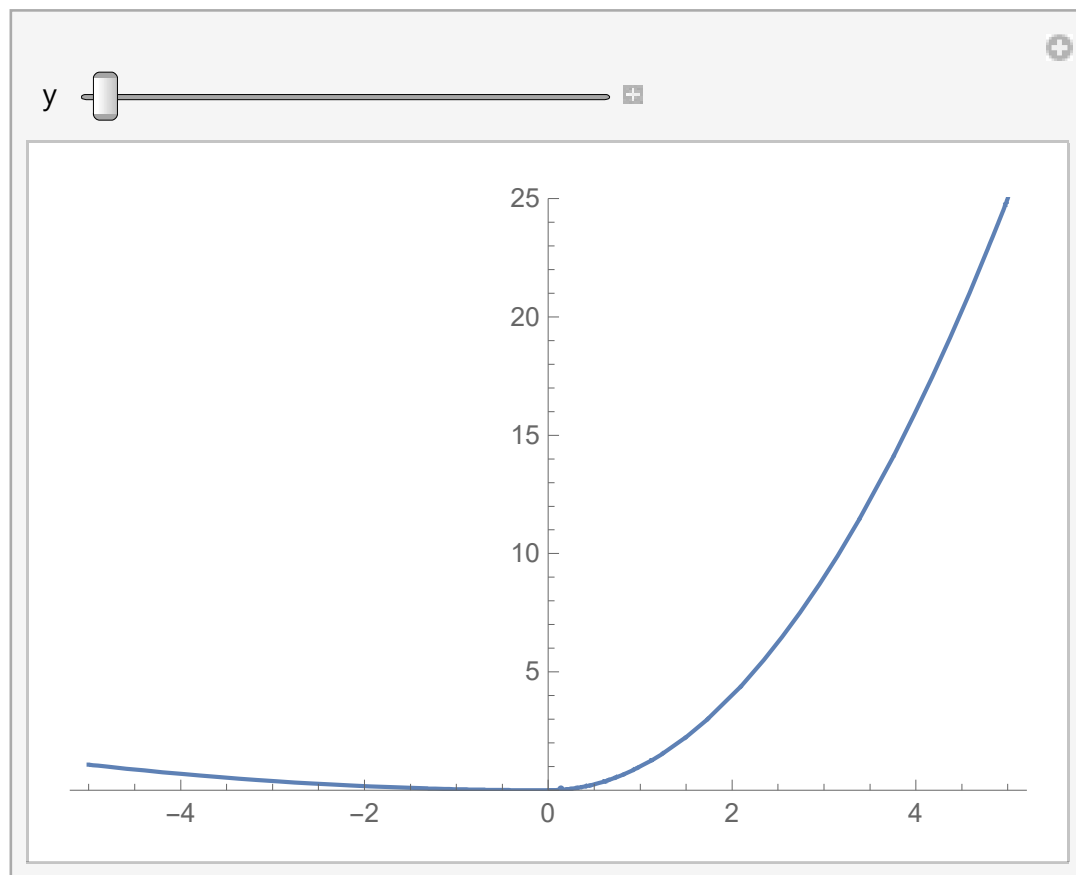


Below I use manipulate to show how the initial condition changes in time, here y is time,

```
In[5]:= Manipulate[Plot[(x^2 + y^2) Exp[-ArcCos[x/sqrt(x^2 + y^2)]]], {x, -5, 5},
```

```
PlotRange -> {0, 25}], {y, 0, 10}]
```

Out[5]=



```
In[6]:= FullSimplify[uu[x, 0], And[x > 0]]
```

Out[6]= x^2

Below I get better graphs of characteristics and the surfaces which I posted on the website. Often in one picture I show several objects which I plot in Mathematica separately, then I use `Show[]` to tell Mathematica to show different objects in one picture.

```
In[7]:= Clear[ff1]; ff1[x_] := (x)^2;
```

```
In[8]:= VP = {1.3, -2.4, 2.} (*ViewPoint->Dynamic[VP]*)
```

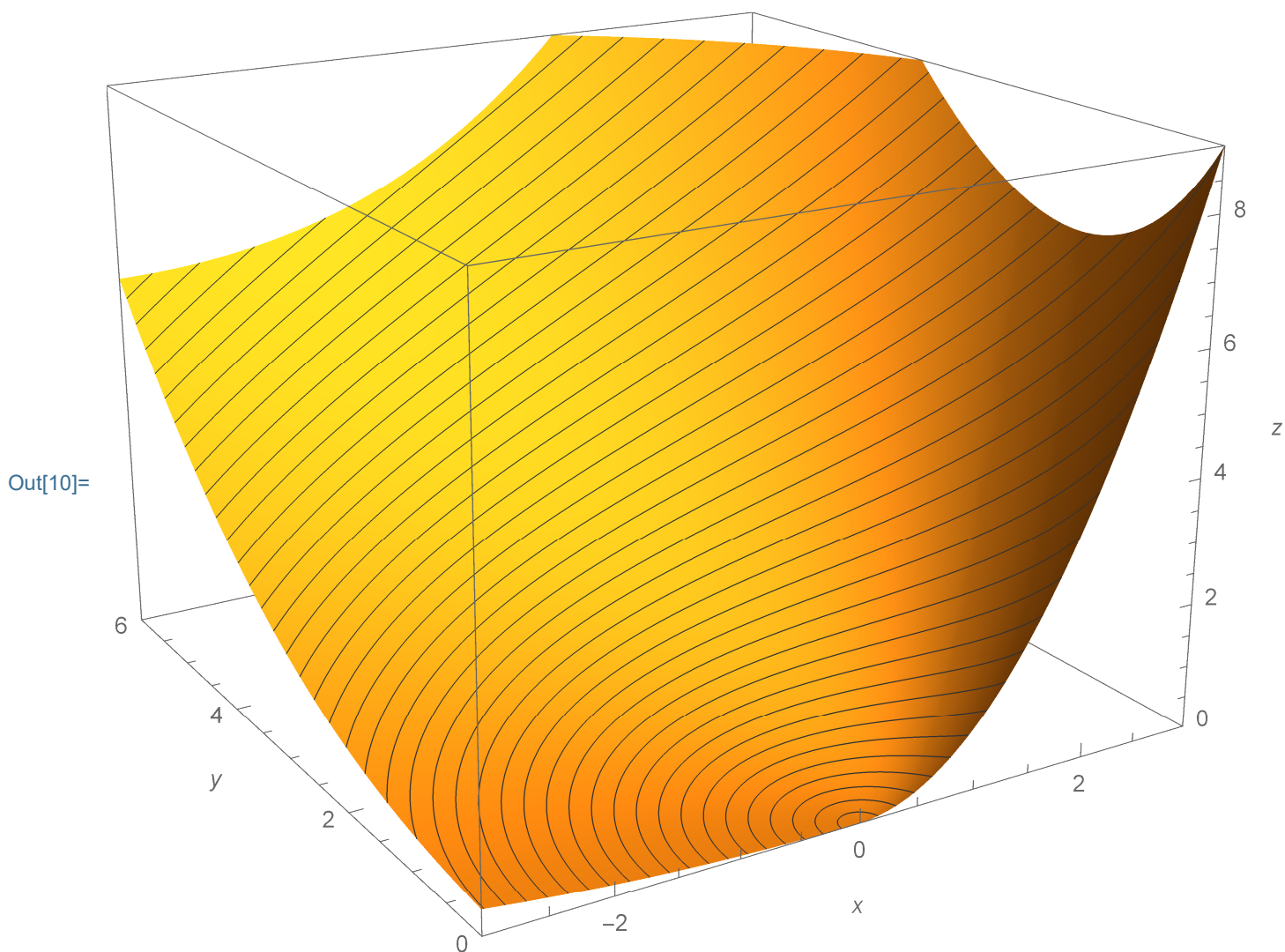
Out[8]= {1.3, -2.4, 2.}

```
In[9]:= VP1 = {-1.6608720911023165, -2.7404767391528453, 1.086872181610885}
```

Out[9]= {-1.66087, -2.74048, 1.08687}

In[10]:= MoCCirclesSur1f1 =

```
Show[ParametricPlot3D[{ξ, 0, ff1[ξ]}, {ξ, 0, 5},
  PlotStyle → {{Thickness[0.01], RGBColor[0, 0.6, 0]}},
  PlotPoints → {100}],
ParametricPlot3D[{ξ Cos[s], ξ Sin[s], ff1[ξ] Exp[-s]}, {s, 0, Pi},
  {ξ, 0, 8}, PlotPoints → {100, 100}, Mesh → {0, 50}],
PlotRange → {{-3, 3}, {-0, 6}, {0, 9}}, PlotRangePadding → None,
BoxRatios → {2, 2, 1.5}, AxesLabel → {x, y, z},
AxesEdge → {{-1, -1}, {-1, -1}, {1, -1}}, ImageSize → 500, ViewPoint → VP1]
```

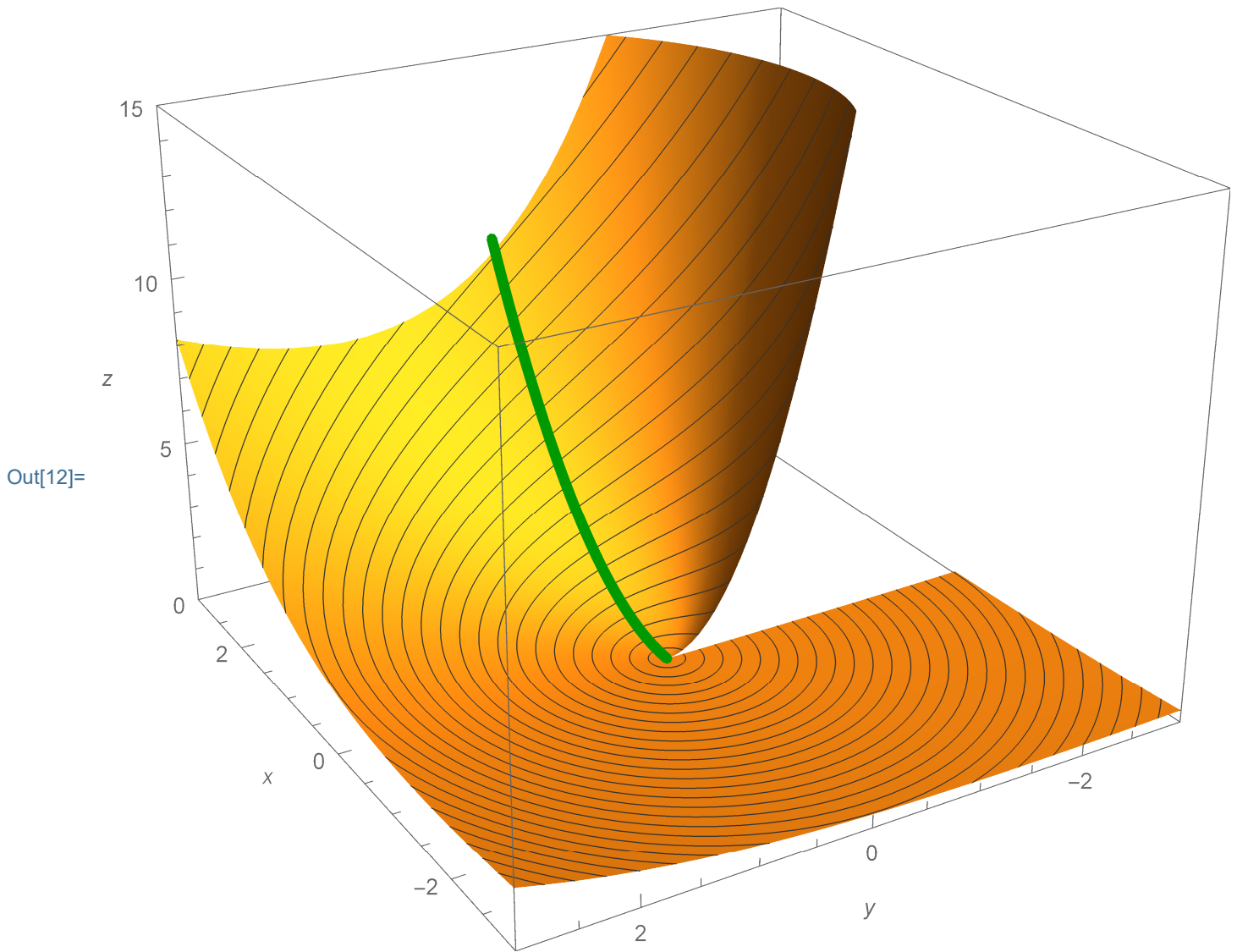


In[11]:= VP1e = {-2.6622328739536525`, 1.5833298913814935`, 1.3621977022071134`}

Out[11]= {-2.66223, 1.58333, 1.3622}

6 | MoC_w_circle.nb
In[12]:= MoCCirclesSur1ef1 =

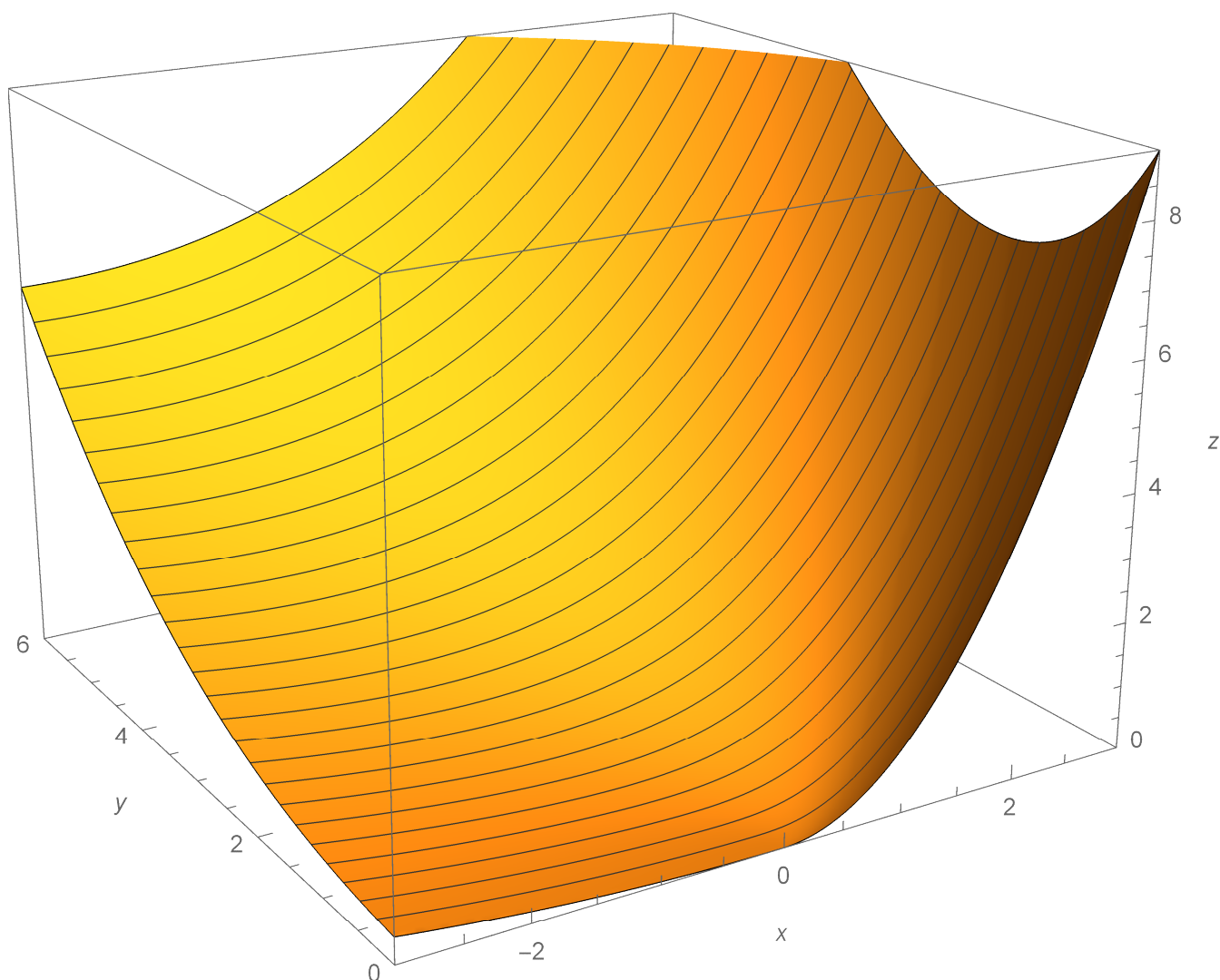
```
Show[ParametricPlot3D[{ $\xi$ , 0, ff1[ $\xi$ ]}, { $\xi$ , 0, 3.05},  
PlotStyle → {{Thickness[0.01], RGBColor[0, 0.6, 0]}},  
PlotPoints → {100}],  
ParametricPlot3D[{ $\xi$  Cos[s],  $\xi$  Sin[s], ff1[ $\xi$ ] Exp[-s]},  
{s, -Pi/2, 3 Pi/2}, { $\xi$ , 0, 8}, PlotPoints → {100, 100}, Mesh → {0, 50}],  
PlotRange → {{-3, 3}, {-3, 3}, {0, 15}}, PlotRangePadding → None,  
BoxRatios → {2, 2, 1.5}, AxesLabel → {x, y, z},  
AxesEdge → {{1, -1}, {-1, -1}, {1, 1}}, ImageSize → 500, ViewPoint → VP1e]
```



In[13]:= MoCCirclesSur2f1 =

```
Show[ParametricPlot3D[{ξ, 0, ff1[ξ]}, {ξ, 0, 5},
  PlotStyle → {{Thickness[0.01], RGBColor[0, 0.6, 0]}},
  PlotPoints → {100}], Plot3D[ff1[√(x² + y²)] Exp[-ArcCos[x/√(x² + y²)]]],
  {x, -3, 3}, {y, 0, 6}, PlotPoints → {100, 100}, Mesh → {0, 25}],
PlotRange → {{-3, 3}, {-0, 6}, {0, 9}}, PlotRangePadding → None,
BoxRatios → {2, 2, 1.5}, AxesLabel → {x, y, z},
AxesEdge → {{-1, -1}, {-1, -1}, {1, -1}}, ImageSize → 500, ViewPoint → VP1]
```

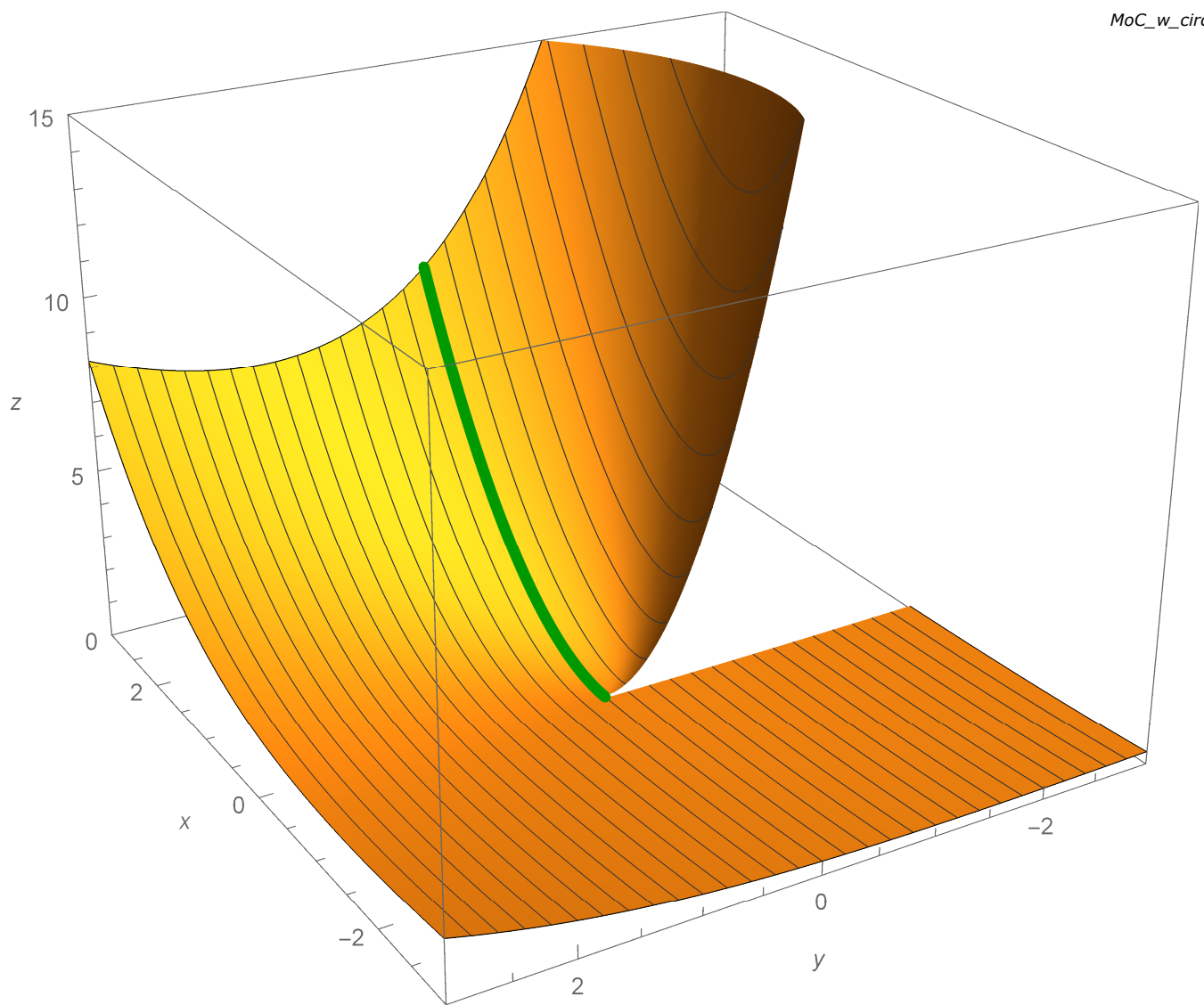
Out[13]=



In[14]:= MoCCirclesSur2ef1 =

```
Show[ParametricPlot3D[{{ξ, 0, ff1[ξ]}, {ξ, 0, 3.0}},
  PlotStyle → {{Thickness[0.01], RGBColor[0, 0.6, 0]}},
  PlotPoints → {100}],
Plot3D[
  If[y ≥ 0, ff1[√(x² + y²)] Exp[-ArcCos[x/√(x² + y²)]]],
  If[x > 0,
    ff1[√(x² + y²)] Exp[-(-ArcCos[x/√(x² + y²)])],
    ff1[√(x² + y²)] Exp[-(2 Pi - ArcCos[x/√(x² + y²)])]
  ]], {x, -3, 3}, {y, -3, 3}, Exclusions → {{x == 0, y ≤ 0}},
  PlotPoints → {100, 100}, Mesh → {0, 29}],
PlotRange → {{-3, 3}, {-3, 3}, {0, 15}}, PlotRangePadding → None,
BoxRatios → {2, 2, 1.5}, AxesLabel → {x, y, z},
AxesEdge → {{1, -1}, {-1, -1}, {1, 1}}, ImageSize → 500, ViewPoint → VP1e]
```


Out[14]=



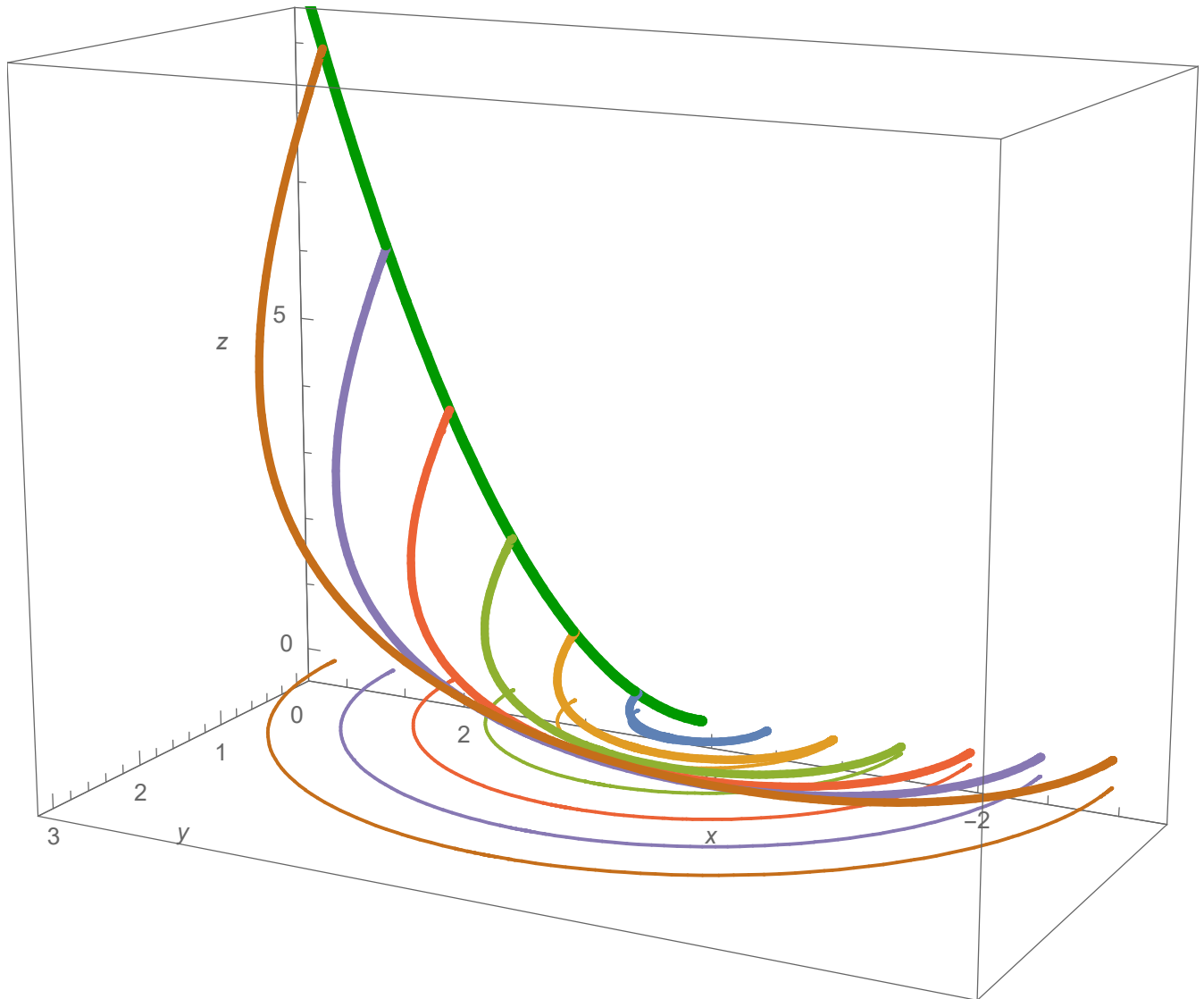
```
In[15]:= VP1a = {-1.5329095616109654`, 2.9047381415820985`, 0.8140544237087928`}
```

```
Out[15]= {-1.53291, 2.90474, 0.814054}
```

10 | MoC_w_circle.nb
In[16]:= MoCCirclesCha1f1 =

```
Show[ParametricPlot3D[{ $\xi$ , 0, ff1[ $\xi$ ]}, { $\xi$ , 0, 4},  
  PlotStyle → {{Thickness[0.009], RGBColor[0, 0.6, 0]}},  
  PlotPoints → {100}],  
ParametricPlot3D[  
  Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], ff1[ $\xi$ ] Exp[-s]}, { $\xi$ , 0.5, 3, .5}],  
  {s, 0, Pi}, PlotStyle → {Thickness[0.007]}, PlotPoints → {100, 100}],  
ParametricPlot3D[Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], 0}, { $\xi$ , 0.5, 3, .5}],  
  {s, 0, Pi}, PlotStyle → {Thickness[0.003]}, PlotPoints → {100}],  
PlotRange → {{-3, 3}, {0, 3}, {0, 9}}, BoxRatios → {2, 1, 1.5},  
AxesLabel → {x, y, z}, AxesEdge → {{-1, -1}, {1, -1}, {1, -1}},  
ImageSize → 500, ViewPoint → VP1a]
```

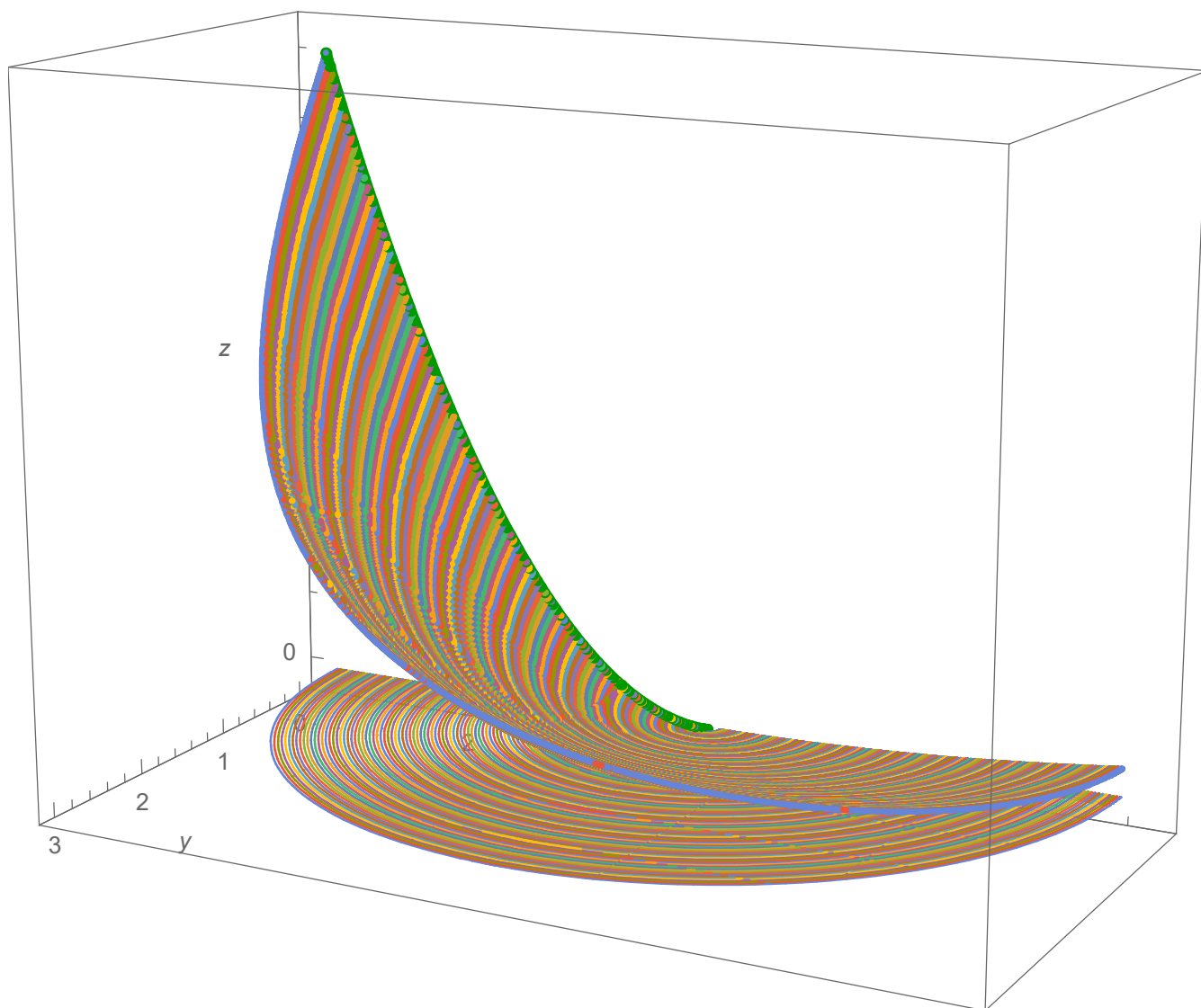
Out[16]=



In[17]:= MoCCirclesCha2f1 =

```
Show[ParametricPlot3D[{ $\xi$ , 0, ff1[ $\xi$ ]}, { $\xi$ , 0, 3},
  PlotStyle → {{Thickness[0.009], RGBColor[0, 0.6, 0]}},
  PlotPoints → {100}],
ParametricPlot3D[
  Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], ff1[ $\xi$ ] Exp[-s]}, { $\xi$ , 0.1, 3, .025}],
  {s, 0, Pi}, PlotStyle → {Thickness[0.005]}, PlotPoints → {100}],
ParametricPlot3D[
  Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], 0}, { $\xi$ , 0.1, 3, .025}],
  {s, 0, Pi}, PlotStyle → {Thickness[0.002]}, PlotPoints → {100}],
PlotRange → {{-3, 3}, {0, 3}, {0, 9}}, BoxRatios → {2, 1, 1.5},
AxesLabel → {x, y, z}, AxesEdge → {{-1, -1}, {1, -1}, {1, -1}},
ImageSize → 500, ViewPoint → VP1a]
```

Out[17]=



In[18]:= NotebookDirectory[]

Out[18]= C:\Dropbox\Work\myweb\Courses\Math_pages\Math_430\

```
In[19]:= SetDirectory["C:\\Dropbox\\Work\\myweb\\Courses\\Math_pages\\Math_430"]
```

```
Out[19]= C:\\Dropbox\\Work\\myweb\\Courses\\Math_pages\\Math_430
```

I commented out (* *) the commands which export the pictures to be displayed on the website.

```
In[20]:= Directory[]
```

```
Out[20]= C:\\Dropbox\\Work\\myweb\\Courses\\Math_pages\\Math_430
```

```
In[21]:= (* Export["MoCCirclesCha1f1.png", MoCCirclesCha1f1, "PNG",
  ImageResolution->1200];
  Export["MoCCirclesCha2f1.png", MoCCirclesCha2f1, "PNG",
  ImageResolution->1200];
  Export["MoCCirclesSur1f1.png", MoCCirclesSur1f1, "PNG",
  ImageResolution->1200];
  Export["MoCCirclesSur2f1.png", MoCCirclesSur2f1, "PNG",
  ImageResolution->1200];
  Export["MoCCirclesSur1ef1.png", MoCCirclesSur1ef1, "PNG",
  ImageResolution->1200];
  Export["MoCCirclesSur2ef1.png", MoCCirclesSur2ef1, "PNG",
  ImageResolution->1200] *)
```

$$y u_x - x u_y = u, \quad u(x, 0) = (\sin[x])^2$$

In this section I repeat the previous section with a different initial condition.

```
In[22]:= uu2[x_, y_] := (Sin[Sqrt[x^2 + y^2]])^2 Exp[-ArcCos[x/Sqrt[x^2 + y^2]]]
```

```
In[23]:= FullSimplify[y D[uu2[x, y], x] - x D[uu2[x, y], y] - uu2[x, y],
  And[y > 0, x ∈ Reals]]
```

```
Out[23]= 0
```

```
In[24]:= FullSimplify[(y D[#, x] - x D[#, y] - #) & [
  (Sin[Sqrt[x^2 + y^2]])^2 Exp[ArcCos[x/Sqrt[x^2 + y^2]]], And[y < 0, x > 0]]
```

```
Out[24]= 0
```

```
In[25]:= FullSimplify[(y D[#, x] - x D[#, y] - #) &[
  (Sin[Sqrt[x^2 + y^2]])^2 Exp[-2 Pi + ArcCos[x/Sqrt[x^2 + y^2]]], And[y < 0, x > 0]]
```

```
Out[25]= 0
```

```
In[26]:= FullSimplify[uu2[x, 0], And[x > 0]]
```

```
Out[26]= Sin[x]^2
```

```
In[27]:= Clear[ff2]; ff2[x_] := (Sin[x])^2;
```

```
In[28]:= VP = {1.3`, -2.4`, 2.`} (*ViewPoint→Dynamic[VP]*)
```

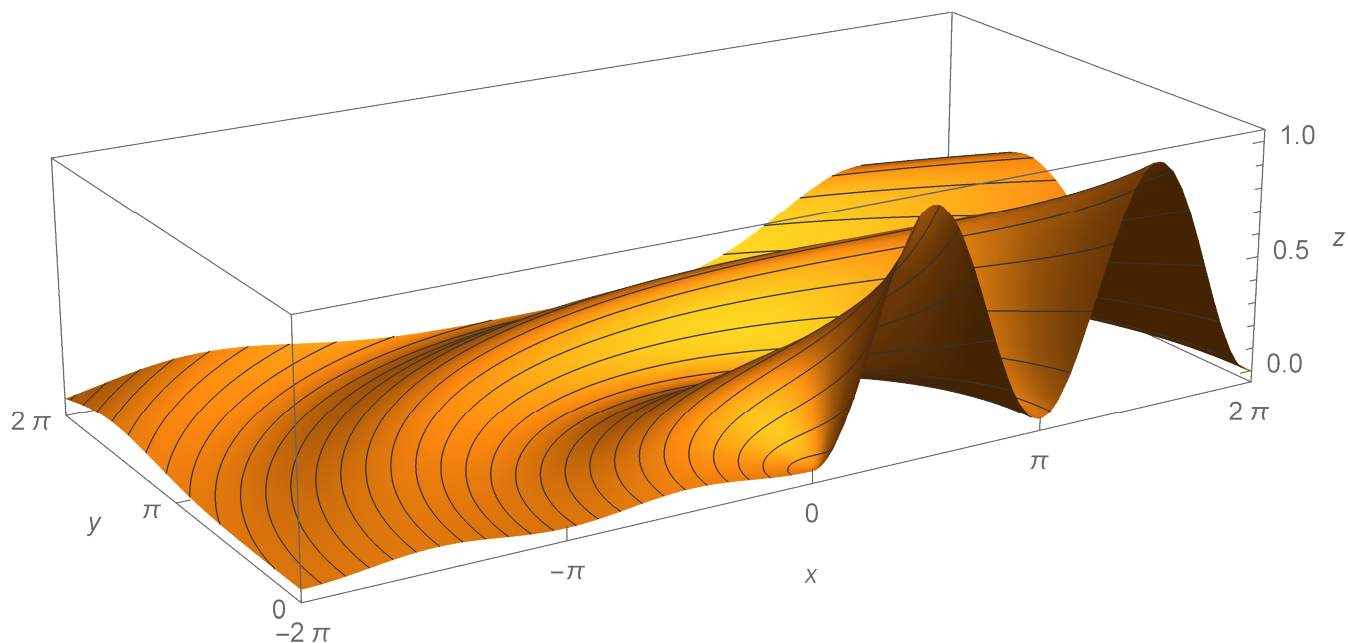
```
Out[28]= {1.3, -2.4, 2.}
```

```
In[29]:= VP1 = {-1.6608720911023165`, -2.7404767391528453`, 1.086872181610885`}
```

```
Out[29]= {-1.66087, -2.74048, 1.08687}
```

```
Show[ParametricPlot3D[{ $\xi$ , 0, ff2[ $\xi$ ]}, { $\xi$ , 0, 4 Pi},  
PlotStyle -> {{Thickness[0.01], RGBColor[0, 0.6, 0]}},  
PlotPoints -> {100}},  
ParametricPlot3D[{ $\xi$  Cos[s],  $\xi$  Sin[s], ff2[ $\xi$ ] Exp[-s]}, {s, 0, Pi},  
{ $\xi$ , 0, 14}, PlotPoints -> {100, 100}, Mesh -> {0, 50}},  
PlotRange -> {{-2 Pi, 2 Pi}, {0, 2 Pi}, {-0.05, 1.05}},  
PlotRangePadding -> None,  
Ticks -> {Range[-4 Pi, 2 Pi, Pi], Range[-4 Pi, 2 Pi, Pi], Automatic},  
BoxRatios -> {2, 1, 0.5}, AxesLabel -> {x, y, z},  
AxesEdge -> {{-1, -1}, {-1, -1}, {1, -1}}, ImageSize -> 500, ViewPoint -> VP1]
```

Out[30]=



In[31]:= (MoCCirclesSur1f2 // Options)

```
Out[31]= {PlotRange → {{-2 π, 2 π}, {0, 2 π}, {-0.05, 1.05}},
  PlotRangePadding → None, Ticks → {{-4 π, -3 π, -2 π, -π, 0, π, 2 π},
    {-4 π, -3 π, -2 π, -π, 0, π, 2 π}, Automatic}, BoxRatios → {2, 1, 0.5},
  AxesLabel → {x, y, z}, AxesEdge → {{-1, -1}, {-1, -1}, {1, -1}},
  ImageSize → 500, ViewPoint → {-1.66087, -2.74048, 1.08687},
  DisplayFunction → Identity, PlotRange →
    {{Automatic, Automatic}, {Automatic, Automatic}, {Automatic, Automatic}},
  PlotRangePadding → {{Scaled[0.05], Scaled[0.05]},
    {Scaled[0.05], Scaled[0.05]}, {Scaled[0.05], Scaled[0.05]}},
  ImagePadding → Automatic, DisplayFunction → Identity, Axes → True,
  DisplayFunction ⇒ Identity, FaceGridsStyle → Automatic,
  Method → {DefaultGraphicsInteraction →
    {Version → 1.2, TrackMousePosition → {True, False}, Effects →
      {Highlight → {ratio → 2}, HighlightPoint → {ratio → 2}, Droplines →
        {freeformCursorMode → True, placement → {x → All, y → None}}}},
  PlotRange → {{0., 12.5664}, {-1., 1.}, {0., 1.}},
  PlotRangePadding → {Scaled[0.02], Scaled[0.02], Scaled[0.02]},
  Ticks → {Automatic, Automatic, Automatic}}
```

```
In[ ]:= {PlotRange → {{-2 π, 2 π}, {0, 2 π}, {-0.05`, 1.05`}},
```

```
PlotRangePadding → None,
```

```
Ticks → {{-4 π, -3 π, -2 π, -π, 0, π, 2 π}, {-4 π, -3 π, -2 π, -π, 0, π, 2 π},
```

```
Automatic}, BoxRatios → {2, 1, 0.5`}, AxesLabel → {x, y, z},
```

```
AxesEdge → {{-1, -1}, {-1, -1}, {1, -1}}, ImageSize → 500,
```

```
ViewPoint → {-1.6608720911023165`, -2.7404767391528453`,
```

```
1.086872181610885`}, DisplayFunction → Identity,
```

```
PlotRange → {{Automatic, Automatic}, {Automatic, Automatic},
```

```
{Automatic, Automatic}},
```

```
PlotRangePadding → {{Scaled[0.05`], Scaled[0.05`]},
```

```
{Scaled[0.05`], Scaled[0.05`]}, {Scaled[0.05`], Scaled[0.05`]}},
```

```
ImagePadding → Automatic, DisplayFunction → Identity, Axes → True,
```

```
DisplayFunction := Identity, FaceGridsStyle → Automatic,
```

```
Method →
```

```
{"DefaultGraphicsInteraction" →
```

```
{"Version" → 1.2`, "TrackMousePosition" → {True, False},
```

```
"Effects" → {"Highlight" → {"ratio" → 2},
```

```
"HighlightPoint" → {"ratio" → 2},
```

```
"Droplines" → {"freeformCursorMode" → True,
```

```
"placement" → {"x" → "All", "y" → "None"}}}}},
```

```
PlotRange → {{0.`, 12.566370487426136`}, {-1.`, 1.`,
```

```
{0.`, 0.999999550504007`}},
```

```
PlotRangePadding → {Scaled[0.02`], Scaled[0.02`], Scaled[0.02`]},
```

```
Ticks → {Automatic, Automatic, Automatic}}
```



```

Out[*]= {PlotRange → {{-2 π, 2 π}, {0, 2 π}, {-0.05, 1.05}},
PlotRangePadding → None, Ticks → {{-4 π, -3 π, -2 π, -π, 0, π, 2 π},
{-4 π, -3 π, -2 π, -π, 0, π, 2 π}, Automatic}, BoxRatios → {2, 1, 0.5},
AxesLabel → {x, y, z}, AxesEdge → {{-1, -1}, {-1, -1}, {1, -1}},
ImageSize → 500, ViewPoint → {-1.66087, -2.74048, 1.08687},
DisplayFunction → Identity, PlotRange →
{{Automatic, Automatic}, {Automatic, Automatic}, {Automatic, Automatic}},
PlotRangePadding → {{Scaled[0.05], Scaled[0.05]},
{Scaled[0.05], Scaled[0.05]}, {Scaled[0.05], Scaled[0.05]}},
ImagePadding → Automatic, DisplayFunction → Identity, Axes → True,
DisplayFunction := Identity, FaceGridsStyle → Automatic,
Method → {DefaultGraphicsInteraction →
{Version → 1.2, TrackMousePosition → {True, False}, Effects →
{Highlight → {ratio → 2}, HighlightPoint → {ratio → 2}, Droplines →
{freeformCursorMode → True, placement → {x → All, y → None}}}},
PlotRange → {{0., 12.5664}, {-1., 1.}, {0., 1.}},
PlotRangePadding → {Scaled[0.02], Scaled[0.02], Scaled[0.02]},
Ticks → {Automatic, Automatic, Automatic}}

```

```

In[32]:= VP1e = {-2.6622328739536525`, 1.5833298913814935`, 1.3621977022071134` }

```

```

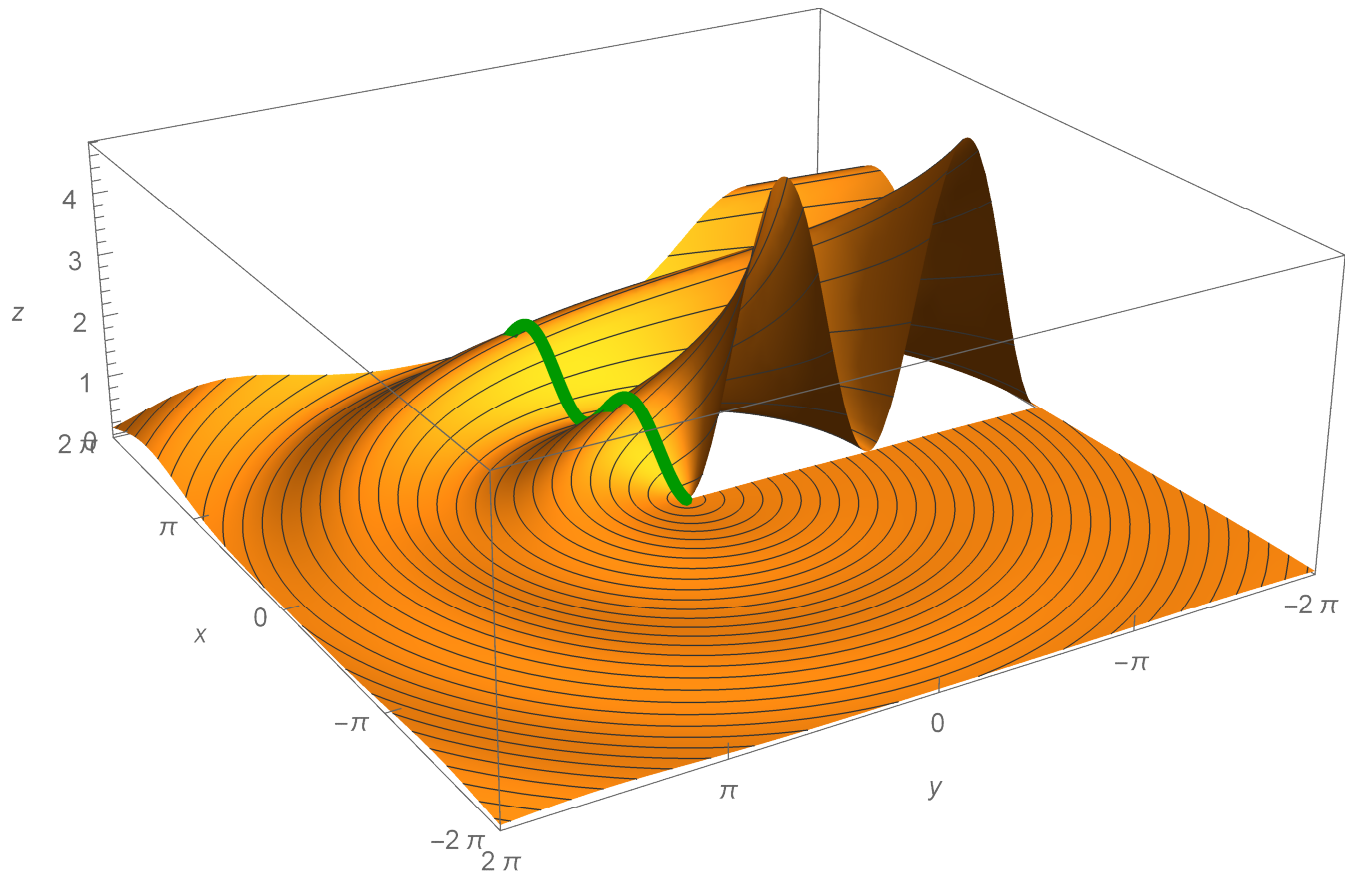
Out[32]= {-2.66223, 1.58333, 1.3622}

```

18 | MoC_w_circle.nb
In[33]:= MoCCirclesSur1ef2 =

```
Show[ParametricPlot3D[{ $\xi$ , 0, ff2[ $\xi$ ]}, { $\xi$ , 0, 2 Pi},  
PlotStyle -> {{Thickness[0.01], RGBColor[0, 0.6, 0]}},  
PlotPoints -> {100}],  
ParametricPlot3D[{ $\xi$  Cos[s],  $\xi$  Sin[s], ff2[ $\xi$ ] Exp[-s]},  
{s, -Pi/2, 3 Pi/2}, { $\xi$ , 0, 14}, PlotPoints -> {100, 100}, Mesh -> {0, 50}],  
PlotRange -> {{-2 Pi, 2 Pi}, {-2 Pi, 2 Pi}, {-0.05, 4.82}},  
PlotRangePadding -> None,  
Ticks -> {Range[-4 Pi, 2 Pi, Pi], Range[-4 Pi, 2 Pi, Pi], Automatic},  
BoxRatios -> {2, 2, 0.75}, AxesLabel -> {x, y, z},  
AxesEdge -> {{1, -1}, {-1, -1}, {1, 1}}, ImageSize -> 500, ViewPoint -> VP1e]
```

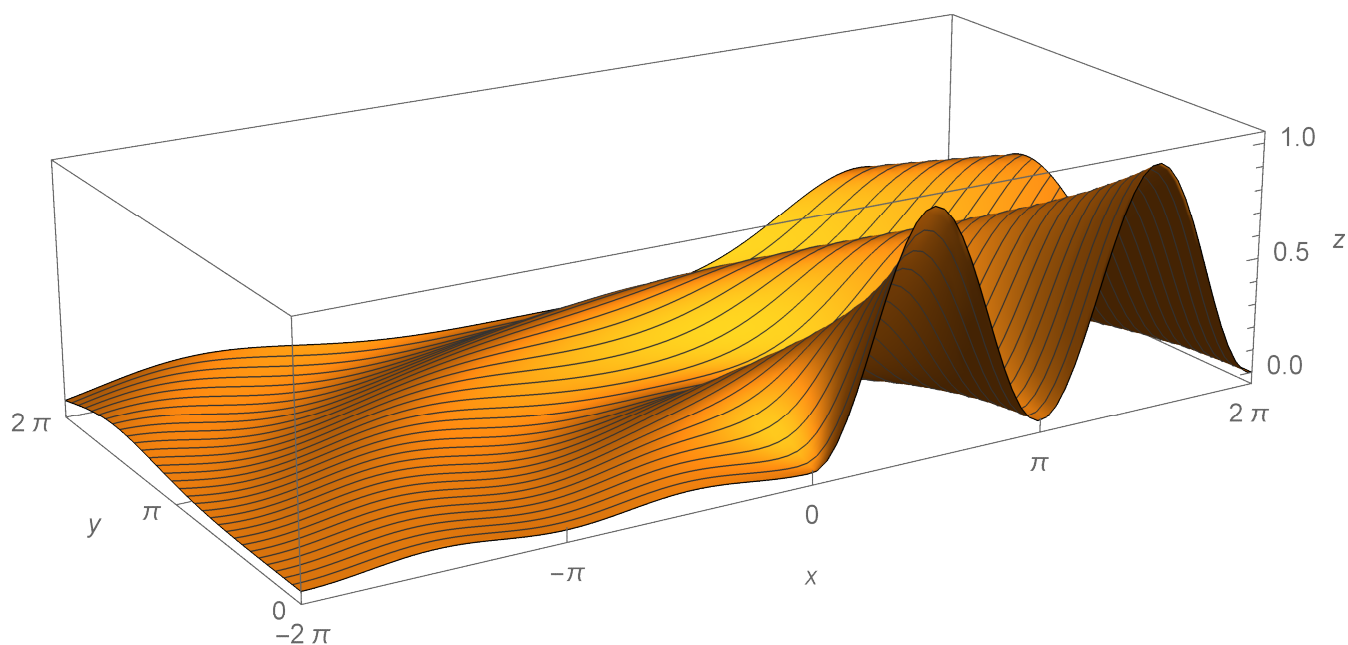
Out[33]=



In[34]:= MoCCirclesSur2f2 =

```
Show[ParametricPlot3D[{ξ, θ, ff2[ξ]}, {ξ, θ, 4 Pi},
  PlotStyle → {{Thickness[0.01], RGBColor[0, 0.6, 0]}},
  PlotPoints → {100}], Plot3D[ff2[√(x² + y²)] Exp[-ArcCos[x/√(x² + y²)]],
  {x, -2 Pi, 2 Pi}, {y, 0, 2 Pi}, PlotPoints → {100, 100}, Mesh → {0, 30},
  PlotRange → All], PlotRange → {{-2 Pi, 2 Pi}, {0, 2 Pi}, {-0.05, 1.05}},
  PlotRangePadding → None,
  Ticks → {Range[-4 Pi, 2 Pi, Pi], Range[-4 Pi, 2 Pi, Pi], Automatic},
  BoxRatios → {2, 1, 0.5}, AxesLabel → {x, y, z},
  AxesEdge → {{-1, -1}, {-1, -1}, {1, -1}}, ImageSize → 500, ViewPoint → VP1]
```

Out[34]=



20 | MoC_w_circle.nb
 In[35]:= uu2e[x_, y_] =

$$\text{Piecewise}\left[\left\{\left\{\text{ff2}\left[\sqrt{x^2 + y^2}\right] \text{Exp}\left[-\text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right], y \geq 0\right\},\right.\right.$$

$$\left.\left\{\text{ff2}\left[\sqrt{x^2 + y^2}\right] \text{Exp}\left[\text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right], y < 0 \ \&\& \ x > 0\right\},\right.$$

$$\left.\left\{\text{ff2}\left[\sqrt{x^2 + y^2}\right] \text{Exp}\left[-\left(2 \text{Pi} - \text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right)\right], y < 0 \ \&\& \ x < 0\right\}\right]$$

Out[35]=

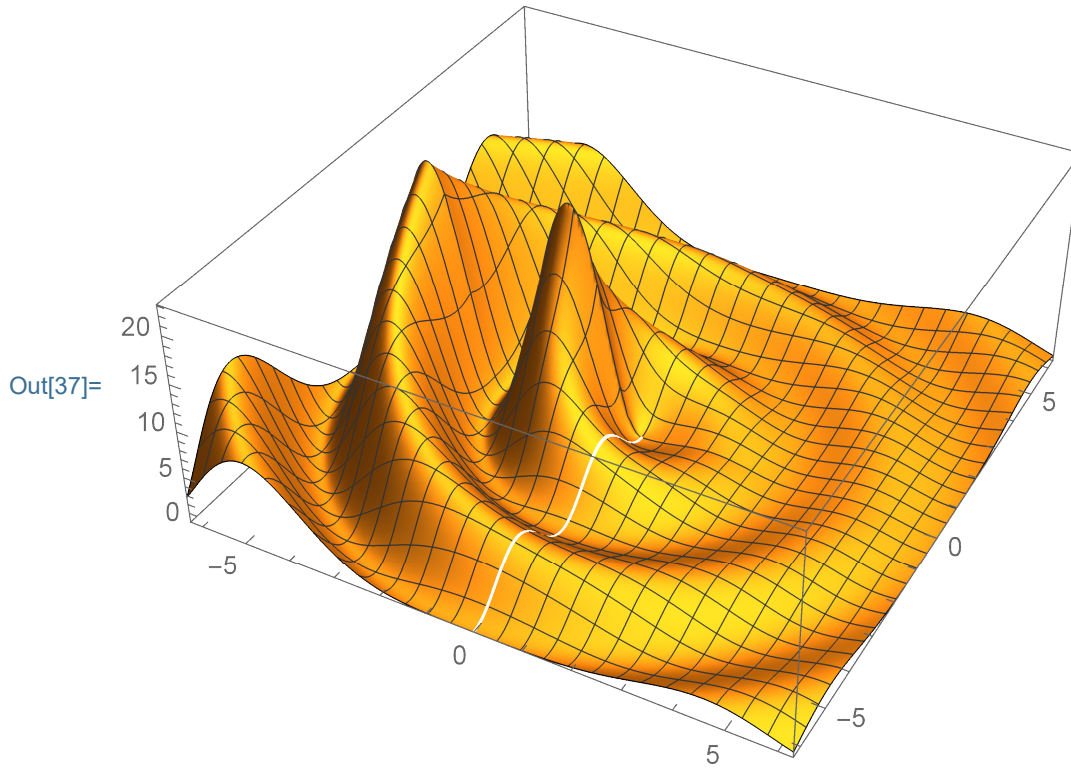
$$\left\{\begin{array}{ll} \text{Exp}\left[-\text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right] \text{Sin}\left[\sqrt{x^2 + y^2}\right]^2 & y \geq 0 \\ \text{Exp}\left[\text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right] \text{Sin}\left[\sqrt{x^2 + y^2}\right]^2 & y < 0 \ \&\& \ x > 0 \\ \text{Exp}\left[-2 \pi + \text{ArcCos}\left[\frac{x}{\sqrt{x^2 + y^2}}\right]\right] \text{Sin}\left[\sqrt{x^2 + y^2}\right]^2 & y < 0 \ \&\& \ x < 0 \\ 0 & \text{True} \end{array}\right.$$

In[36]:= uu2e[0.00001, -Pi / 2]

Out[36]= 4.81045

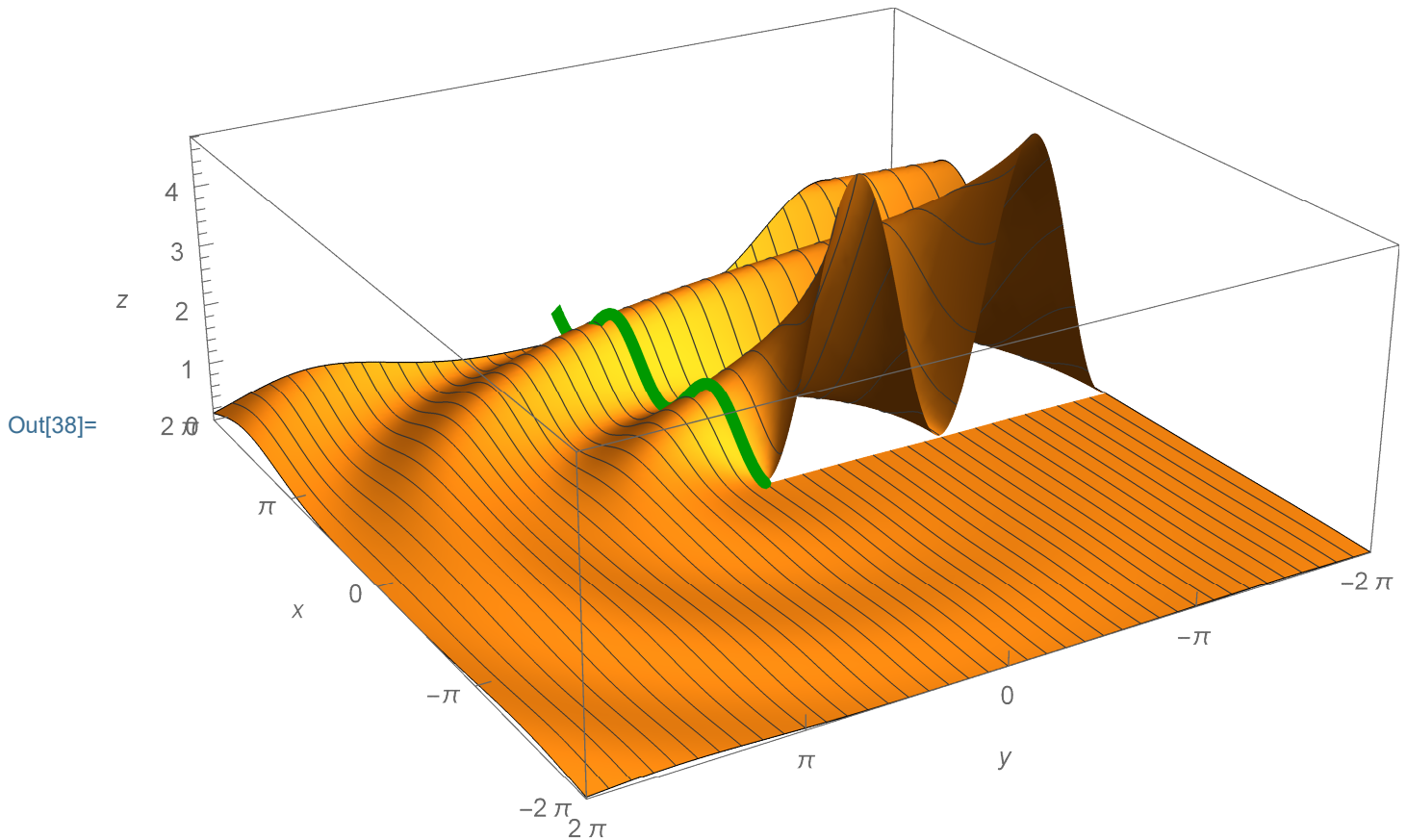
```
In[37]:= Plot3D[ff2[ $\sqrt{x^2 + y^2}$ ] Exp[ArcCos[ $\frac{x}{\sqrt{x^2 + y^2}}$ ]], {x, -2 Pi, 2 Pi},
```

```
{y, -2 Pi, 2 Pi}, Exclusions -> {{x == 0, y <= 0}}, PlotPoints -> {100, 100},  
Mesh -> 25, PlotRange -> {{-2 Pi, 2 Pi}, {-2 Pi, 2 Pi}, All},  
PlotRangePadding -> None]
```



22 | MoC_w_circle.nb
In[38]:= MoCCirclesSur2ef2 =

```
Show[ParametricPlot3D[{ $\xi$ , 0, ff2[ $\xi$ ]}, { $\xi$ , 0, 4 Pi},  
  PlotStyle -> {{Thickness[0.01], RGBColor[0, 0.6, 0]}},  
  PlotPoints -> {100}],  
Plot3D[Evaluate[uu2e[x, y]], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi},  
  Exclusions -> {{x == 0, y <= 0}}, PlotPoints -> {100, 100}, Mesh -> {0, 39},  
  PlotRange -> {{-2 Pi, 2 Pi}, {-2 Pi, 2 Pi}, All}],  
PlotRange -> {{-2 Pi, 2 Pi}, {-2 Pi, 2 Pi}, {0, 4.82}},  
PlotRangePadding -> None,  
Ticks -> {Range[-4 Pi, 2 Pi, Pi], Range[-4 Pi, 2 Pi, Pi], Automatic},  
BoxRatios -> {2, 2, 0.75}, AxesLabel -> {x, y, z},  
AxesEdge -> {{1, -1}, {-1, -1}, {1, 1}}, ImageSize -> 500, ViewPoint -> VP1e]
```

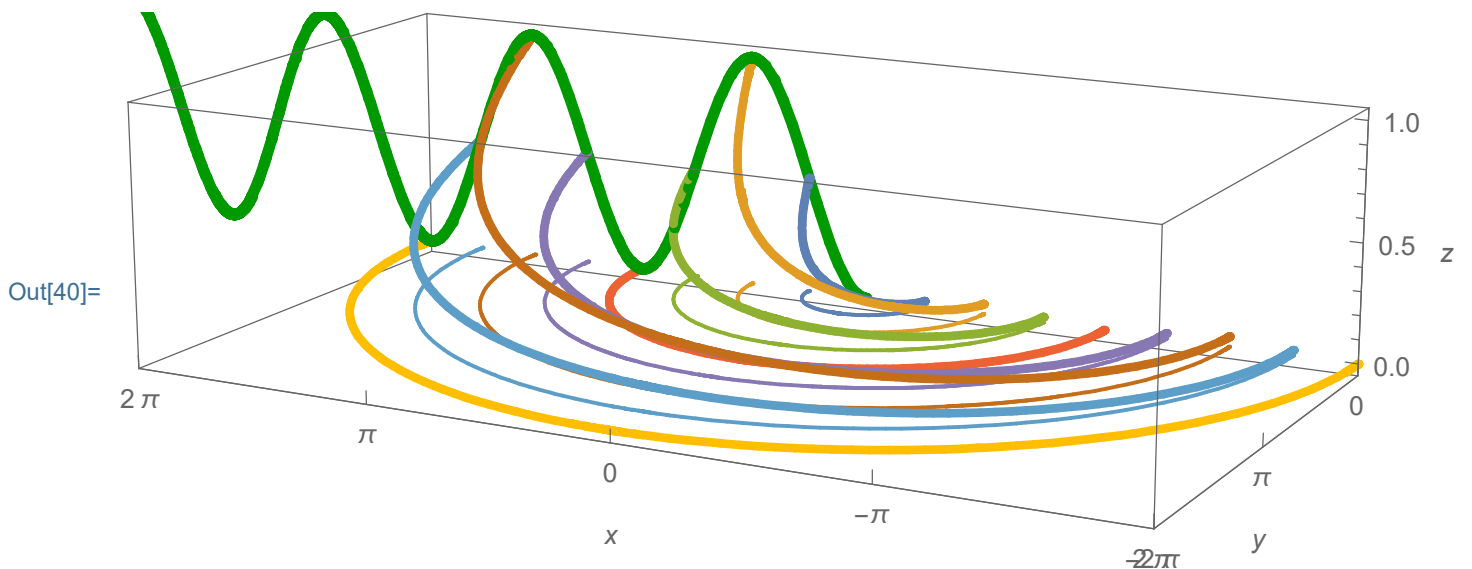


In[39]:= VP1a = {-1.5329095616109654`, 2.9047381415820985`, 0.8140544237087928`}

Out[39]= {-1.53291, 2.90474, 0.814054}

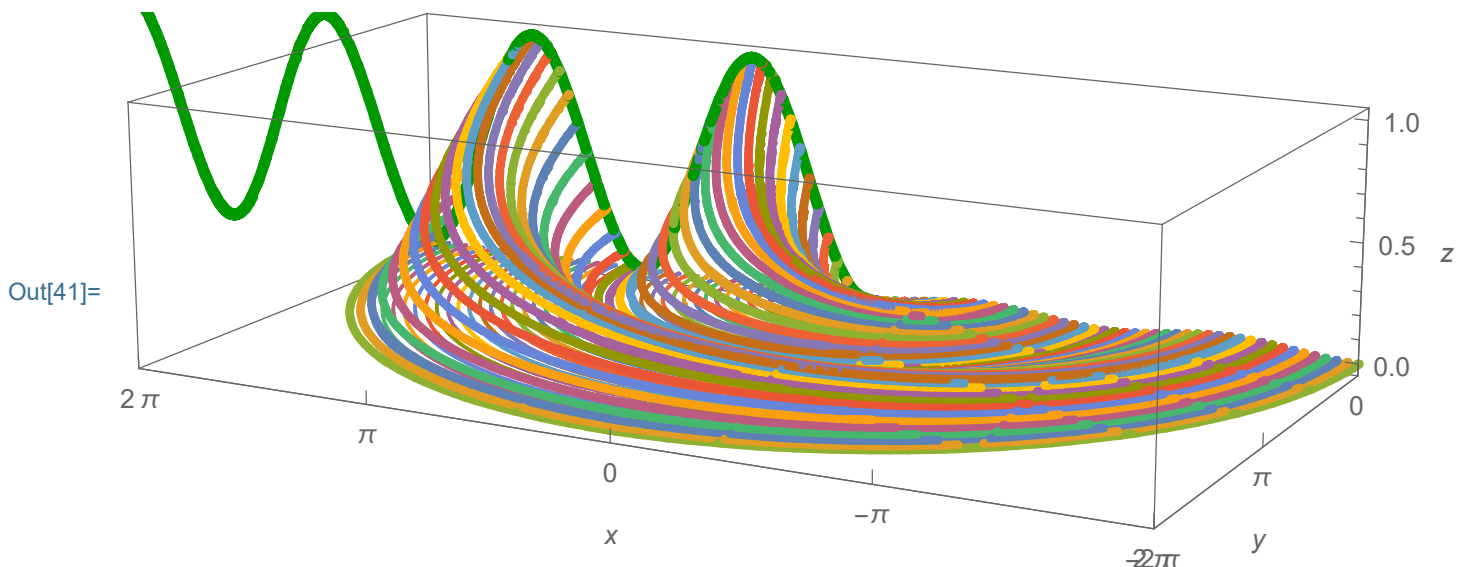
In[40]:= MoCCirclesCha1f2 =

```
Show[ParametricPlot3D[{ $\xi$ , 0, ff2[ $\xi$ ]}, { $\xi$ , 0, 4 Pi},
  PlotStyle -> {{Thickness[0.009], RGBColor[0, 0.6, 0]}},
  PlotPoints -> {100}],
ParametricPlot3D[
  Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], ff2[ $\xi$ ] Exp[-s]},
    { $\xi$ , Pi/4, 2 Pi, Pi/4}], {s, 0, Pi}, PlotStyle -> {Thickness[0.007]},
  PlotPoints -> {100}],
ParametricPlot3D[
  Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], 0}, { $\xi$ , Pi/4, 2 Pi, Pi/4}],
  {s, 0, Pi}, PlotStyle -> {Thickness[0.003]}, PlotPoints -> {100}],
PlotRange -> {{-2 Pi, 2 Pi}, {0, 2 Pi}, {-0.05, 1.05}},
PlotRangePadding -> None,
Ticks -> {Range[-4 Pi, 2 Pi, Pi], Range[-4 Pi, 2 Pi, Pi], Automatic},
BoxRatios -> {2, 1, 0.5}, AxesLabel -> {x, y, z},
AxesEdge -> {{1, -1}, {-1, -1}, {-1, -1}}, ImageSize -> 500,
ViewPoint -> VP1a]
```



24 | MoC_w_circle.nb
In[41]:= MoCCirclesCha2f2 =

```
Show[ParametricPlot3D[{ $\xi$ , 0, ff2[ $\xi$ ]}, { $\xi$ , 0, 4 Pi},  
PlotStyle -> {{Thickness[0.009], RGBColor[0, 0.6, 0]}},  
PlotPoints -> {100}],  
ParametricPlot3D[  
Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], ff2[ $\xi$ ] Exp[-s]},  
{ $\xi$ , Pi / 24, 2 Pi, Pi / 24}], {s, 0, Pi}, PlotStyle -> {Thickness[0.007]}],  
PlotPoints -> {100}],  
ParametricPlot3D[  
Evaluate@Table[{ $\xi$  Cos[s],  $\xi$  Sin[s], 0}, { $\xi$ , Pi / 24, 2 Pi, Pi / 24}],  
{s, 0, Pi}, PlotStyle -> {Thickness[0.003]}, PlotPoints -> {100}],  
PlotRange -> {{-2 Pi, 2 Pi}, {0, 2 Pi}, {-0.05, 1.05}},  
PlotRangePadding -> None,  
Ticks -> {Range[-4 Pi, 2 Pi, Pi], Range[-4 Pi, 2 Pi, Pi], Automatic},  
BoxRatios -> {2, 1, 0.5}, AxesLabel -> {x, y, z},  
AxesEdge -> {{1, -1}, {-1, -1}, {-1, -1}}, ImageSize -> 500,  
ViewPoint -> VP1a]
```



In[42]:= Length[Range[Pi / 24, 2 Pi, Pi / 24]]

Out[42]= 48

In[43]:= NotebookDirectory[]

Out[43]= C:\Dropbox\Work\myweb\Courses\Math_pages\Math_430\

In[44]:= (* SetDirectory[
"C:\\Dropbox\\Work\\myweb\\Courses\\Math_pages\\Math_430"] *)

In[45]:= Directory[]

Out[45]= C:\Dropbox\Work\myweb\Courses\Math_pages\Math_430


```
In[46]:= (* Export ["MoCCirclesCha1f2.png",MoCCirclesCha1f2,"PNG",  
ImageResolution→1200];  
Export ["MoCCirclesCha2f2.png",MoCCirclesCha2f2,"PNG",  
ImageResolution→1200];  
Export ["MoCCirclesSur1f2.png",MoCCirclesSur1f2,"PNG",  
ImageResolution→1200];  
Export ["MoCCirclesSur2f2.png",MoCCirclesSur2f2,"PNG",  
ImageResolution→1200];  
Export ["MoCCirclesSur1ef2.png",MoCCirclesSur1ef2,"PNG",  
ImageResolution→1200];  
Export ["MoCCirclesSur2ef2.png",MoCCirclesSur2ef2,"PNG",  
ImageResolution→1200] *)
```